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Student Achievement and Principal Value Added: The Case of Brazilian Schools

Desempenho escolar e valor adicionado de diretor: o caso de escolas brasileiras

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Abstract

School leadership and management are discerning factors between schools with great influence over other school characteristics. We study principal effects in Brazil using value-added models that introduce the joint estimation of principal, school and teacher effects in a parsimonious manner. First, we detail the construction of a student panel linking test scores to teacher, classroom and principal allocation by merging several datasets from different sources, paving the way for future research requiring such information. We propose two classes of models for this endeavour: a group of parametric fixed effects models, and a semi-parametric model exploring the within-school variance in effects following principal turnover, for which we propose an extension to encompass teacher effects. Additionally, we seek to investigate a possible cause of principal value-added variation by studying the association of these estimates to a management practice instrument based on principal interviews. While our parametric model estimation stumbles on a series of difficulties, rendering frail principal value-added estimates, our semi-parametric model points to a result aligned with the literature: a higher principal effect variance is associated with a score 8% to 12% of a standard deviation higher in mathematics and Portuguese. Our management practice analysis inherit the frail results from the principal value-added measure estimates, on which they were contingent. Estimation setbacks, originating both in data and in our modelling, are discussed and paths ahead in this research agenda are presented.

Key-words: Principal effects; Value-added; Management practices; Student achievement.

Abstract

A liderança e a gestão escolar são fatores que diferenciam escolas e detém grande influência sobre outras características escolares. Nós estudamos o efeito de diretores no Brasil através de modelos de valor adicionado que incorporam a estimação conjunta de efeitos de diretor, escola e professor de forma parsimoniosa. Primeiro, detalhamos a construção de um painel de alunos que liga o desempenho destes com a alocação de professores, turmas e diretores por meio do cruzamento de vários bancos de dados de origens diferentes, oferecendo um guia para pesquisas futuras que demandem essas informações. Propomos duas classes de modelos: um grupo de modelos paramétricos de efeitos fixos, e um modelo semiparamétrico que explora a variância intra-escolar nos efeitos de diretor usando trocas de liderança, e para o qual propomos uma extensão para considerar efeitos de professores. Ainda, investigamos uma possível causa para variação nas estimativas de valor aicionado de diretores por meio de uma associação destes com um instrumento de práticas gestoras baseado em intrevistas com diretores. Enquanto nosso modelo paramétrico enfrenta uma série de restrições, rendendo estimações frágeis de valor adicionado de diretores, nosso modelo semiparamétrico apresenta resultados em linha com a literatura: uma variância maior de efeitos de diretor está associada a resultados 8% a 12% de um desvio-padrão maiores em matemática e português. Nossa análise de práticas gestoras herda o caráter frágeil das estimativas de valor adicionado de diretores, das quais dependia. Discutimos os problemas enfrentados na estimação, com origem tanto nos dados quanto na modelagem, e apresentamos direcionamentos para pesquisas futuras nesse tema.

Palavras-chaves: Efeitos de diretor; Valor adicionado; Práticas gestoras; Desempenho escolar.

List of Figures

List of Tables

Contents

1 Introduction

Brazil figures among a number of developing countries that managed to greatly expand its secondary education over the past decades, but whose students' achievement have remained stagnant (FILMER et al., 2018), something reiterated in the latest PISA exam analysis for the country: both mathematics and reading scores moved sideways (AVVISATI; ILIZALITURRI, 2023).[1] The same report indicates a glaring discrepancy: only 20% of Brazilian principals whose schools took part in the 2022 PISA exam had responsibilities in teacher selection, compared to an average of 60% in OECD economies. Considering the different factors affecting student learning, recent literature has begun to shine a light on the effects principals exert on the learning outcomes of students (GRISSOM et al., 2021). Schools very similar to one another – in terms of budget and student body composition, for example – exhibit different results in student examinations, and have in their management position one of the main discerning factors among them. Understanding and measuring the value added by principals to students' learning outcomes is yet another important step in improving education, especially in a system with underperforming results as Brazil's.

In this research, we look at the influence principals have on student learning in Brazil. In a literature dominated by empirical studies in developed countries, we seek to estimate principal value-added measures in a developing country. Making use of five years of data on teacher and principal allocation and standardized test scores from last-year high school students from Minas Gerais, the second most populous state in Brazil, we aim at estimating not only principal effects but also to introduce teachers into this analysis, something both coveted and cautioned in the literature.

Many school-related factors influence learning outcomes, but given the variety of tasks principals are charged with, few professionals exert such a transversal and broad influence on students. Principals' effects on students have many particularities that researchers need to account for (GRISSOM et al., 2021). Extensive research has been conducted on how to best disentangle principal effects, that is, principal value-added, from teacher and school effects, while also seeking to understand their intricate relationships (COELLI; GREEN, 2012; BRANCH et al., 2012; DHUEY; SMITH, 2014; GRISSOM et al., 2015; CHIANG et al., 2016; DHUEY; SMITH, 2018).

The discussion on the inclusion of teachers in value-added analysis is careful to consider the role played by this group as mediation channels for principal influence.

[1] Considering the COVID-19 pandemic that occurred between the previous PISA exam, in 2018, and the latest 2022 one, and the reduction in average scores in OECD countries, the Brazilian result could be interpreted as attesting to the few losses in education to be had from an already low starting point.

Furthermore, teacher hiring and retention is often an institutional attribution of principals, posing a threat of effect collinearity (GRISSOM et al., 2015). In this sense, the Brazilian case is of special interest. In Brazil, principal selection varies across Brazilian states, with elections among school teachers being the official and most adopted method in Minas Gerais state public schools (SIMELLI et al., 2023). However, just as with any public service post, teacher selection is mainly held through public tender and is outside of principals' authority. At the same time this limits personnel management, often indicated as the most impactful management area on student achievement (BLOOM et al., 2015), it also means principal and teacher allocation are not as correlated as in usual cases in the literature. We exploit this difference to introduce teacher effects into our principal value-added estimation.

Assessment of school management has been guided by several instruments seeking to map management practices adopted in schools, linking these markings to student outcomes in efforts to comprehend principal mediation channels (BLOOM et al., 2015; LEMOS et al., 2021). However, these links are mostly established with student outcomes directly. We proposed a stricter notion for comparison: an association using principal value-added estimates, thus selecting only the delimited portion of principal influence in student learning. This is done using a management practice instrument adapted to the Brazilian institutional context in order to better capture institutional realities and inequalities in school management (HENRIQUES et al., 2020). This additional step is also motivated by still precarious explanations for principal value-added variance in the literature, with characteristics such as principal education background providing only partial elucidation (DHUEY; SMITH, 2018).

We propose two different approaches to principal value-added estimation. Our parametric approach, prevalent in the literature, incurs many estimation difficulties, which we are not able to completely overcome. Despite frail estimates from this class of models, we both examine the limitations faced, both in data and modelling and indicate a possible path to introduce teachers in principal effect analysis in future research. Since these are the results obtained on the principal level, the association with management practice instruments were made but inherited the fragility from our value-added measures. Our semi-parametric exploits theoretical within-school variance in principal effects (COELLI; GREEN, 2012). This approach has a more indirect interpretation but presents results in line with principal effects found in the international literature: higher principal effect variance in schools is associated with an impact of 12% to 8% of a standard deviation in standardized exam scores in mathematics and Portuguese.

Several contributions can be pointed out in this thesis. To the best of our knowledge, it is the first estimation of principal value-added with Brazilian data. The linking of students' testscores and teacher, classroom and principal allocations required merging several datasets from different sources, circumventing incompatibilities, is in itself a noteworthy achievement for Brazilian education data and establishes a blueprint for similar work in the future. Secondly, to the extent of the literature review undertaken, this is the first effort to introduce a joint estimation of principal, school and teacher composition value-added measures. Despite the fragility of these estimates, a path forward is discussed and alternatives for possible setbacks – as faced in this research – are presented. Furthermore, the extension proposed for the semi-parametric model to include variance in average teacher effects displayed interesting results: it showcases teachers as mediation channels while also underscoring the importance of principal effects acting through other means. Lastly, the association between the management practice instrument and principal value-added ranks several management practice domains as exhibiting statistically significant correlation with principal value-added measures, pointing directions for future research on the topic.

This thesis is structured in nine chapters overall. After this introduction, Chapter 2 presents a review of relevant topics in the literature engaged in this research, whilst Chapter 3 discusses the institutional background faced by school principals in Brazil and Minas Gerais. Chapter 4 showcases the conceptual frameworks employed in this work, making explicit the difference between the two classes of models. Chapter 5 introduces the data made available to us, details data manipulation steps and presents several descriptive statistics. Chapter 6 makes explicit the application of the discussed models to our data, detailing adaptations made and highlighting identification assumptions. Afterwards, Chapter 7 presents results for both model estimation and management practice association, whilst Chapter 8 discusses the limitations faced in obtaining these results. Finally, Chapter 9 concludes our research.

2 Literature Review

This study seeks to abridge two strands of research within the economics of education literature. Albeit complementary to one another, these two topics have thus far, and to the extent of our knowledge, not been connected. In this effort to associate principal effect measures with management practices adopted, a renewed level of scrutiny is brought to the literature. Instead of searching for links between different management practice agendas and students' test scores, here student outcomes are decomposed among factors that structure the schooling system to obtain the specific contribution of school principals to these outcomes. This, in turn, allows for an association of the respective fraction of student outcomes under principal influence and, therefore, the fraction of learning outcomes most susceptible to variation following differences in management practice adoption. This contrasts with the association currently dominant in the literature, which does not adequately separate students' outcomes influenced by school-level factors (like principals, but also teachers, schools, and peers, among other things) from home-bound factors, that are not susceptible to many of the school management decisions and practices.

In order to carry out such a connection, a thorough review of both strands of literature is necessary. In this chapter, a review of the many approaches in the literature to conceptualize and measure principal effects is first presented, detailing the common models employed in such tasks. This is followed by a review of the school management literature and associations to management practices made thus far.

2.1 Principal effects

The study of principals' influence on student learning has been the focus of a consolidated strand of literature for decades. Following analysis of factors directly impacting students, like teachers and classrooms, research began incorporating highly relevant but indirect factors, like principals. As Hallinger & Heck (1998) document, the research on principal effects in student learning has focused on big themes, like leadership. Both the connotations associated with school leadership and the methods increasingly used to study its effects have changed over the years.

On the conceptual metamorphosis of school leadership, Hallinger & Heck (1998) note that earlier works from the 1980s conceptualized it drawing from the effective schools literature, in which principal effectiveness was defined based on instructional leadership, that is, the concentration of roles in the principal figure in a top-down manner, like school-wide goal setting and instruction (curriculum) program management, for example.[1]

[1] For an outline of characteristics associated with instructional leadership, see Hallinger & Murphy

Later studies in the 1990s began to conceptualize leadership differently, following what Leithwood (1994) calls transformational leadership. This new conceptualization questions the centrality of principals' roles in bringing about effective schools and seeks to bring other school staff into the task by empowering them in a distributed leadership fashion, focusing on teacher leadership and recognizing the importance of school communities.[2]

This conceptual change over time was accompanied by methodological advances in research. Hallinger & Heck (1998) categorize studies into two main approaches and document the transition the literature underwent from the Direct-effects model, as the authors coined it, into the Mediated-effects model. This transition meant researchers parted from models in which principals influenced school outcomes directly (direct-effects model) to those in which principals affected school outcomes indirectly mediated by other variables. The similarity between this transition and the different conceptualizations of school (and principal) leadership is no coincidence: more sophisticated models came to be implemented with the necessity to investigate the aspects and channels formulated by the transformational leadership literature.

2.1.1 Value-Added Models

An even more recent methodological framework being employed is value-added modelling. These models work with the exact gains in student outcomes and are being used to specify the roots and attribute the causes of the measured educational growth.

These models, as with the aforementioned ones, were first adapted to education-based applications to study the impact of teachers on students' outcomes. Since teacher effects are more easily controlled for and identified, various specifications were tested, and a robust literature on such models for teacher effects surfaced. Koedel et al. (2015) provide an excellent review of such literature[3], as well as a synthetic characterization of the best models currently in use. In this sense, value-added models are characterized for evaluating the growth in outcomes between two measured periods (for example, two consecutive standardized test scores). By focusing on the growth in outcomes in a specific period, researchers are able to investigate specific influences at play during this time and determine factors that effectively add value to such outcomes. A key assumption made in this value-added literature (not restricted to teacher value-added) is that the earlier outcome measure can fully capture previous influences, that is, that earlier test scores, for example, fully capture students' education attainment, from which incremental gains can lead to growth calculations.

(1985).

2 For other aspects of transformational leadership, refer to Leithwood & Jantzi (2008).

3 Special note is given to the debate between Rothstein (2010) and Chetty et al. (2014) on estimation bias, student sorting, and model assumptions, all relevant to our principal application.

Implementing value-added models to investigate principal influences on student learning proves a difficult task. This is due to principals' effects on students having many particularities, as Grissom et al. (2021) put. Firstly, principals affect students via indirect channels, often referred to as mediation, examples of which are teachers and school organization. This mediation complicates principal value-added estimation because it entangles principal effects with direct, mediation effects influencing students. A second complicating aspect is timing: not everything accomplished by school administrators has an immediate impact on students' learning, and some actions taken may only have perceivable effects later on, maybe even after the enacting tenure. Lastly, the abundance of factors beyond principals' control is another challenge; some of these are shared with teachers, such as the whole baggage students bring with them to school and class from their communities; however, some are specific to principals, like the school body upon tenure start.

Researchers deal with the first peculiarity by exploiting principal transitions. As Branch et al. (2012) detail, by exploiting principal transitions between schools, researchers are able to observe principals in two distinct environments. This allows researchers to distinguish between principal effects mediated through schools and school effects (those the school would exert independently of the principal in charge).[4] The importance this has is underscored by the findings of Chiang et al. (2016) that school value-added is a poor indicator of principal value-added.

Regarding factors outside of principals' control, a combination of control variables and assumptions is used. Typically, a comprehensive set of controls is implemented to account for students' demographic and socioeconomic characteristics, as well as community influences, where possible. Nevertheless, school factors that are beyond principals' control and not covered by these variables, such as teachers, staff, and school infrastructure at the beginning of their tenure, are difficult to account for and often place stringent data demands. The literature frequently circumvents these difficulties by assuming such factors as an integral part of principals' administration, with changes in the school community and improvements in schools' infrastructure to be viewed as mediation channels for principal impacts.

Finally, simplifying assumptions are used to deal with the timing problem faced in value-added estimations. As Grissom et al. (2021) and Koedel et al. (2015) affirm in their literature syntheses for principal and teacher value-added, respectively: studied effects are assumed to be immediate. This implies that principals already exert effects upon tenure start and that no additional effects are transmitted after tenure ends. This assumption

[4] The study by Branch et al. (2012) and many of the other principal value-added studies are conducted in the United States, where principals' tasks include teacher hiring and retention, and relatively broad and unrestricted people management areas. This is not the case for Brazilian principals, as is detailed in Chapter 3. For this same reason, it is important to disentangle teacher effects from principal effects, which is proposed in Chapter 4.

is fairly strong, as it isn't difficult to imagine management practices that, once adopted, take time to impact students' achievement. However, it is an assumption widely used in the value-added literature. The works of Grissom et al. (2015) and Coelli & Green (2012) are among the few attempts at relaxing this hypothesis.[5]

As reviewed by Koedel et al. (2015) on the teacher case, value-added can be modelled quite differently, taking on different assumptions. Grissom et al. (2015) provide an excellent example by modelling three different specifications on how principals might affect students based on different assumptions on their interaction with schools: a first model attributes all school effects to principals, not considering factors outside their control; a second model compares schools' effectiveness under different principals and is able to take into consideration outside born factors, but is vulnerable to principal sorting among schools and tiny comparison groups formed in this approach, which render results fruitless; lastly, a model that seeks to capture improvements in school effectiveness during principals' tenure, instead of the average effect during the same period, which allows for non-immediate, incremental effects. The authors compare each model's results to non-test measures (external evaluations; student, staff and parent reports; self-evaluation) for their sample of schools. Their goal here is to evaluate which model exhibits better adherence to these non-test measures and discuss what these varying degrees of correlation imply for each model's teacher effects behaviour assumptions. In this exercise, they find that the simplest model has the highest correlation to such non-test measures, partly because the more reasonable models face difficulties due to stringent data demands.

On the same note as the second model proposed by Grissom et al. (2015) that investigates the within-school change in effectiveness, Coelli & Green (2012) conduct a similar conceptual analysis on more restrictive data. The authors adapt a means-centered approach proposed by Rivkin et al. (2005) to study within-school teacher effects and apply it to data with few principal transitions between schools and no student longitudinal data.[6] Recall that these two characteristics were central in principal value-added studies, be it for the necessity to delimit the period-specific improvement in outcome, or the fashion in which principal effects are distinguished from other effects. By analyzing the rate of principal turnover in the same school, Coelli & Green (2012) are able to estimate the within-school variance in principal effectiveness. With this, they investigate if schools with higher variance in principal effects exhibit lower graduation rates or English test scores.

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[5] The model proposed by Grissom et al. (2015) is commented further along in this Chapter. On the other hand, we forego an analysis of the model with a relaxed timing assumption developed by Coelli & Green (2012). Briefly, they develop a dynamic principal effects model, wherein principals exert cumulative effects on student outcomes over time and principal effects are modelled as an average of past and current individual principal effects, weighed by tenure durations. They use both this dynamic model and a static model to study within-school principal effects variance and its impact on student learning outcomes. In this research, we focus and draw heavily on their static principal effects model, which will be greatly detailed in Chater 4.

[6] As will be explained in Chapter 5, this is also the case for our data on Brazilian public schools.

The means-centred approach employed by Coelli & Green (2012) departs from a similar structure present in other more common value-added models (BRANCH et al., 2012; DHUEY; SMITH, 2014; GRISSOM et al., 2015; CHIANG et al., 2016), but does not impose a specific parametric form. By making use of differences in averages in schools and cohorts, the model is able to eliminate school fixed effects, and this attenuates the problem of having to disentangle principal effects. Given this development, principal transitions between schools do not carry the same importance as in other approaches. The within-school analysis underscores this, with the main comparison group being other principals at the same school at different times. Overall, this enables a wider range of applications, especially in contexts with more precarious or unavailable data. However, this comes at the cost of precision in estimations: what Coelli & Green (2012) effectively estimate is the lower bound of the variance in principal effects. This is a consequence of assuming across school average principal effects are zero, which is to say that all schools are assumed to hire principals from a common pool of candidates. Nevertheless, if certain schools can hire from a larger pool of candidates because they can offer better living or working conditions, for example, then the average quality of these principals should be higher, meaning there may be considerable variation in across school principal quality.[7]

One difficulty faced in this literature is the collective analysis of both principals and teachers in value-added models. This stems in part from the added data requirements of such analysis; but also from the manner principals and teachers interact. Due to the limited institutional contexts in which principal research has been conducted, all studies consider scenarios in which principals have extensive freedom in personnel management, especially teacher hiring and retention. This renders principal and teacher effects seemingly indistinguishable, and Grissom et al. (2015) go so far as to say they are collinear. In fact, one does not find a similar treatment bestowed upon teacher effects as those on school effects (BRANCH et al., 2012; DHUEY; SMITH, 2014; GRISSOM et al., 2015; CHIANG et al., 2016; DHUEY; SMITH, 2018). The approach by Coelli & Green (2012) may yet deal with this additional challenge in literature, were it not for a repetition of the same institutional context and data unavailability on teachers.

A less pervasive analysis in the literature is the study of determinants of principal value-added variation. Dhuey & Smith (2018) investigate whether characteristics related to principals' educational background help explain such variation. Previous research explored the relationship between school performance and principal characteristics outside the value-added methodology. Clark et al. (2009) study the relationship between school performance and principal experience, while Loeb et al. (2010) investigate such relationship with many other principal characteristics, also discussing principal allocations among schools with heterogeneous performances. We found no studies seeking systematic associations between

[7] We formalize this assumption made in the Coelli & Green (2012) model in the end of Chapter 4.

principal value-added and management practices adopted by these principals.

2.2 Management and Student Achievement

Another strand of literature that seeks to unveil the mechanisms behind principal influence in students' achievement focuses on studying management practices adopted in schools. This follows a wide-ranging literature on management in various applications (BLOOM; VAN REENEN, 2007), which was famously applied to school management by Bloom et al. (2015). The authors developed the World Management Survey (WMS) to measure well-documented management practices in several dimensions and applied the survey to different school systems in a handful of countries. Their main goal was to document varying degrees of principal management characteristics across these dimensions and relate them to students' school outcomes. They find that practice adoption varies significantly across countries and school systems (public and private) within countries, and also that student achievement has strong links to the number of practices adopted. A special interest in the research is investigating how much principal autonomy influences outcomes, and results show that these are indeed linked to higher management scores in the WMS and to higher student achievement. The authors highlight the differences between charter schools[8] in the US and normal public and private schools, with higher WMS scores even in the latter comparison. This is a trend that persists after similar analysis in other OECD[9] countries, and even in Brazil, where autonomous government schools[10] score slightly higher than both public and private schools.

In their analysis of WMS results, Bloom et al. (2015) highlight the importance of the institutional context to explain within-country differences. Indeed, while the difference in the WMS score between OECD countries and Brazil and India is eye-grabbing, the difference between public and private systems does not seem to explain the achievement gap as expected. Lemos et al. (2021) point out problems in using WMS to analyze developing countries: the survey only captures formal practice adoption, which conveys little information on informal practices that may affect student outcomes. According to the authors, formal practices may be partially incorporated into informal practices due to institutional or financial constraints. In this sense, they propose the D-WMS, an adapted version of the WMS for developing countries, one that is more sensible to informal practices implemented by principals. This new instrument is tested on public and private school systems in India, yielding comparable results to previous WMS applications, and

8 Charter schools receive special funding and extensive experimental freedom in management, often acting as a development engine for school administration in the USA (DOBBIE; FRYER JR, 2013).

9 Bloom et al. (2015) present results for the US, United Kingdom, Germany and Sweden, which are part of OECD, and Brazil and India, which are not members.

10 *Escolas de Referência*, public schools with autonomous management only found in Pernambuco state.

expanding on within-country comparisons: private schools increased the management score gap relative to public schools.

Both studies results emphasize the importance of people management. This is not restricted to principals' ability to select, hire, and retain high-quality teachers, but also to motivate teachers and other staff to work towards common goals. Lemos et al. (2021) also find that people management scores are strongly correlated with both external evaluation scores and internal measures of teaching practices.

3 Institutional Background

Much of the within-country variation in management indexes is not adequately explained. This is due to different institutional contexts, which are not precisely captured in surveys intended to compare school systems in various countries (BLOOM et al., 2015; LEMOS et al., 2021). In this research, we make use of a management practice survey adapted to the Brazilian institutional context to better explore these differences in Brazil. Taking this into account, we briefly contextualize the reader in what the Brazilian institutional context entails for high school education, broadly explaining public education structure in the first section of this chapter, as well as informing on institutional constraints principals face in the public school system. In the second and last section, we discuss the management practice mapping used in this study, emphasizing the various validity stages it underwent.

3.1 Schools and Principals in Brazil

The provision of public education in Brazil is divided among all three government levels: federal, state, and municipal. High school level education, in particular, is mainly under state jurisdiction, meaning all 27 Brazilian states develop education policies for this schooling level following federal government guidelines. In 2019, over 7.45 million students were enrolled in a high school program in Brazil, with 83.9% of these students attending state public schools[1], whilst 12.5% of them attended private schools (INEP, 2021).

Besides these federal guidelines, public schools under state jurisdiction are also subject to public sector legislation, which applies legal and administrative constraints, as well as limited financial control. This directly impacts principals' management, limiting projects and planning, especially due to financial restrictions. However, the main restriction principals face is towards human resource management. As with any other public servant in Brazil, public teachers are hired through public tender and face job stability. This essentially eliminates teacher hiring and dismissal by principals, and the financial constraints coupled with flat public servant earnings make retaining good teachers extremely difficult. Moreover, school principals cannot use school financial resources for payroll expenses.

Another institutional difference extremely relevant to our analysis is that Brazilian public schools face different principal selection methods to those commonly analyzed in

[1] Over the period analyzed in this research, the number of total pupils attending high schools fell from just over 8.1 million to around 7.45 million. The share of students attending state high schools shows a slight drop over the period, departing from 84.8% in 2015 to 83.9% in 2019, while the share of students in private schools rises slightly from 12.4% to 12.5% over the same period. The remaining students attend either municipal or federal high schools, which are the exception (INEP, 2021).

the principal effects literature. This selection method is also heterogenous in Brazil, but three main methods are commonly employed: public tender; appointment (usually political, either by mayors or governors or by the local education bureau); and community election (usually encompassing school staff and school community) (MUñOZ et al., 2021). Pereda et al. (2019) find that elected principals tend to be more qualified in terms of leadership and managerial skills, also showing that principals selected via election processes or public tender have a higher probability of promoting continued education programs to teachers compared to appointed principals.

Simelli et al. (2023) conduct a diagnosis on principal selection and training in Brazilian schools for all levels using official data and interviews with state-level public servants.[2] According to their findings, 23% of state public school principals are chosen exclusively via appointment, whilst 38% are chosen exclusively via school community elections, with another 14% being chosen via school community elections after some sort of qualified selection process.[3] Specifically for Minas Gerais, the main form of access to the principal position is via school community election following a qualification exam, along with a prerequisite of a 120 hours management course (not offered in the selection process).[4]

Although principal elections were introduced in Minas Gerais in 1991 (BORGES, 2004), numerous resolutions have since been enacted altering details of principal selection. These definitions are under the State Education Bureau's (*Secretaria Estadual de Educação de Minas Gerais* - SEE-MG) jurisdiction and three different resolutions were active during the 2015-2019 period: one enacted in December/2011 (MINAS GERAIS, 2011), replaced by the one enacted in September/2015 (MINAS GERAIS, 2015), which was in turn replace by the one enacted in April/2019 (MINAS GERAIS, 2019). The election process itself has remained much the same during this period, the reason for which we describe it based on the 2015 resolution (MINAS GERAIS, 2015), which was in place during most of the studied period. All principal candidates must place an electoral ticket jointly with their vice-principal candidates, and an electoral commission consisting of three to five members from both school staff (teachers and other staff) and school community is assigned to organize the election process.[5] A valid principal candidate must be a public school teacher employed in that school for two years within the five years prior to the election, have an

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[2] Beyond mapping principal selection and training, Simelli et al. (2023) also detail the principal demographic in Brazil. State public school principals are, on average, white women with university degrees, aged 40 or older, with over five years of teaching experience, and responsible for one school.

[3] These are the three biggest principal selection mechanisms in Brazil. Please refer to Simelli et al. (2023) for the whole process list.

[4] Borges (2004) briefly reviews the 1991 school system reform in Minas Gerais that instituted community election as the main method of principal selection and documents that students and their families exhibited low participation rates in these elections.

[5] The current principal and all principal candidates are vetted from participating in this electoral commission.

education bachelor degree or an education specialization following a bachelor degree in another area, plus teacher training. The elections are held at the school and all staff are eligible to vote; members of the school community may also vote, with students aged 14 or over allowed to vote themselves and guardians of students younger than 14 voting in their place. A ticket that receives the highest share of valid votes is considered the winner.[6]

A relevant change regarding principal tenure is present in newer resolutions. The 2011 resolution (MINAS GERAIS, 2011) stated (Art. 43) that elected principals would remain in charge of schools until the next elections took place, which is explicitly written as only being considered from 2013 onward, meaning at least one year of administration but no effective tenure time enunciated.[7] The other two resolutions (MINAS GERAIS, 2015; MINAS GERAIS, 2019) are much clearer (Art. 45 in both resolutions), establishing three-year tenure limits for elected principals. These same articles include the possibility of one consecutive reelection for principals, follwoing the same election procedure already described. This again constrasts with the 2011 resolution, that did not limit the number of consecutive tenures explicitly, but disavowed tickets from staff with 4 years experience as principal at the school.[8] In other words, tenure limits and reelection possibilities were rather complex under the 2011 resolution, but its two successors established much clearer rules that prevailed during most of the years analyzed in this research.

Minas Gerais state is Brazil's second most populous federal unit, with over 19.5 million inhabitants according to the 2010 Census, and Figure 2(a) shows the logged population for all 853 municipalities, highlighting the five main cities in more concentrated areas. The state's socioeconomic disparities mirror Brazil's, with poorer municipalities on the northern portion of the state, near the semi-arid *sertão*, and richer municipalities in the south and around Belo Horizonte, the capital, as can be seen in Figure 2(b). Figure 2(c) presents the number of schools per municipality, with 358 (41.96%) having a single state public school, and 704 (82.53%) having five or fewer schools.[9] The last panel, Figure 2(d) displays the average Management Complexity Index (ICG) for schools in every municipality. The ICG is a metric developed by INEP to evaluate the difficulty of managing different processes in schools, taking into account not only the number of students but the number of grades, nocturnal classes, young adult education programs (EJA), number of teachers, among other factors, with lower scores associated with more complex school administration

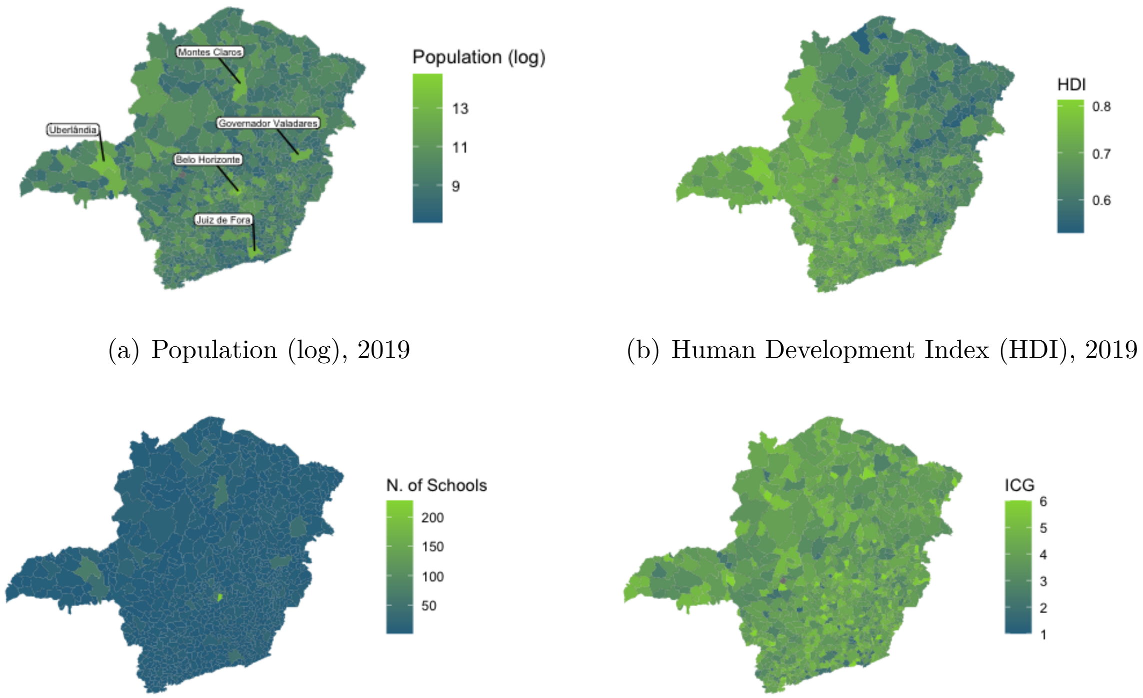
6 Lone tickets in schools are required to obtain over 50% of the vote. Schools in which single tickets fail to obtain such margin or that receive no principal tickets have their principals appointed by the school council, which must favour local teachers.

7 Notice that, since resolution Minas Gerais (2011) was enacted in December, it entails that elected principals entenring in 2012 have only one year of guaranteed administration.

8 The wording in Minas Gerais (2011), Article 43, is imprecise, but we gather that it entails the current principal cannot present a reelection ticket if she has four or more years in that school‘s top administrative position.

9 The obvious outlier here, as seen from Figure 2(c) is Belo Horizonte, with over 2 million inhabitants and 228 unique school codes registered.

Figure 1 – Log population, Human Development Index, Management Complexity Index and Number of Schools across Minas Gerais municipalities in 2019



(a) Population (log), 2019

(b) Human Development Index (HDI), 2019

(c) Number of schools, 2019

(d) Management Complexity Index (ICG), 2019

(INEP, 2014).

Overall, authors documenting these processes register significant changes in the periods prior to their analysis. Indeed, federal guidelines establish broad criteria, with the Brazilian Constitution (BRASIL, 1988) enlisting democratic management as an organizing principle of public education, something reinforced in the *Lei de Diretrizes de Bases da Educação Nacional* (LDB - National Education Guidelines) (BRASIL, 1996). More recently, the *Plano Nacional de Educação* (PNE - National Education Plan) (BRASIL, 2014) goes a step further in standardizing school management by conditioning additional fund transfers to management improvement in schools, including technical principal selection criteria (MUñOZ et al., 2021). These recent efforts are backed by a recent approval of the *Base Nacional Curricular - Formação* (BNC-F) (BRASIL, 2019), guidelines specifying principal action for Brazilian public schools of all levels.

3.2 School Management Practice Mapping

We make use of a management practice adoption survey developed to assess *Instituto Unibanco*'s[10] *Jovem de Futuro* program, an educational initiative active since 2007 and

[10] *Instituto Unibanco* is a private institute focused on improving Brazilian education through research, partnerships and events promoting better educational management. It has ties to *Itaú Unibanco* Holding

focused on mapping, evaluating and improving school management. *Jovem de Futuro* has its goals and theory of change set out by Henriques et al. (2020), and essentially aims at promoting improvements in school management by incorporating a results-oriented mentality. Barros et al. (2019) document the inequality in Brazilian public education and what they deem inefficiency in the use of public resources, with substantial growth in per-student annual spending but stagnant results in (2015) PISA scores. They find that the *Jovem de Futuro* has an estimated impact of increasing, on average, 12% of a standard deviation in students' mathematics learning (following the SAEB scale[11]), and 8% of a standard deviation in students' Portuguese learning. This is an estimated 11% of total estimated gains from improvements in management, according to the authors.

A fundamental part of the *Jovem de Futuro* program is mapping management practices implemented in schools, achieved using an instrument developed solely for this goal. Borges et al. (2023) detail the construction of a 2017 version of this instrument, focusing on its various robustness steps. Even though we make use of a slightly different, more recent version of the instrument, the authors' description of its methodology serves us greatly.[12] In this section, we briefly review this methodology and discuss some of the instrument's characteristics and validity measures.

Despite being heavily inspired in the renowned international surveys (BLOOM et al., 2015; LEMOS et al., 2021), the instrument used in this research differentiates itself by incorporating the Brazilian institutional context. This addresses much of the commonly discussed setbacks in using the WMS or D-WMS, being that institutional factors played a major role in similarities in scores between public and private school systems within studied countries. By constructing the instrument within the Brazilian context, it hinders international comparisons but enables more precise mapping and scoring of practices adopted in Brazilian schools. Such a moulding to the Brazilian context was achieved by incorporating official principal normatives into instrument construction through iterated rounds of application and feedback (BORGES et al., 2023).

The management practice mapping used by Borges et al. (2023) was tested in 297 schools in the states of Espírito Santo and Pará[13], in which schools from various achievement and socioeconomic backgrounds were selected. School principals were interviewed in a

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and operates with an endowment, allowing partnerships with state and municipal education bureaus. *Instituto Unibanco* provided financial support during the last third of this research's development, as well as logistical support in contacting SEE-MG public officials.

[11] The SAEB, or *Sistema de Avaliação da Educação Básica*, is a grading guideline developed by INEP to infer both a comparative level and the specific skills a student attained given the standardized scores in mathematics and Portuguese.

[12] The instrument used is explained in Chapter 5. Here, we cover the methodology employed in data collection and the robustness steps taken to ensure the data is grounded in reality, which are the exact same as the one described in Borges et al. (2023).

[13] No schools from Minas Gerais, the state public school system being used in this research, were used in this robustness checking phase.

Figure 2 – Management Practice Domains

Groups	#	Practices	Description
Pedagogical Domain	1	Pedagogical project	Evaluates school elaboration of the pedagogical project and how (and if) it is used to guide decisions.
	2	Teaching planning process	Evaluates the quality of the pedagogical planning process.
	3	Teaching and learning customization	Evaluates how principals identify pedagogical strategies to work with different levels of students' learning.
	4	New teaching practices	Evaluates if principals encourage the improvement of teaching practices and the search for innovative learning strategies.
	5	Intenal learning assessment	Evaluates how the school evaluates student performance internally.
Use of Data and Monitoring Domain	6	Student flow analysis	Evaluates how principals deal with absence, grade repetition and dropout.
	7	External learning assessment	Evaluates how principals use national and state-level external evaluations to analyze students' learning conditions.
	8	School targets	Evaluates whether there is a management focused on goals for student learning.
Human Resources Domain	9	Leadership	Evaluates how principals recognises the leadership and responsability of school actors.
	10	Worker evaluation	Identifies and qualifies how principals evaluate the performance of school pofessionals.
	11	Worker prformance management and retention	Analyze how (and if) principals deal with both great and poor staff performance.
Administrative Management Domain	12	Infrastructure	Verifies how principals act as custodians of good school infrastructure and incentivize its use.
	13	Financial aspects	Evaluates principals' awareness of the school's financial situation and measures.
Values Domain	14	School values	Evaluates if principals comprehend the school context and develop an institutional school identity.

Notes: Own elaboration, based on management domains in Henriques et al. (2020).

double-blind format[14] and had their answers recorded. The interviews followed a script designed to avoid inducing specific answers, and indeed Borges et al. (2023) present evidence of substantial variation in the overall management index scores. Transcribers were trained to identify cues for management practices from principal recordings and correctly mark the management index. Markings were tested by assigning two different transcribers to each interview and comparing markings, allowing the authors to reject random markings (at 5% confidence interval) for all domains.

The instrument we have at hand is constructed based on interviews with high school principals in Minas Gerais in 2019. It covers 14 domains of school management salient in Brazil, which are shown in Figure 2. Each domain compromises of 10 management practices, meaning each domain can score up to 10 points.[15] Markings came from double-blind interviews just as in the instrument described by Borges et al. (2023). More information on instrument statistics will be presented in Chapter 5.

[14] In a double-blind interview, neither the interviewer nor the interviewee know the other's identity. Additionally, principals were told their answers would not be used for evaluation purposes, to ensure no incorrect incentives in answers.

[15] We reiterate that this is a description of the instrument we use, which is different from the one described by Borges et al. (2023). Both are inspired on the WMS and both follow the same management practice mapping process shown to be reliable. More information on the instrument itself is available in Chapter 5.

4 Conceptual Framework

4.1 Schools

In much of the economics of education literature, an education production function is taken as a basal conceptual framework. This education production function, at its simplest level, decomposes student achievement into various effects, selecting those of interest for every research. These selected effects influence students and shape their education experience and output.

$$A_{ist} = F(X_{ist}, S(P_{ps}, O_s)). \tag{4.1}$$

Grissom et al. (2015) use the above equation to explicit the school production function. Students' achievement A depends crucially on two factors: their own abilities, X, and school effectiveness, S. This last factor can be decomposed into what is within principal control and can be written as a function of principal performance, P, and that which is outside of principal influence, O.

Generally, an additive education production function is considered when analyzing principal effects on student achievement:

$$A_{ist} = \delta_s + \theta_{st} + \gamma_{ist} + \upsilon_{ist}, \tag{4.2}$$

in which student achievement is a function of school (δ), principal (θ) and individual (γ) effects. Here, υ contains all unobserved factors affecting achievement. We assume that this non-observable term is independent of all other terms.[1]

In its simplest form, this function indicates the effects influencing students‘ education. Parametric approaches propose specific forms for these components to exert such effects on student learning, whereas semi-parametric approaches opt to follow through without specifying such behaviours. The choice between these approaches is often contingent on available (observable) data to properly measure desired effects.

Besides the terms in this education production function, another important aspect is the function‘s additive nature. School and principal effects have an additive interaction, which enables researchers to distinguish them and propose their individual (or joint) estimation. In the same manner that this allows for the estimation of such effects, a consequence of this additive nature is that it may require special assumptions on how these components interact with one another. After all, principals are a key part of the

[1] Coelli & Green (2012) deem this assumption innocuous, seeing as any shock correlated with the other effects is absorbed into those effects, and not into υ.

transversal management of schools and have an impact on students, teachers, school and community. Therefore, each model must shine a light on how these interactions are treated.

4.2 Principal Effects

4.2.1 Parametric Value-Added Models

The first model we explore is a variation of that widely used in the literature for studying principal effects.[2] These models depart from the education production function in equation (4.2) and propose specific decompositions for each of its components. Essentially, this determines paths through which each education input influences student achievement, and must thus be grounded on observational data.

Principal value-added models are fixed effects models that seek to estimate the time-invariant effect principals have on student achievement. Since principals only influence students indirectly, as discussed in Chapter 2, it is crucial to be able to control for autonomous effects exerted by other educational factors, like teachers and the school itself. Common approaches include a level of fixed effects for schools and seek to distinguish principal effects from school effects, while also controlling for time-varying characteristics. Such is the case in the models proposed by Grissom et al. (2015), the second of which is the basal starting point for the value-added model we employ.

Equation (4.3) presents an example.[3]

$$A_{ijcpst} = X_{ijcpst}\beta_1 + S_{st}\beta_2 + \theta_p + \delta_s + \upsilon_{ijcpst}. \tag{4.3}$$

In this model, A is the achievement of student i in classroom c, taught by teacher j, in school s, led by principal p in year t. Controls for school (S) are often included. Our main interest lies in the principal and school fixed effects, θ_p and δ_s, respectively. These fixed effects estimates act as the value-added measures for their respective educational factor.

A key, if not the central role of principals highlighted in the literature, is their responsibility and authority in teacher hiring and retention, which of course pervades teacher motivation and school climate.[4] Given this influence in teacher body composition,

[2] Per the literature review in Chapter 2, see, for example, Grissom et al. (2015), Branch et al. (2012).

[3] Parametric value-added models commonly make use of the difference in achievement over time in their specifications. See Koedel et al. (2015) for an explanation with teacher value-added models and Grissom et al. (2021) for an overview of common principal value-added models. Here, we forego a difference in achievement approach due to unavailable longitudinal data on students, with only one test score observed per student for each subject, as will be detailed in Chapter 5.

[4] Most of the literature in principal value-added concerns analysis made in North America or Western Europe. The principal effects literature also has a similar heavy geographical bias.

and the immediate and evident nature teachers play as mediation channels for principal influence in student outcomes, models seldom include teacher fixed effect terms.[5]

Such principal authority is not present in Brazilian schools, where teacher hiring is done through public tender and, as with all public servants, faces employment stability, as explained in Chapter 3. This does not mean principals do not play a fundamental role in retaining good teachers: motivation and school climate are still key. Nevertheless, principal authority in Brazilian public schools is still very reduced when compared to the context studied in other school systems in the literature. Because of this, we believe it is possible to consider teacher value-added jointly with principal and school value-added in models studying principal influence in Brazilian public schools. This significant difference in principal and teacher interactions with respect to school body composition pivots our attempt at expanding the commonly used model to include all three effects simultaneously.

We conduct our analysis employing three value-added models. The first is the usual school-principal model widely used in the literature. It follows closely equation (4.3), but adds more control options and a period fixed effect to control for time trends.

$$A_{ijcpst} = X_{ijcpst}\beta_1 + S_{st}\beta_2 + J_{jpst}\beta_3 + C_{jcst}\beta_4 + \theta_p + \delta_s + \eta_t + \upsilon_{ijcpst}. \tag{4.4}$$

In this specification, J are teacher characteristics, C are classroom characteristics, and η_t is a period fixed effect.

We then add teacher fixed effects, and do so parsimoniously: first, we consider a model with principal and teacher fixed effects, adding the usual controls.

$$A_{ijcpst} = X_{ijcpst}\beta_1 + S_{st}\beta_2 + J_{jpst}\beta_3 + C_{jpst}\beta_4 + \theta_p + \pi_j + \eta_t + \upsilon_{ijcpst}. \tag{4.5}$$

In this case, π_j is the teacher composition fixed effect, and acts as our value-added measure for teachers. This is different from an individual teacher fixed effect, and instead considers as a teacher composition the collective teachers (or individual teacher, in a special case) who minister classes to a given grade of interest. This approach is inspired on the treatment Abowd et al. (1999) employ on workers to analyze changes in firm employees over time. We adopt such a technique due to the difficulty in tracking principal-teacher pairings in data, especially when several teachers are assigned simultaneously.

Essentially, teacher compositions are formed out of all teachers of a subject in a given grade (for example, all math teachers supervising a 3º EM classroom form the teacher composition that year). We consider that there is a change in teacher composition when a different group of teachers minister classes for the same subject and grade in another year. Notice that this includes a range of possible changes: there may be a composition

5 An identification problem is also prevalent in this topic. Distinguishing principal and teacher effects is extremely complicated exactly because principals administer the school body as one of their main tasks.

expansion, where the number of teachers increases, or a composition reduction, where the number of teachers decreases. Additionally, teacher substitution also changes compositions, and may even be coupled with expansions or reductions to form new compositions. It may be easier to consider what characterizes the permanence of a teacher composition over time: the same teachers are responsible for teaching a given subject for a given grade. Anything different is considered a change in teacher composition and establishes a new unit.

A third and final value-added model includes all three fixed effects: principal, teacher and school. Again, teacher fixed effects are constructed to consider teacher compositions.

$$A_{ijcpst} = X_{ijcpst}\beta_1 + S_{st}\beta_2 + J_{jpst}\beta_3 + C_{jpst}\beta_4 + \theta_p + \pi_j + \delta_s + \eta_t + \upsilon_{ijcpst}. \tag{4.6}$$

We discuss our identification strategy and the necessary assumptions for these models (Equations (4.4), (4.5) and (4.6)), along with more details on controls and fixed effects characteristics, in Chater 6.

4.2.2 Semi-Parametric Within School Variance Models

In contrast with the models presented in the previous section, we also consider a less parameterized approach to education production. With a semi-parametric model of principal effects, we theorize less on the specific channels of principal influence on student achievement and instead focus on the expected variance in factor quality in education production. Here, the work of Coelli & Green (2012) is a central inspiration on how to investigate principal quality under these circumstances. We first review the approach proposed by these authors, and then propose an extension to consider teacher quality in the analysis.

4.2.2.1 Within School Principal Effect Variance

Our second approach to studying principal effects in student learning is a semi-parametric model, also inspired in the education production function in equation (4.2). Instead of specifying the paths this influence takes, we follow Coelli & Green (2012) by using a within-school variance analysis of principal effects. Let us first explore the model they developed.[6]

The authors depart from the education production function (4.2). Calculating a school's average student achievement for period t.

$$\bar{A}_{st} = \delta_s + \theta_{st} + \bar{\gamma}_{st} + \bar{\upsilon}_{st}. \tag{4.7}$$

[6] In what follows, a few adaptations in notation were made in relation to Coelli & Green (2012) to better accommodate the extensions presented later in this section.

Now taking the average of student achievement of a school s for all periods.

$$\bar{A}_s = \delta_s + \bar{\theta}_s + \bar{\gamma}_s + \bar{\upsilon}_s. \tag{4.8}$$

Subtracting (4.8) from (4.7) to obtain the cohort deviation from school average achievement.

$$(\bar{A}_{st} - \bar{A}_s) = (\theta_{st} - \bar{\theta}_s) + (\bar{\gamma}_{st} - \bar{\gamma}_s) + (\bar{\upsilon}_{st} - \bar{\upsilon}_s). \tag{4.9}$$

Notice that the school effect term, δ_s, is eliminated in this last expression by subtracting mean school effects. This is crucial in developing a model focused on within-school variation.

Squaring both sides of expression (4.9).

$$\begin{aligned}(\bar{A}_{st} - \bar{A}_s)^2 =& (\theta_{st} - \bar{\theta}_s)^2 + (\bar{\gamma}_{st} - \bar{\gamma}_s)^2 + 2(\bar{\gamma}_{st}\theta_{st} + \bar{\gamma}_s\bar{\theta}_s - \bar{\gamma}_s\theta_{st} - \bar{\gamma}_{st}\bar{\theta}_s) + \\ &+ 2(\bar{\upsilon}_{st}\theta_{st} + \bar{\upsilon}_s\bar{\theta}_s + \bar{\upsilon}_{st}\bar{\gamma}_{st} + \bar{\upsilon}_s\bar{\gamma}_s - \bar{\upsilon}_s\theta_{st} + \bar{\upsilon}_{st}\bar{\theta}_s - \bar{\upsilon}_{st}\bar{\gamma}_s - \bar{\upsilon}_s\bar{\gamma}_{st}) + (\bar{\upsilon}_{st} - \bar{\upsilon}_s)^2.\end{aligned} \tag{4.10}$$

As explained by Coelli & Green (2012), this term characterizes the squared deviation in mean student achievement in terms of the within-school variation in principal effects ($(\theta_{st} - \bar{\theta}_s)^2$), the within-school variation in average student quality ($(\bar{\gamma}_{st} - \bar{\gamma}_s)^2$), the covariance between principal quality and average student quality, and the variation of υ ($(\bar{\upsilon}_{st} - \bar{\upsilon}_s)^2$). Here, we make all the covariance terms explicit, but notice that any term accompanied by υ terms (be it υ_s or υ_{st}) vanishes upon taking its expectation due to the independence of the error term.

To obtain their estimator, we take the average within each school and then apply expectations to equation (4.10).

$$E\left[\frac{1}{T}\sum_{t=1}^{T}(\bar{A}_{st} - \bar{A}_s)^2\right] = \underbrace{E\left[\frac{1}{T}\sum_{t=1}^{T}(\theta_{st} - \bar{\theta}_s)^2\right]}_{\star} + \sigma^2_{\bar{\gamma}_s} + \sigma_{\bar{\gamma}_s\theta_{st}} + \sigma^2_{\upsilon}. \tag{4.11}$$

Here, $\sigma^2_{\bar{\gamma}_s}$ is the variance of cohort average quality and $\sigma_{\bar{\gamma}_s\bar{\theta}_{st}}$ is the covariance between deviations in cohort average quality and deviations in principal effects in school s, whilst σ^2_{υ} is the variance of the error term. Notice that the error term interactions vanish because υ is defined such that it is non-correlated to the other factors in the model.

The main interest in estimation lies in the first term on the right-hand side of equation (4.11), which is highlighted with a star marking. They develop this term further by analyzing a specific case, and then generalizing its results.[7] The final expression for

[7] For a comprehensive derivation of this term, please refer to Coelli & Green (2012), specifically Appendix B. Their derivation is mirrored in Appendix A of this research, along with that of the proposed extension with teacher effects.

deviation in principal effectiveness can be written generally as:

$$E\left[\frac{1}{T}\sum_{t=1}^{T}(\theta_{st}-\bar{\theta}_s)^2\right]=\sigma^2_{\theta_s}\left[\frac{1}{T}\sum_{p=1}^{P}q_p\left(1+\frac{1}{T^2}\sum_{k=1}^{J}q_k^2-\frac{2}{T}q_p\right)\right], \tag{4.12}$$

in which q_p is the number of years principal p is in charge of the school and $\sigma^2_{\theta_s}$ denotes the within-school variance in principal effectiveness and constitutes the term of interest. The term that accompanies it is a deterministic number based on principal turnover, that is, the number of different principals heading the school over the observed period. If each principal p, with $p = 1, \ldots, P$, stays in charge of school s for q_p years, then $\sum_{p=1}^{P} q_p = T$, which is the entire observed period. This term's temporal average weights the variance term based on how much principal turnover a school experiences. For example, if a single principal is in charge of the school during the whole period, this term collapses to zero. This is intuitive: if there is no principal turnover in a school, no variation in principal effect is expected. If there is more than one principal over the analyzed period, then this term is positive and increases with principal turnover.

The whole intuition behind this term is that schools with higher principal turnover have shorter principal tenure spells, thus more years with different principal effects at play. This means that the within-school variance in principal quality should be higher in schools with higher principal turnover. Recall that the parametric model described in the previous section proposed an analysis of individual principal effects. In the model proposed by Coelli & Green (2012), principal effects are not analyzed individually, and the variance in principal effects at the school level due to principal turnover is at the core instead.

This approach requires quite a few assumptions. As with the parametric model shown in the previous section, this semi-parametric approach also depends crucially on the additive education production, equation (4.2), specifically on additive school effects in this case. Another crucial assumption is that principal effects are immediate, implying a principal already exerts an effect on student achievement upon tenure spell start. However, some assumptions made on this semi-parametric model were not in place before. Although it allows for systematic sorting of principals to perennially better schools based on principal characteristics, this approach does not allow for systematic principal sorting based on trends in school quality. This would imply the presence of a school-specific term in the regression idiosyncratic error correlated with principal effects.

Subtituting equation (4.12) in (4.11), we obtain the equation Coelli & Green (2012) seek to estimate. The authors discuss the further identifying assumptions, specifically on the covariance term ($\sigma_{\bar{\gamma}_s\theta_{st}}$). We engage with this discussion and enunciate other assumptions in Chapter 6.

4.2.2.2 Within School Teacher Effect Variance Extension

One of the limitations Coelli & Green (2012) point to in their research is the absence of student-teacher matched data. As such, no teacher effects are inserted in the decomposition in equation (4.2) or developed in further steps. Their whole model, however, is based on that developed by Rivkin et al. (2005) for evaluating within-school variance in teacher effects. These authors, in turn, possessed not only student and teacher links, but also longitudinal cohort data, and developed their model using within-cohort difference of within-school effects, something unavailable to Coelli & Green (2012) even if the authors had access to matched student-teacher observations in data.

Since we do possess matched student-teacher data, we seek to further develop the model proposed by Coelli & Green (2012) to incorporate teacher effects. We do refer to the work of Rivkin et al. (2005), but since longitudinal student data was also unavailable to us, we follow the differences in school averages approach presented so far.

We begin by incorporating a teacher effect term into the education production function (4.2).

$$A_{istc} = \delta_s + \theta_{st} + \pi_{stc} + \gamma_{istc} + \upsilon_{istc}. \tag{4.13}$$

In this case, A defines student achievement for student i from cohort t, in school s and classroom c. This achievement is decomposed into school (δ), principal (θ), teacher (π) and individual (γ) effects, together with an error term. Since we are working with only one grade, its subscript is suppressed for clarity.

Following the same route as outlined by Coelli & Green (2012), we begin by calculating the average achievement for students in school s and cohort t.

$$\bar{A}_{st} = \delta_s + \theta_{st} + \bar{\pi}_{st} + \bar{\gamma}_{st} + \bar{\upsilon}_{st}. \tag{4.14}$$

We also calculate the average student achievement in school s (across cohorts).

$$\bar{A}_s = \delta_s + \bar{\theta}_s + \bar{\pi}_s + \bar{\gamma}_s + \bar{\upsilon}_s. \tag{4.15}$$

Now we subtract equation (4.15) from (4.14) in order to obtain the deviation from average school achievement.

$$(\bar{A}_{st} - \bar{A}_s) = (\theta_{st} - \bar{\theta}_s) + (\bar{\pi}_{st} - \bar{\pi}_s) + (\bar{\gamma}_{st} - \bar{\gamma}_s) + (\bar{\upsilon}_{st} - \bar{\upsilon}_s). \tag{4.16}$$

With this step, we can eliminate the school effect term from our expression. Notice also that, since schools may have more than one teacher allocated across classrooms in the same grade, the average teacher effect in school s and cohort t can be written as a weighted mean of all teacher effects at play in a grade:

$$\bar{\pi}_{st} = \sum_{c=1}^{C} \frac{n_{ct}}{N_t} \pi_{stc}, \tag{4.17}$$

where π_{stc} is the effect the teacher allocated to classroom c in school s in period t exerts; n_{ct} is the number of students in classroom c in period t; and N_t is the total number of students in the schooling year being analyzed, such that $\sum_{c-1}^{C} n_{ct} = N_t$, with $c = 1, \ldots, C$.

Returning to equation (4.16) and squaring both sides.

$$\begin{aligned}(\bar{A}_{st} - \bar{A}_s)^2 =& (\theta_{st} - \bar{\theta}_s)^2 + (\bar{\pi}_{st} - \bar{\pi}_s)^2 + (\bar{\gamma}_{st} - \bar{\gamma}_s)^2 + \\ &+ 2(\theta_{st}\bar{\pi}_{st} + \theta_{st}\bar{\gamma}_{st} + \bar{\theta}_s\bar{\pi}_s + \bar{\theta}_s\bar{\gamma}_s + \bar{\pi}_{st}\bar{\gamma}_{st} + \bar{\pi}_s\bar{\gamma}_s - \\ &- \theta_{st}\bar{\pi}_s - \theta_{st}\bar{\gamma}_s - \bar{\theta}_s\bar{\pi}_{st} - \bar{\theta}_s\bar{\gamma}_{st} - \bar{\pi}_{st}\bar{\gamma}_s - \bar{\pi}_s\bar{\gamma}_{st}) + (\bar{\upsilon}_{st} - \bar{\upsilon}_s)^2.\end{aligned} \tag{4.18}$$

Equation (4.18)[8] expresses the squared deviation in average student achievement in terms of the within-school variation in principal effects ($(\theta_{st} - \bar{\theta}_s)^2$), the within-school variation in average teacher quality ($(\bar{\pi}_{st} - \bar{\pi}_s)^2$), the within-school variation in average student quality ($(\bar{\gamma}_{st} - \bar{\gamma}_s)^2$), the distributed interactions between principal effect, average teacher effect and average student quality, plus the variation of the idiosyncratic error. For notation brevity, we hereon refer to these interaction terms in parentheses as Φ.

We calculate the average of these terms within each school to effectively measure the within-school variance. Applying expectations:

$$\begin{aligned}E\left[\frac{1}{T}\sum_{t=1}^{T}(\bar{A}_{st} - \bar{A}_s)^2\right] = E\left[\frac{1}{T}\sum_{t=1}^{T}(\theta_{st} - \bar{\theta}_s)^2\right] + E\left[\frac{1}{T}\sum_{t=1}^{T}(\bar{\pi}_{st} - \bar{\pi}_s)^2\right] + E\left[\frac{1}{T}\sum_{t=1}^{T}(\bar{\gamma}_{st} - \bar{\gamma}_s)^2\right] + \\ + E\left[\frac{1}{T}\sum_{t=1}^{T}\Phi\right] + E\left[\frac{1}{T}\sum_{t=1}^{T}(\bar{\upsilon}_{st} - \bar{\upsilon}_s)^2\right].\end{aligned} \tag{4.19}$$

The above expression defines the average squared deviation from mean student outcomes in each school as a sum of five terms. We follow Coelli & Green (2012) in proposing the same expressions for the first, third and fifth right-hand side elements: equation (4.12) denotes the term put forth by the authors to capture the within-school variance in principal effects; the third term is the within school variance of average student quality, defined as $\sigma^2_{\bar{\gamma}_s}$; and the fifth term is the within school variance in the idiosyncratic error, σ^2_{υ}. The fourth term captures the pairwise within-school covariance effects between principal effects, average teacher quality and average student quality.[9] Thus, we can write the Φ element as:

$$E\left[\frac{1}{T}\sum_{t=1}^{T}\Phi\right] = \sigma_{\bar{\gamma}_s\theta_{st}} + \sigma_{\bar{\gamma}_s\bar{\pi}_s} + \sigma_{\bar{\pi}_s\theta_{st}}, \tag{4.20}$$

in which $\sigma_{\bar{\gamma}_s\theta_{st}}$ is the covariance between principal effect and average student quality, as in the Coelli & Green (2012) formulation; $\sigma_{\bar{\gamma}_s\bar{\pi}_s}$ is the covariance between average student

[8] In this equation, we suppress the interactions with υ for clarity. These interactions vanish once the expectation is taken due to υ independence. Please refer to equation (4.10) where we made these interactions explicit. It is analogous, with the added teacher effect interactions.

[9] Once again, the error term interactions, suppressed in equation (4.16), lead to covariance terms with the other effects, which are zero by definition.

quality and average teacher quality; and $\sigma_{\bar{\pi}_s\theta_{st}}$ is the covariance between principal effect and average teacher quality.

To obtain our estimator, we must further develop the teacher effect term in the right-hand side of equation (4.19). We begin by rewriting the expectation of variation in average teacher effects.[10]

$$E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] = E[\bar{\pi}_{st}^2+\bar{\pi}_s^2-2\bar{\pi}_{st}\bar{\pi}_s]. \tag{4.21}$$

Using the fact that $\bar{\pi}_s = \frac{1}{T}\sum_{t=1}^{T}\bar{\pi}_{st}$.

$$E[\bar{\pi}_s^2t+\bar{\pi}_s^2-2\bar{\pi}_{st}\bar{\pi}_s] = E[\bar{\pi}_{st}^2]+E\left[\left(\frac{1}{T}\sum_{t=1}^{T}\bar{\pi}_{st}\right)^2\right]-2E\left[\bar{\pi}_{st}\left(\frac{1}{T}\sum_{t=1}^{T}\bar{\pi}_{st}\right)\right]. \tag{4.22}$$

Applying the $\bar{\pi}_{st}$ definition from equation (4.17).

$$E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] = E\left[\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}\right)^2\right]+E\left[\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}\right)^2\right]-2E\left[\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}\left(\frac{1}{T}\sum_{t=1}^{T}\frac{n_{ct}}{N_t}\pi_{stc}\right)\right]. \tag{4.23}$$

To further develop this term, we make three simplifying assumptions on teacher behaviour and interaction. First, we assume teachers are drawn from a common distribution, such that two teachers are independent draws from one another. First, this implies that the average teacher effect across all schools is zero, that is, $E[\pi_{stc}]=0$. Notice how this assumption mirrors that of no across-school variation in principal effects made by Coelli & Green (2012) and discussed in Chapter 2. This also implies that teacher quality covariance is zero:

$$E[\pi_j\pi_k]=0. \tag{4.24}$$

The second assumption is that teachers have time-invariant effects. This means there is a variance between teacher effects, but no variance in a teacher's effect over time.

$$E[\pi_{jt_1}]=E[\pi_{jt_2}] \Rightarrow E[\pi_{jt_1}^2]=E[\pi_{jt_2}^2]. \tag{4.25}$$

Lastly, we assume teachers exert the same effect in different classrooms. That is, teachers have an immediate effect on all classrooms they teach, much like the effect we assume for principals on school students in our models.

$$E[\pi_{jc_1}]=E[\pi_{jc_2}]. \tag{4.26}$$

Notice that this last assumption essentially transforms the classroom subscript c into a teacher subscript. This is clear to see when recalling equation (4.17), which will now

[10] We begin our analysis with the variation in average teacher effects first, not considering the school average, for notation clarity.

be an average of individual teacher effects weighted by the proportion of students taught by that teacher (effectively concatenating classrooms at the teacher level). Assumptions about time and classroom invariant teacher effects are commonplace in value-added models, and additive school effects models in general.

With these assumptions, we can rewrite expression (4.23) as:

$$E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] = E[\pi_{stc}^2]\left(\sum_{c=1}^{C}\frac{n_{ct}^2}{N_t^2}\right) + E[\pi_{stc}^2]\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)^2 + E[\pi_{stc}^2]\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right). \tag{4.27}$$

Denoting $E[\pi_{stc}^2] = \sigma_{\bar{\pi}_{st}}^2$ as the within-school variance in average teacher quality, we can rewrite the above expression as:

$$E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] = \sigma_{\bar{\pi}_{st}}^2\left[\left(\sum_{c=1}^{C}\frac{n_{ct}^2}{N_t^2}\right) + \left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)^2 + \left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\right]. \tag{4.28}$$

Further development leads to the following expression:

$$E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] = \sigma_{\bar{\pi}_{st}}^2\left[\sum_{c=1}^{C}\frac{n_{ct}^2}{N_t^2} + \frac{1}{T^2}\left(\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}^2}{N_t^2} + 2\sum_{c=1}^{C}\sum_{t=1}^{T}\sum_{u\neq t}\frac{n_{ct}}{N_t}\frac{n_{cu}}{N_u}\right) - \frac{2}{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\left(\sum_{v=1}^{T}\frac{n_{cv}}{N_v}\right)\right]. \tag{4.29}$$

The above equation expresses the within-school variance of average teacher quality in school s and year t ($\sigma_{\bar{\pi}_{st}}^2$). We calculate the school average to obtain the within-school variance of average teacher quality:

$$\begin{aligned} E\left[\frac{1}{T}\sum_{t=1}^{T}(\bar{\pi}_{st}-\bar{\pi}_s)^2\right] = \sigma_{\bar{\pi}_s}^2\sum_{t=1}^{T}&\left[\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2 + \frac{1}{T^2}\left(\sum_{t=1}^{T}\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2 + 2\sum_{c=1}^{C}\sum_{t=1}^{T}\sum_{u\neq t}\frac{n_{ct}}{N_t}\frac{n_{cu}}{N_u}\right) - \dots\right] \\ &\left[\dots - \frac{2}{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\left(\sum_{v=1}^{T}\frac{n_{cv}}{N_v}\right)\right]. \end{aligned} \tag{4.30}$$

Expression (4.30) is the term we propose for capturing within-school variation in average teacher quality. It mirrors both the logic and derivation steps employed by Coelli & Green (2012) to capture principal variation. A comprehensive derivation of this result, including comparisons with that of the principal term, is available in Appendix A.

This expression is a deterministic number obtained based on teacher turnover in the grade at hand, as well as the proportion of students in such grade who are taught by each teacher. Despite being fairly complicated, its intuition mirrors that of the deterministic term for principal effect: schools with more teachers will have higher teacher quality variance. Teacher turnover can contribute to this (as it means more teachers acting at the school over the period analyzed), but in this case, another factor takes center stage:

the variation in classroom proportions allocated to teachers over time can still affect the deterministic term accompanying the within-school average teacher quality variance without explicit teacher turnover. This is important, as varying shares of students allocated to the same teachers over time still lead to variation in the average teacher effect at the school level. The added complexity in this term, when compared to the one proposed by Coelli & Green (2012) for within-school principal effect variation in equation (4.12), is not only due to the logic behind the definition in equation (4.17) but also because it specifies a difference in means, thus including temporal average terms (whereas θ_{st} required no such thing).

Taking once again the simplest case available: a single teacher is responsible for all students in a grade during the whole period analyzed, that is, there is neither teacher turnover nor changes in the share of pupils allocated to the teacher. This implies that the within-school variance in average teacher quality is zero.

$$E[(\bar{\pi}_{st} - \bar{\pi}_s)^2] = 0, \tag{4.31}$$

which follows directly from the fact that the average teacher quality in year t and the average teacher quality over the whole period are equal to the single teacher's effect, $\bar{\pi}_{st} = \bar{\pi}_s = \pi$.

This must hold in espression (4.30), and indeed, having $n_{ct} = N_t = N$ under the given conditions:

$$\begin{aligned}
E[(\bar{\pi}_{st} - \bar{\pi}_s)^2] =& \sigma^2_{\bar{\pi}_s} \left[\sum_{c=1}^{C} \left(\frac{n_{ct}}{N_t} \right)^2 + \frac{1}{T^2} \left(\sum_{t=1}^{T} \sum_{c=1}^{C} \frac{n_{ct}^2}{N_t^2} + 2 \sum_{c=1}^{C} \sum_{t=1}^{T} \sum_{u>t} \frac{n_{ct}}{N_t} \frac{n_{cu}}{N_u} \right) - \frac{2}{T} \sum_{c=1}^{C} \frac{n_{ct}}{N_t} \left(\sum_{v=1}^{T} \frac{n_{cv}}{N_v} \right) \right] \\
&= \sigma^2_{\bar{\pi}_s} \left[\sum_{c=1}^{C} \left(\frac{N}{N} \right)^2 + \frac{1}{T^2} \left(\sum_{t=1}^{T} \sum_{c=1}^{C} \frac{N^2}{N^2} + 2 \sum_{c=1}^{C} \sum_{t=1}^{T} \sum_{u \neq t} \frac{N}{N} \bigg|_t \frac{N}{N} \bigg|_u \right) - \frac{2}{T} \sum_{c=1}^{C} \frac{N}{N} \left(\sum_{v=1}^{T} \frac{N}{N} \right) \right] \\
&= \sigma^2_{\bar{\pi}_s} \left[\sum_{c=1}^{C} 1 + \frac{1}{T^2} \left(\sum_{t=1}^{T} \sum_{c=1}^{C} 1 + 2 \sum_{c=1}^{C} \frac{T(T-1)}{2} \right) - \frac{2}{T} \sum_{c=1}^{C} 1 \left(\sum_{t=1}^{T} 1 \right) \right] \\
&= \sigma^2_{\bar{\pi}_s} \left[1 + (T + T(T-1)) - 2 \right] \\
&= \sigma^2_{\bar{\pi}_s} [1 + 1 - 2] = 0.
\end{aligned}$$

We can now return to equation (4.19) and substitute expressions (4.12), (4.30) and other elements discussed.

$$\begin{aligned}
E\left[\frac{1}{T} \sum_{t=1}^{T} (\bar{A}_{st} - \bar{A}_s)^2 \right] =& \sigma^2_{\theta_s} \left[\frac{1}{T} \sum_{p=1}^{P} q_p \left(1 + \frac{1}{T^2} \sum_{k=1}^{P} q_k^2 - \frac{2}{T} q_p \right) \right] + \\
& + \sigma^2_{\bar{\pi}_s} \left[\sum_{c=1}^{C} \left(\frac{n_{ct}}{N_t} \right)^2 + \frac{1}{T^2} \left(\sum_{t=1}^{T} \sum_{c=1}^{C} \frac{n_{ct}^2}{N_t^2} + 2 \sum_{c=1}^{C} \sum_{t=1}^{T} \sum_{u \neq t} \frac{n_{ct}}{N_t} \frac{n_{cu}}{N_u} \right) - \dots \right] \\
& \left[\dots - \frac{2}{T} \sum_{c=1}^{C} \frac{n_{ct}}{N_t} \left(\sum_{v=1}^{T} \frac{n_{cv}}{N_v} \right) \right] + \sigma^2_{\bar{\gamma}_s} + \sigma_{\bar{\gamma}_s \theta_{st}} + \sigma_{\bar{\gamma}_s \bar{\pi}_s} + \sigma_{\bar{\pi}_s \theta_{st}} + \sigma^2_{\upsilon}.
\end{aligned} \tag{4.32}$$

This approach requires both the three assumptions on teacher behaviour spelt in expressions (4.24), (4.25) and (4.26), and the assumptions made by Coelli & Green (2012), discussed in their model's review. The teacher assumptions introduced in this formulation seek to emulate those made on principal effect behaviour. Two assumptions made deserve emphasis. Take the annual within-school variance in the teacher quality definition we employed, that is, $E[\pi_{stc}^2] = \sigma_{\bar{\pi}_{st}}^2$. Applying the variance definition, have that $E[\pi_{stc}] = 0 \Rightarrow \sigma_{\bar{\pi}_{st}}^2 = E[\pi_{stc}^2] - (E[\pi_{stc}])^2$. This means that the average teacher effect across schools and periods is zero.[11] As already mentioned, this comes from the assumption of independent teacher effects, wherein teachers are hired from a common pool of candidates.[12] This follows from the definition of principal effect variance used by Coelli & Green (2012) that we seek to emulate on teacher effects, in which $E[\theta_{st}] = 0 \Rightarrow E[\theta_{st}^2] = \sigma_{\theta_s}^2$, which reveals the underlying hypothesis on average principal effects.[13]

With said collection of assumptions, we are able to obtain expression (4.32) based on observed parameters. This model's estimation, however, depends crucially on the identification hypothesis made to deal with the covariance terms. This is discussed in length in Chapter 6.

[11] One might inquire about different definitions for variance in teacher effects. One possible example is the definition $E[\bar{\pi}_{st}^2] = E[\sum_{c=1}^{C} \frac{n_{ct}}{N_t} \pi_{stc}] = \sigma_{\bar{\pi}_s}^2$, which implements the within-school variance of average teacher effects over the whole period directly. However, this follows from the underlying hypothesis that $E[\bar{\pi}_{st}] = 0$, alternatively, that the average teacher effects in school s is zero, which is a further reaching assumption than the one employed in our model. This would be equivalent to assuming that average teacher composition effects are zero for every school, following the composition approach mentioned earlier in the value-added model review and further explored ahead in Chapter 6. As will be discussed in the referred chapter, while such an assumption made sense given the difficulty in tracking teacher and principal pairings in data, it becomes too strong an assumption in this semi-parametric approach, where individual teachers need not be tracked among classrooms (which enables a weaker assumption on teacher effects).

[12] Empirically, principal or teacher effects are expressed relative to one another, often without omitted categories, which implies that average effects are zero by construction.

[13] Recall this was briefly discussed in Chapter 2, and is the reason the authors' estimates represent a lower bound of the actual impact of within-school variation in principal effects in student outcomes.

5 Data

In order to carry out this research, a panel connecting students and their test scores to their respective teachers, schools and principals is needed. Additionally, linking each student to relevant demographic and socioeconomic information is necessary to provide adequate controls for regressions. This is also accomplished for teachers, principals and schools, with varying degrees of informational depth. Once this panel has been constructed, model samples have been drawn and the relevant VA measures have been estimated, management practice adoption markings are coupled with principal information to proceed with the intended association. This chapter is planned as follows: We first explain our data sources and the raw data structure to which we had access. Then, we document the necessary work to obtain the student panel linking all information, along with some descriptive statistics. The last section turns to the management instrument and explores the information it brings.

5.1 Data Sources

Three main data sources were used to obtain the necessary information for this task: the Minas Gerais State Education Bureau (*Secretaria Estadual de Educação de Minas Gerais* – SEE-MG); the identified *Censo Escolar* database, an annual school census in custody of *Instituto Nacional de Estudos e Pesquisas Educacionais Anísio Teixeira* (INEP), a government education research agency; and a management practice adoption instrument developed by Henriques et al. (2020) on *Jovem de Futuro* program evaluation. In this section, we touch on each of these sources. We start with the administrative records and the necessity to utilize two different sources and then discuss the data obtained from the management practice mapping in MG state, the structure of which has already been explained in Chapter 3.

5.1.1 Administrative Records

Administrative data records ceded by the SEE-MG concerned two central themes: principal allocation into schools and student test scores for mathematics and Portuguese. With respect to the first topic, we had access to a database specifying principals' contractual details, with fields for principal identification, school identification, tenure start and end dates, along with employment details and characteristics. Students' mathematics and Portuguese test scores come from the PROEB exam, an Item Response Theory (IRT) based exam that structures the SIMAVE[1], the state-level external evaluation of public education, applied to students in all high-schools in Minas Gerais; as well as context survey responses from students and a sample of teachers and principals, containing relevant demographic and socioeconomic information. PROEB mathematics and Portuguese test scores are for students in the third year of *ensino médio* (3º EM), students in their last year of secondary education.

SIMAVE data is fragmented into several different datasets, with two datasets containing relevant information for students' PROEB test scores, and separate sets for student, teacher and principal survey responses, totalling five datasets per year and 25 non-standardized datasets overall. These non-standardized datasets not only contain varying information (with overlapping information often labelled differently) but also non-longitudinal identificators, meaning one individual present in a dataset cannot be identified in any other year. Despite being able to link students and principals using the principal allocation panel constructed, no way was found to link students and teachers using these administrative records. Of important note is that, in the Brazilian educational registry, students are matched with teachers and peers in data using classroom (*turmas*) identifiers[2], which are present in student-level SIMAVE data, but absent in teachers' survey answers.

Student and teacher pairings are central to our investigation, and so we resort to the identified *Censo Escolar*[3] database and successfully match classrooms to students' test score data. Since common identifiers between SIMAVE datasets and those in *Censo Escolar* differ, we use a combination of classroom names and school identifiers to match the classroom identifiers present in the latter to students in the SIMAVE datasets.[4] In

[1] The *Sistema Mineiro de Avaliação e Equidade da Educação Pública* (SIMAVE) is organized by the *Centro de Políticas Públicas e Avaliação da Educação* (CAEd) from the Juiz de Fora Federal University (UFJF) in partnership with the Minas Gerais state government. It has been in place evaluating state and municipal public education across Brazil since 2000 and evaluates students in the 5th, 9th and 12th year of formal education annually through the *Programa de Avaliação da Rede Pública de Educação Básica* (PROEB).

[2] Unlike classroom divisions commonly seen in the USA and Europe, students with varying levels of performance within one grade are grouped randomly into classrooms in the Brazilian public school system. These students are peers in all subjects throughout the year, while teachers are allocated to these mixed-performance classrooms.

[3] As mentioned, we use the identified version of *Censo Escolar*, which includes sensitive demographic information on students, teachers and school administrators, both public and private, and thus requires special manipulation. INEP, the database custodian, requires all work and result extraction to be approved by its Data Access team to prevent identified or identifiable data from being leaked. A non-identified *Censo Escolar*, containing only aggregate information on schools, is publicly available online. No statistics or results presented in this dissertation identify or allow for individual identification, following both INEP's and the University of São Paulo's ethical code.

[4] All individual identifiers in CAEd data are internally masked, and no usable keys were ceded to researchers. This means we were not able to identify factors longitudinally in CAEd databases and, ultimately, that no student, teacher or principal individual identifiers can identify individuals in both CAEd and INEP data simultaneously. All interactions between these two sources of data are either done through school codes or classroom identifiers obtained via classroom name matching.

Section 5.2 we detail this classroom name matching, as well as other data manipulation stages.

5.1.2 Management Practice Instrument Data

Data on management practice adoption by principals comes from the instrument developed by Henriques et al. (2020) for the *Jovem de Futuro* program partnership between the Minas Gerais state government and *Instituto Unibanco*. Instrument design and practice mapping were detailed in Chapter 3.

Regarding the instrument's data structure, it contains information on 14 domains of school management, presenting a summarized score obtained from cumulative markings from 10 practices in each domain following principal interview transcription. We have information on these individual markings for every principal interviewed. From these markings, we are able to construct the instrument score for each domain analyzed. These markings are linked to the school identifiers, instead of the schools' principals; but we can link principals to schools using the principal allocation panel constructed from the SEE-MG administrative data. More details on the instrument data are available further in this chapter.

5.2 Data Manipulation and Student Panel Construction

In this section, we detail the data manipulation steps employed to construct the complete student panel, linking last year high-school students' PROEB test scores to students' respective teachers and principals over the period of 2015 to 2019. This includes the classroom name matching used to abridge the SEE-MG data and the *Censo Escolar*, but also previous work harmonizing all 25 different SIMAVE datasets and later work on teacher-to-classroom allocation using the *Censo Escolar* classroom identifiers.

The first step was to adapt the database containing principals' contractual information into a principal-to-school allocation panel. Although variables for principal start and end tenure dates were present in the data, around two-thirds of tenure end date entries were missing values, meaning schools had several principals allocated to them simultaneously.[5] To work around this problem, we use solely tenure start dates to create the principal allocation panel. This is done by ranking each school's principals, beginning with the principal with the earliest tenure start date up until the principal with the last recorded tenure start marking. We then separate the five-year period into trimesters and create an algorithm that fills all of the school's trimesters after a principal's tenure start

[5] This is an obvious data record mistake and contradicts what Simelli et al. (2023) and Muñoz et al. (2021) document in their research. Since these authors' work was based on a dataset different from ours, we took extra measures to deal with this mistake.

date. Since this is done for all of the school's principals chronologically following their tenure start markings, we manage to fill each trimester with the correct principal in charge of the school.

As an illustrative example, consider a school s in which principal j is the first to have a tenure start marking, which corresponds to April/2016. Since April is in the second trimester of 2016, the algorithm fills the second, third and fourth trimesters of 2016 and all trimeters in 2017-2019 with principal j's unique identifiers. If a second principal, k, is the second recorded principal for school s and has a tenure start date marked as January/2019, then principal j's markings in all four trimesters in 2019 are replaced with principal k's unique identifier. Four things need to be highlighted in this example. First, since principal k only became an administrator in January/2019, none of principal j's markings in previous years are replaced. Second, if a third principal (or even principal j again) has a tenure start marking after 2019, it is discarded by the algorithm. Third, since no principals were registered as in charge of school s before April/2016, the algorithm leaves all trimesters in 2015 and the first trimester in 2016 blank (missing values).[6] Lastly, since the panel is constructed using trimesters, if more than one principal has a tenure start date assigned to the same trimester, the algorithm considers the more recent tenure start date, regardless of the amount of time each principal spent acting as administrator during the three months.

This algorithm assumes that only one principal can be allocated to a school at a time, meaning that a principal's tenure start date also marks her predecessor's tenure end date. Having constructed this trimestral panel, we then select only entries registered in the second trimester of every year, transforming it into an annual panel. The second trimester is chosen because Brazilian states' education bureaus are required to send school information for the *Censo Escolar* to INEP in May, and so we chose to filter for the second trimester to maintain this timing consistency.[7] The number of principals observed in each data step described is available in Table 1.

Original PROEB scores data came in three batches: one for 2015, one spanning from 2016 to 2018, and one for 2019. Neither of the first two batches contained any information on classrooms, whilst the 2019 batch contained only classroom name information. A supplementary dataset containing classroom names was merged for every yearly set (except for 2019) using the masked student identifiers. This step implied almost no friction, as Table 2 shows.

[6] As will be shown in Chapter 5, a significant portion of schools have missing values. Missing values in the annual principal allocation panel are interpreted as a school not having yet opened or having been closed.

[7] We test if our results are robust to this second trimester annual panel filter by estimating our full value-added model for principal allocation panels constructed using other bimesters. These results are presented in Section 7.1.4.

Table 1 – Principal Statistics

Principals in SEE-MG Contractual Database	7841
Principals allocated in bimestral principal panel	5249
Principals allocated in annual principal panel	5052
Principals in initial PROEB match	3121
Principals in whole student panel	3112
Principals in filtered student panel	3107

Notes: Principal data comes from contractual administrative records from SEE-MG. These principals are allocated to schools for every bimester in 2015-2019 following an algorithm based on their tenure start date (errors in the tenure end date variable made its use improper). This panel is transformed into an annual panel by selecting principals in charge during the second bimester of each year. This principal allocation panel receives annual PROEB information via school identifiers, thus matching students to principals. "Initial student panel" refers to "Student Data I" and "whole student panel" refers to "Student Data V" from Table 6. Lastly, "filtered student panel" refer to filters that guarantee control variables used in our models, and are described in Table 13.

Table 2 – PROEB Scores and Classroom Names Merge Statistics

	2015 (1)	2016 (2)	2017 (3)	2018 (4)	2019 (5)
PROEB data	145,327	144,798	143,951	152,326	175,474
Classroom name data	626,950	672,365	143,951	698,795	—
Student Data I	138,992	144,798	143,951	152,326	175,474
Perc. merged (%)	95.64	100	100	100	—

Notes: Columns (1) through (5) present merge statistics for the years 2015 through 2019, respectively. PROEB data refers to the number of unique students observed in the database containing students' PROEB maths and Portuguese scores. Classroom name data refers to the students observed in the supplementary SIMAVE database with classroom names. The dataset generated form this step is called Student Data I for clarification purposes. No merge was conduced for the 2019 data because PROEB data already contained classroom names.

Once this is accomplished, all years have information linking students' scores and their respective classroom names. Before we proceed to classroom name matching, since it involves the *Censo Escolar* and, therefore, implies several limitations on data work, we choose to couple students' survey answers at this point. Since both the scores database just manipulated and the survey answers database come from SEE-MG, they share CAEd masked individual identifiers, and we can simply merge them year-by-year. We reiterate that CAEd unique identifiers do not identify students in data longitudinally, but succeed in doing so across databases from the same year. Table 3 presents the merge results: over 90% of students had their survey answers coupled for all years. Two clarifications are necessary: First, notice that survey answers contain a much greater number of students than the PROEB score database for some years. This is because SIMAVE is applied yearly at three different stages, and these years with more students include data for these other schooling stages. Second, SEE-MG student databases contain two individual identifier columns; however, these aren't all filled out every year.

Table 3 – Students' PROEB Scores and SIMAVE Context Survey Answers Merge Statistics

	2015 (1)	2016 (2)	2017 (3)	2018 (4)	2019 (5)
Survey Answers Data	175,231	366,922	183,108	359,010	631,386
Filtered Answers	140,018	140,256	143,335	150,131	143,709
Student Data I	138,992	144,798	143,951	152,326	175,474
First masked student ID as merge key					
Successful Merges	133,995	140,199	—	—	143,694
Perc. PROEB Data (%)	96.40	96.82			92.85
Perc. Survey Data (%)	95.69	99.95			99.98
Second masked student ID as merge key					
Successful Merges	—	—	139,943	146,161	—
Perc. PROEB Data (%)			97.21	95.95	
Perc. Survey Data (%)			97.63	97.35	
Sudent Data II	133,995	140,199	139,943	146,161	143,694

Notes: Columns (1) through (5) show survey merge statistics for the years 2015 through 2019, respectively. The filter applied to survey answers data is simply a step to guarantee all data to be merged has student identifiers filled and non-blank survey answers. The two student identifiers present in the data are not compatible with one another. A unique student identifier will be created from these two separate identifiers further into the research. The data resulting from this step is called Student Data II for clarification purposes.

Afterwards, we proceed with the classroom name matching between this merged database and *Censo Escolar*'s classroom information. Classroom names are how schools differentiate classrooms, which then receive identifiers once this is reported to SEE-MG or

INEP. These are school-level names and may follow some patterns, like attributing letters or numbers (*3º A* and *3º B*, or *Turma 301* and *Turma 302*) but may also be arbitrary, taking to homeroom teachers' names or some inspiring word (for example: *Turma Murilo* ou *Turma Amizade*). Given how some patterns may be repeated across schools, we use both classroom names and school codes to match names in the student data to names in *Censo Escolar*. Also due to these usually small patterned names, we opt for a sharp matching approach with several rounds of marginal modifications to classroom names, instead of specifying an acceptance threshold in a fuzzy matching approach. Overall, we employ 24 rounds of matching, with the first attempt made without any alterations to classroom names.[8] All alterations are made to classroom names in the student database, maintaining names in the *Censo Escolar* intact. For every round, a list of unmatched classroom names is submitted to a merge attempt with the *Censo Escolar* database, and any classroom names matched are removed from this list and stored, with the remaining unmatched classroom moving on to another round. Statistics on classroom name and school code matching are presented in Table 4.

Table 4 – Classroom name and school code matching to *Censo Escolar* statistics

	2015 (1)	2016 (2)	2017 (3)	2018 (4)	2019 (5)
Student Scores Data	133,995	140,199	139,943	146,161	143,694
N. Classrooms	5754	5626	5907	6218	6021
N. Schools	2197	2231	2239	2268	2324
Single Classroom Schools (%)	33.45	34.51	33.41	30.03	33.56
Classrooms matched – 1st round (%)	66,4	65,5	67,0	65,4	15,3
Classrooms matched – 24th round (%)	92,6	92,2	93,1	93,4	91,6
Student Data III	125,724	129,492	130,942	137,111	132,609
N. Classrooms	5330	5187	5501	5806	5517

Notes: Columns (1) through (5) show classroom matching statistics for each respective annual database. Information on alterations made for every round of iterative matching can be found in Table B.1. Data resulting from this step, with classroom identifiers, is called Student Data III for clarity purposes.

Over 90% of classrooms are identified for each year using this iterative matching process. This is the link we can establish between data from SEE-MG administrative records and the various *Censo Escolar* databases. Since this match is carried out to obtain the corresponding classroom identifiers, any communication between these two data sources is done using either classroom or school codes. Despite having access to *Censo Escolar*'s student panels, which contain supplementary demographic information on students, we are not able to make use of them unless aggregated at the classroom and school levels.

[8] Further information on classroom name matching is available in Appendix Table B.1, which contains information on the marginal alterations made to classroom names at every round of matching.

As a final step in our annually separated data work, we use the newly acquired classroom identifiers to allocate teachers among these classrooms. For this, we use *Censo Escolar*'s teacher panel by selecting maths and Portuguese teachers, their respective demographic information from the panel, and adding a few markings indicating some characteristics of their work outside selected classrooms. These include, for example, if they taught other subjects besides maths or Portuguese, the number of total schools and total classrooms they taught simultaneously, and if they taught simultaneously in any private schools.[9] Table 5 presents merge statistics for this last step.

Table 5 – Teacher Allocation to Classrooms Statistics

	2015 (1)	2016 (2)	2017 (3)	2018 (4)	2019 (5)
Student Data III	125,724	129,492	130,942	137,111	132,609
N. Classrooms	5330	5187	5501	5806	5517
Matched Classrooms:					
N. Classrooms	5238	5165	5470	5801	5505
N. Math Teachers	3108	2968	3704	3235	3173
N. Port. Teachers	3170	3025	3147	3321	3215
Unmatched classrooms:					
N. Classrooms	91	22	31	5	12
Perc. Students (%)	1.72	0.40	0.53	0.08	0.20
Duplicated Classrooms:					
N. Classrooms	127	56	72	60	32
N. Math Teachers	61	25	32	30	7
N. Port. Teachers	68	37	48	35	25
Student Data IV	123,555	128,971	130,246	137,000	132,343

Notes: Columns (1) through (5) present merge statistics for the years 2015 through 2019, respectively. "Matched Classrooms" refer to classrooms which had both mathematics and Portuguese teachers allocated through the merge process. "Unmatched Classrooms" are those in which at least one subject did not have a teacher allocated. "Duplicated Classrooms" are a subgroup of matched classrooms which had more than one teacher allocated for at least one of the subjects. Mathematics and Portuguese teachers are unique within each year and appear in smaller numbers than classrooms due to simultaneous allocation to multiple classrooms in this schooling stage. Data resulting from this step is called Student Data IV for clarification purposes.

The second part of Table 5 shows us classrooms with successful teacher allocation, whereas the third part shows the number of classrooms for which at least one teacher

[9] Not all SIMAVE context survey markings for teachers contained individual teacher identifiers, and since these CAEd identifiers are masked separately every year, we were not able to extend the identifiers available to other databases. Consequently, we were not able to incorporate teacher SIMAVE survey answers into our analysis, using only (more restrictive) information available in the *Censo Escolar*'s teacher panel.

(maths of Portuguese) wasn't found. As can be seen, this represents a very small share of total students per year, with the highest being less than 2% of our annual sample, in 2015. The final part of Table 5 presents the number of duplicated classrooms, that is, the number of classrooms to which more than one maths or Portuguese teacher were allocated. This double allocation has two possible explanations: First, these classrooms had one or both subjects split into two fronts[10], and two teachers taught these subjects simultaneously, but registry in *Censo Escolar* does not distinguish this special case. A second explanation would be teacher changes during the school year, which would mean both teachers were registered, but did not teach the same classroom simultaneously. No markings point to any clues as to which is the case at hand; additionally, there is no timestamp in the case of departing teachers, so that no analysis on the share of school year taught can be conducted. Since only a small number of classrooms incur in this anomaly, our approach here is to draw one of these teachers and allocate this individual in our final panel[11].

A concise summary of every step in manipulating annual data can be found in Table 6. An additional step was taken to guarantee all necessary information concerning this panel construction work: observations without PROEB mathematics or Portuguese scores, or student, school, principal, classroom or teacher identification codes, are filtered out. This step characterizes the final database and is described as Student Data V in Table 6. Additional information on this sample, including the share of students present from the original PROEB scores database, is also presented.

The last column in Table 6 presents information on the stacked annual databases, and represents our final student panel. In the second panel, we can see that there are 639,863 identified students across 2294 public schools in the Minas Gerais state. This is the panel considered in Chapter 6 for sample delimitation following the identification strategies there enunciated.

In this final dataset, we manage to link control variables for students and teachers. For students, we have demographic variables (gender and race), socioeconomic variables (mother and father education variables, *Bolsa Família*[12] program participation, household amenities, school reproval history and previous private school attendance history), and peer variables ("leave-me-out" peer variables constructed from the other two by analyzing students' peers in their classrooms). For teachers, we obtain demographic variables (gender, race, age and education history). All these variables are described in Appendix Table B.2.

[10] This is a common practice in many private schools for this schooling stage, seeing as students are apt to take university entrance exams at the end of the school year. An example of this would be dividing Portuguese into two separate fronts: grammar and literature.

[11] To ensure our results are robust to teacher allocation, we conduct a total of five draws and discuss our full value-added model results at the end of Chapter 7.

[12] *Bolsa Família* is a cash transfer program that attends vulnerable families across Brazil. Family eligibility status is tied to school attendance.

Table 6 – Annual Student Databases Construction Steps

Construction Steps	2015 (1)	2016 (2)	2017 (3)	2018 (4)	2019 (5)	All Years (6)
PROEB Scores Data	145,327	144,798	143,951	152,326	175,474	761,876
Student Data I	138,992	144,798	143,951	152,326	175,474	755,541
Student Data II	133,995	140,199	139,943	146,161	143,694	703,992
Student Data III	125,778	129,492	130,942	137,111	132,609	655,878
Student Data IV	123,555	128,971	130,246	137,000	132,343	652,115
Student Data V	123,114	128,508	129,808	127,050	131,383	639,863
Total students (%)	84.71	88.75	90.17	83.41	74.87	83.98
Schools	2025	2043	2064	2111	2106	2294
Principals	2012	2031	2053	2099	2093	3112
Classrooms	5214	5136	5453	5782	5461	27,045
Math Teachers	3081	2958	3060	3229	3156	7863
Portuguese Teachers	3149	3014	3139	3314	3201	8629

Notes: Columns (1) through (5) present merge statistics for years 2015 through 2019, respectively, whilst column (6) contains data with respect to the stacked panel. Database names refer to those created after every manipulation step. An additional step is taken here in order to guarantee all necessary information to link students' PROEB scores to their school, principal, classroom and teacher allocations are filled. The results from this extra step are called Student Data V for clarification purposes.

5.3 Management Practices Instrument Statistics

The data on management practice adoption comes from a school management practice mapping undertaken in 2019 in Minas Gerais. In Chapter 3, we briefly explained the data collection methodology employed by Borges et al. (2023) in an older version of this management practice instrument. The principal interviews and management practice markings processes are the same, but instrument interpretation is different.[13] In our instrument, 1038 high school principals in Minas Gerais were interviewed, their answers recorded and transcribed according to a management practice instrument. This instrument was divided into 14 management domains, each composed of 10 practices mapped. We have data on these practice markings for all domains. Table 7 presents some descriptive statistics on these management practice domains.

The first line in Table 7 shows the average number of practices adopted in each domain for all schools in the sample. We see that domains 10 (Worker evaluation) and 8 (School targets) have the lowest average number of practices adopted in interviewed schools, whilst domains 13 (Financial aspects) and 3 (New teaching practices) have the

[13] Besides the geographical differences (recall the instrument used in Borges et al. (2023) was applied in Espírito Santo and Pará), the older version of the instrument was based on a 5-point scale covering 13 domains of management. As will be explained in this section, our instrument covers 14 domains with a maximum of 10 points each, apart from other more nuanced information on practice adoption probability.

Table 7 – Management Practice Instrument Statistics

Management Practice Domain	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Average Domain Score	5.03	5.19	5.62	5.08	4.26	4.21	5.14	3.05	3.29	2.20	3.44	4.42	5.70	4.52
Number of Commonplace Practices	2	4	5	3	2	2	4	1	1	1	1	0	5	3
Number of Occasional Practices	3	1	0	2	2	2	1	2	2	1	2	4	0	1
Number of Rare Practices	5	5	5	5	6	6	5	7	7	8	7	6	5	6

Notes: The table presents descriptive statistics for the management practice instrument, based on recorded interviews of 1.038 high school principals from Minas Gerais.

highest[14].

The instrument also delves deeper than management practice marking and provides analytical tools to study management journeys. Journeys can be understood as practices often adopted jointly, and their delimitation depends on underlying practice adoption probabilities. These adoption probabilities are calculated using information from all interviews, and an adoption score is assigned to each practice in every domain. This adoption score has a very practical interpretation: the number of practices that must already have been adopted so that the practice at hand has a 0.66 probability of being adopted next in that domain.[15] By calculating these adoption scores for all practices in every domain, the instrument allows for a more refined analysis of practice adoption.

Practices are classified into three journey categories based on their adoption score and the average number of adopted practices per domain. If a practice's adoption score is lower than the average number of practices adopted in the domain, then said practice is deemed commonplace. The difference between the average number of practices and the number of commonplace practices in each domain defines occasional practices. Practices not deemed commonplace or occasional are classified as rare.[16] The number of commonplace, occasional and rare practices in each domain is shown in Table 7.

[14] Please refer to Figure 2 for information on management domains.

[15] Take the case of domain 09 (Leadership) as an example. In this domain, the management practice with the lowest associated adoption score is "Administrator is aware of actions developed by the leadership" (which is referenced as practice 01 in domain 09 because of this score ranking). The score associated with this practice is 1.70, which means that when a school already has 1.70 practices adopted, there is a 66% chance that practice 01 is the one being adopted next.

[16] Continuing with the previous example from domain 09 (Leadership), practice 01 has an adoption score (1.70) lower than the average number of adopted practices in domain 09 (3.29). Therefore, practice 01 in domain 09 is deemed commonplace. The practice with the second lowest adoption score in this domain (practice 02) has a score of 4.45. Since this score is higher than 3.29, it is not deemed commonplace, and domain 09 only has one commonplace practice. Since, on average, three practices are adopted in domain 09, and only one is considered commonplace, as just explained, then the next two practices with the lowest adoption scores (practices 02 and 03) are deemed occasional. All other seven practices in domain 09 are classified as rare.

6 Empirical Strategies

We propose two kinds of models to study principal effects on student achievement. In the first section, we detail the empirical specifications and identifying assumptions used in the parametric value-added models. As discussed in Chapter 4, we use three different approaches to try to distinguish between principal, school and teacher effects, seeking to better capture the true share of student achievement that can be attributed to principals. In the second section, we turn to our semi-parametric approaches, where the focus becomes the within-school variation in principal and teacher effects. This takes a step back and aims at understanding and quantifying the effect of principal changes on student achievement. Finally, the last section details the association we make with management practice scores as measured by the instrument detailed in Chapter 3.

6.1 Value-Added Models

We implement a class of value-added models that seek to isolate the specific chunk of student achievement that can be attributed to school principals, as discussed in Chapter 4. This is experimented with in the form of three different fixed effects models, each aiming at making different distinctions between principal, school and teacher effects. These estimated fixed effects are precisely the value-added measures we aim to estimate. Put in other words, the principal value-added we seek is the time-invariant effect principals have in student achievement that is unrelated to time-invariant school and teacher composition characteristics.

All models rely crucially on variation in the form of staff changes. A change in principal positions allows us to observe schools under (at least) two different administrations. Similarly, changes in the teacher compositions permit analysis of school outcomes under distinct conditions. By exploring these changes in school actors, we employ three fixed effects models. Since our main interest is in understanding the influence principals have on student achievement, we first regress model specifications with principal fixed effects considering school and teacher composition fixed effects separately, and lastly, a model aggregating principal, school and teacher composition fixed effects.

6.1.1 School-Principal Model

Our first model implements principal and school fixed effects, making use of principal changes in Minas Gerais schools from 2015 to 2019. Equation (6.1) defines the model regression:

$$A_{ijcpst} = X_{ict}\beta_1 + S_{st}\beta_2 + J_{jst}\beta_3 + \theta_p + \delta_s + \eta_t + \upsilon_{ijcpst}, \tag{6.1}$$

where A is the achievement of student i in classroom c, taught by teacher composition j, and in school s, with principal p in charge in period t. X is a set of student control variables, S is a set of school control variables and J is a set of teacher control variables.[1] Our terms of interest are the principal fixed effects, θ_p, and the school fixed effect, δ_s. We also add a period fixed effect, η_t, to our specification. Students' PROEB scores are used as achievement measures, with regressions for mathematics and Portuguese estimated separately.[2]

Our identification strategy for these sets of principal and school fixed effects requires principal changes in school management, meaning we can estimate equation (6.1) only in schools that underwent such change in administration. Table 8 depicts the number of principal changes in the schools in our sample. We observe different movement characteristics, with few principal transitions between schools and the bulk of movement characterized by principal substitutions in schools, with old tenures ending and new tenures beginning. This is in stark contrast with the principal value-added literature in other countries, where school systems with explicit principal rotation policies were studied to ensure principal transitions. In this research, we use a broader definition of principal changes to estimate our models.

Table 8 – Principal Tenure Movements

Principals	3112
Principals with Tenure Start/End	1507
Principals with School Transitions	28
Principals with Simultaneous Tenures	148
Overall Principal Changes	1683
Principals with Multiple Tenures	580
Average Number of Tenures	1.11
Average Total Tenure Years	11.00

Notes: Principals who start or end their tenure during the 2015-2019 period are taken as (one of the) conditions to delimit samples 1 and 3. Neither principals with longitudinal movement between schools (principal transitions) nor with simultaneous tenures are excluded from the panel or samples.

These substitutions characterize our sample for the School-Principal model. Table 9 presents statistics for what we call Sample 1. Of the 2294 schools in our panel, only 848

[1] Control variables are presented in Appendix Table B.2, along with prevalence statistics in the complete student panel and all specified samples.

[2] For clarity, we omit the subscript for the subject and instead clarify to the reader that all equations apply to both mathematics and Portuguese modelling.

experienced at least one change in their principal position. This represents 1683 principals and over 190 thousand students. PROEB mathematics and Portuguese scores are similar to those observed in the full sample. We only include principals with simultaneous tenures in two or more schools if said schools experienced principal transitions, and even so only schools that fulfil this criteria are included, which may not amount to all schools administered by these principals.

Table 9 – Sample 1 (School-Principal) Statistics

	Student Panel (1)	School-Principal (2)	Filtered Sample (3)
Students	639,863	203,080	196,447
Schools	2294	848	846
Principals	3112	1683	1681
Classrooms	27,045	8909	8904
Math Teachers	7863	3037	3034
Math Compositions	7263	2698	2695
Portuguese Teachers	8629	3298	3295
Portuguese Compositions	7608	2808	2805
Average PROEB Math Score	269.6524	267.6180	268.0661
	(51.4150)	(50.4002)	(50.3852)
Average PROEB Portuguese Score	270.7311	268.9578	269.4916
	(49.8920)	(49.4696)	(49.3257)

Notes: Column (2) presents information on Sample 1, with column (3) ensuring only observations with all controls are retained. Column (1) presents the information on the whole student panel for comparison. Sample 1 concerns the school-principal model (Model 1), and is characterized by schools that experience a principal transition. PROEB scores' standard deviations are in parentheses.

6.1.2 Teacher-Principal Model

In our second model, we distance ourselves from what is commonly explored in the principal value-added literature and propose a model that includes both principal and teacher fixed effects. As explained in Chapter 2, much of the research in principal value-added is conducted in countries where teacher hiring, retention and dismissal figure among principals' main tasks, and teacher effects cannot be effectively distinguished from principal effects. We make use of the restrictions to principal people management in Brazil to explore this relationship unavailable in previous research.

The equation we estimate is as follows:

$$A_{ijcpst} = X_{ict}\beta_1 + S_{st}\beta_2 + J_{jst}\beta_3 + \theta_p + \pi_j + \eta_t + \upsilon_{ijcpst}, \tag{6.2}$$

in which π_j is the teacher composition fixed effect term.

Another discerning factor employed in this model is that we pivot from the previous school-based approach into a tenure-based approach. Since our interest here is to separate principal from teacher effects, we explore changes in teacher compositions during a principal's tenure. This means that we depart from the analysis of changes within a school and explore only changes within principal tenures, put another way, we delimit the identifiable sample by looking at principal and teacher composition pairings, selecting only cases in which a principal is paired with more than one teacher composition.[3] Recall that teacher compositions are here defined as the the group of teachers that supervise a subject's class to students in a given grade, with any changes in composition (be it more teachers, fewer teachers or simply different teachers) resulting in a new composition.[4] Table 10 shows composition dynamics in our student panel.

Panel A of Table 10 presents statistics for mathematics teacher compositions, whilst Panel B shows the same statistics for Portuguese teacher compositions. We find a high number of teacher compositions across all years, with over half of them being composed of a single teacher in all years for both mathematics and Portuguese. A result of observing a higher number of Portuguese teachers than math teachers in our panel is that Portuguese compositions are, on average, bigger than math compositions: 1.54 average Portuguese teachers per composition, as opposed to 1.52 math teachers per composition. More Portuguese teachers also means more room for composition changes, and indeed we do observe an average of 3.41 Portuguese compositions per school over the studied period, while only 3.31 math compositions. Given the higher turnover of Portuguese compositions, we observe Portuguese teachers individually for smaller periods, 1.64 year on average, contrasting to 1.72 year for math teachers.[5] We also track the number of composition changes yearly: with 2015 as a baseline, we observed 1531 new math compositions in 2016.[6] We also decompose these composition changes into composition expansions and reductions[7], that is, compositions with more or fewer teachers than before, while also documenting composition changes in which the number of teachers was kept the same.[8]

3 Notice that this implies that there is a teacher composition continuation (or absence of teacher composition change) when a principal transitions between schools in which the group of teachers both follow the principal to a new school and all teach the same grade at the new school. This highlights how this sample delimitation does not take schools into account whatsoever and focuses solely on distinguishing between principal and teacher composition effects. Although quite illustrative, such a case is not present in our data.

4 This notion of teacher compositions draws inspiration from the work of Abowd et al. (1999), and was explained in Chapter 4. We reiterate that this is a more restrictive analysis, but substantially easier to track pairwise with principals when compared to individual teachers.

5 Recall that we observe only teachers supervising 3º EM classes. This high teacher turnover does not (necessarily) mean teachers leave schools, and may simply mean that other schooling years are taught in subsequent years.

6 1531 new compositions out of a total of 2030 is a turnover of over 75%. Both mathematics and Portuguese composition turnovers yearly are around three-quarters of the observed compositions every year.

7 Composition expansions or reductions may also be coupled with teacher substitution. The clearest example is a teacher supervising all classes in a given year, with two different teachers dividing classes the following year.

8 The majority of these are single-teacher compositions in which the lone teacher was substituted for another.

Table 10 – Teacher Compositions Statistics for Mathematics and Portuguese

	2015 (1)	2016 (2)	2017 (3)	2018 (4)	2019 (5)	All Years (6)
Panel A: Mathematics						
Math Teachers	3081	2958	3060	3229	3156	7863
Math Compositions	2016	2030	2053	2105	2093	7263
Single-teacher Compositions	1180	1309	1268	1254	1277	3660
Average Teachers per Composition	1.55	1.47	1.51	1.56	1.53	1.52
Average Number of Years Observing Teachers						1.72
Average Compositions per School						3.31
Number of Composition Changes	0	1531	1420	1377	1386	5714
Composition Expansions	0	235	294	322	269	1120
1:1 Composition Change	0	945	891	828	823	3487
Composition Reduction	0	351	235	227	294	1107
Panel B: Portuguese						
Portuguese Teachers	3149	3014	3139	3314	3201	8629
Portuguese Compositions	2022	2035	2057	2105	2099	7608
Single-teacher Compositions	1155	1295	1250	1227	1271	3852
Average Teachers per Composition	1.57	1.49	1.54	1.58	1.54	1.54
Average Number of Years Observing Teachers						1.64
Average Compositions per School						3.41
Number of Composition Changes	0	1,514	1,498	1,477	1,468	5,957
Composition Expansions	0	233	339	332	259	1163
1:1 Composition Change	0	912	898	901	882	3593
Composition Reduction	0	369	261	244	327	1201

Notes: A composition is defined as the group of teachers who administer a subject to 3º EM classrooms in a school simultaneously. Since teacher allocations are annual, only one composition is allocated to a school every year. Composition changes occur when the body of teachers that administer a subject to 3º EM classrooms change. This may be an expansion, with more teachers than the previous compositions; or a reduction, with fewer teachers. A composition also changes when teachers who are part of it are substituted. This can happen to a composition expansion or reduction, but may also characterize a 1:1 change. Columns (1) through (5) show teacher and composition statistics for each year in our panel, while column (6) presents statistics for the whole panel. Panel A contains information on math compositions, whilst panel B contains information on Portuguese compositions.

These composition changes characterize the Teacher-Principal model to be estimated. We select principal tenures in which changes in both mathematics and Portuguese compositions were observed. Table 11 describes what we call Sample 2.

We observe 1895 schools from the original 2294. This amounts to 82.06% of schools from the student panel, but 91.40% of students[9], meaning that schools on Sample 2 are bigger on average, which is intuitive considering that smaller schools may not have as many teachers or as much teacher turnover within grades. This is underscored by the fact that this selection method includes 87.63% of original math teachers, but only 84.45% of math compositions (similarly, 86.27% of Portuguese teachers, but 83.08% of Portuguese compositions from the student panel).

[9] Here we consider the student count from column (2) of Table 11, that does not consider the filter we apply for control variables but serves us for comparison purposes, since there is no variation in the number of schools between columns (2) and (3).

Table 11 – Sample 2 (Teacher-Principal) Statistics

	Student Panel (1)	School-Principal (2)	Filtered Sample (3)
Students	639,863	540,852	523,888
Schools	2294	1841	1841
Principals	3112	1895	1895
Classrooms	27045	22575	22573
Math Teachers	7863	6861	6860
Math Compositions	7263	6134	6133
Portuguese Teachers	8629	7445	7444
Portuguese Compositions	7608	6321	6320
Average PROEB Math Score	269.6524	269.8603	270.2740
	(51.4150)	(51.6763)	(51.6596)
Average PROEB Portuguese Score	270.7311	270.9095	271.4228
	(49.8920)	(50.0836)	(49.9426)

Notes: Notes: Column (2) presents information on Sample 2, with column (3) ensuring only observations with all controls are retained. Column (1) presents the information on the whole student panel for comparison. Sample 2 concerns the teacher-principal model (Model 2). PROEB scores' standard deviations are in parentheses.

6.1.3 School-Teacher-Principal Model

Our final value-added model is the synthesis of both previous models. We incorporate principal, teacher composition and school fixed effects into our model specification, defined in equation (6.3).

$$A_{ijcpst} = X_{ict}\beta_1 + S_{st}\beta_2 + J_{jst}\beta_3 + \theta_p + \pi_j + \delta_s + \eta_t + \upsilon_{ijcpst}. \tag{6.3}$$

The identifying assumptions for the two previous models are concatenated for the identification of the fixed effects in equation (6.3). This means that we first select schools that experienced a principal change over the 2015-2019 period, and then further restrict our analysis to principals that underwent teacher composition changes in both mathematics and Portuguese during their tenure. Table 12 presents descriptive statistics for what we call Sample 3.

Only 628 schools figure in our more restricted sample, with just under 130 thousand students. Both the average PROEB scores for mathematics and Portuguese and their variance are similar to those observed in the full panel.

6.1.4 Sample Comparisons

In the end, our value-added approach proposes three different models, rendering three different estimation samples originating from the student panel constructed. The distribution of municipalities with at least one school per sample can be seen in Figure 3. Table 13 presents a comparison between these different (filtered) samples, as well as

Table 12 – Sample 3 (School-Teacher-Principal) Statistics

	Student Panel (1)	School-Principal (2)	Filtered Sample (3)
Students	639,863	137,179	133,074
Schools	2294	628	628
Principals	3112	689	689
Classrooms	27045	5894	5894
Math Teachers	7863	2181	2181
Math Compositions	7263	1849	1849
Portuguese Teachers	8629	2382	2382
Portuguese Compositions	7608	1902	1902
Average PROEB Math Score	269.6524	267.3090	267.7094
	(51.4150)	(50.6171)	(50.6069)
Average PROEB Portuguese Score	270.7311	268.9381	269.4591
	(49.8920)	(49.7669)	(49.6344)

Notes: Notes: Column (2) presents information on Sample 3, with column (3) ensuring only observations with all controls are retained. Column (1) presents the information on the whole student panel for comparison. Sample 3 concerns the school-teacher-principal model (Model 3). PROEB scores' standard deviation are in parentheses.

comparisons to their respective complementaries. Panel A compares sample compositions, with percentages referenced to column (0) in parenthesis; Panel B presents some basic principal tenure statistics; and Panel C shows PROEB scores average and standard deviation (in parenthesis).

We caution readers on the interpretation of some of these variables. Since Samples 1 and 2 are defined differently, we have an unequal division of factors between the samples and their respective counterparts depending on the line being analyzed. For example, in our School-Principal model, sample delimitation is based on school-level changes; this means that the school percentages in Columns (1a) and (1b) sum to 100%, but teacher percentages do not. The opposite is true for information on Sample 2 in Columns (2a) and (2b): school percentages do not add up to 100%, but teacher percentages do because sample delimitation is based on teacher-level changes. Notice that principal and classroom percentages, as well as student percentages, always total 100%.

In Panel B, there is a significant difference between the average total number of tenure years in Samples 1 and 3. This is a direct consequence of our identification strategy for their respective models, in which we require principal changes at the school level, meaning these samples' complementaries do not face tenure interruptions and have higher average tenure year totals.

Figure 3 – Municipalities with school representation in model samples

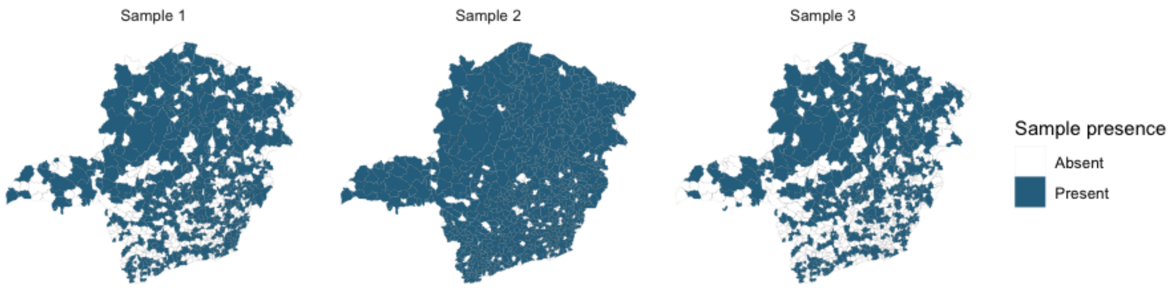


Table 13 – Sample Comparison Statistics

	Student Panel		Sample 1		Sample 2		Sample 3	
		(0)	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)
Panel A: School Statistics								
Students	639,893	619,535	196,447	422,939	523,888	95,647	133,074	486,461
		(100)	(31.733)	(68.267)	(84.561)	(15.439)	(21.480)	(78.520)
Schools	2294	2289	846	1443	1841	979	628	2192
		(100)	(36.959)	(63.041)	(80.428)	(42.770)	(27.436)	(95.762)
Principals	3112	3107	1681	1441	1895	1226	689	2429
		(100)	(54.104)	(45.896)	(60.991)	(39.459)	(22.176)	(78.178)
Classrooms	27,045	27,035	8904	18131	22,573	4462	5894	21141
		(100)	(32.935)	(67.065)	(83.495)	(16.505)	(21.801)	(78.199)
Math Teachers	7863	7858	3034	5357	6860	1707	2181	6508
		(100)	(38.610)	(61.390)	(87.300)	(21.723)	(27.755)	(82.820)
Math Compositions	7263	7257	2695	4698	6134	1386	1849	5681
		(100)	(34.137)	(62.863)	(84.512)	(19.099)	(25.479)	(78.283)
Portuguese Teachers	8629	8622	3295	5763	7444	1885	2382	7042
		(100)	(38.216)	(61.784)	(86.337)	(21.863)	(27.627)	(81.675)
Portuguese Compositions	7608	7601	2805	4878	6320	1522	1902	5938
		(100)	(36.903)	(63.097)	(83.147)	(20.024)	(25.023)	(78.121)
Panel B: Principal Tenure Statistics								
Average Number of Tenure Spells	1.011	1.011	1.002	1.010	1.009	1.004	1.009	1.003
Average Total Tenure Years	10.37	10.38	7.24	14.40	11.47	8.70	6.74	11.41
Average Number of Tenures per School	1.37	1.37	2.00	1.00	1.26	1.03	1.11	1.10
Panel C: PROEB Statistics								
Average Math PROEB Score	269.6524	270.0754	268.4032	270.5983	270.6120	268.5171	267.7094	270.7227
	(51.4151)	(51.3960)	(50.4319)	(51.8530)	(51.7059)	(49.9491)	(50.6069)	(51.5909)
Average Portuguese PROEB Score	270.7311	271.2481	268.8472	271.5559	271.7746	269.7566	269.4591	271.7375
	(49.8921)	(49.7470)	(49.3152)	(50.0659)	(49.9049)	(48.8211)	(46.6344)	(49.7666)

Notes: Column (0) presents information for the filtered student panel, whereas columns (1) through (3) show the same information for the three samples used in this research. The first, unnumbered column presents information on the whole student panel, as first described in Table 6. Columns labelled with an "a" present information regarding the actual samples, whilst columns labelled with a "b" present information on the sample's complementary observations. Panel A contains statistics on school distribution, whereas panels B and C contain information on principal tenure and PROEB scores, respectively. In panel A, percentages are in parentheses and referenced in column (0). Notice not all proportions need sum 100% in all lines, seeing as it depends on how each sample (and its complementary, consequently) was delimited (for example, a school can be allocated in sample 2 for a principal's tenure, but allocated in it's complementary during the following principal's tenure). In panel C, PROEB scores standard deviation is in parentheses.

6.2 Within School Variance Model

The other class of models we study in this research is semi-parametric, focused on understanding the within-school variance of some of the effects analyzed in the value-added models just presented. As explained in length in Chapter 4, we look at how principal and teacher turnover impact student average achievement. In this sense, two models are proposed. The first draws directly from Coelli & Green (2012) and looks exclusively at principal turnover by estimating the components of equation (4.11). The second model incorporates our proposed extension to include teacher effects in the within-school variation analysis and estimates the model described in equation (4.32). In this section, we deal with how to estimate these two models empirically, discussing the necessary identification hypothesis.

6.2.1 Within School Variance of Principal Effects

We begin by copying the empirical specification proposed by Coelli & Green (2012) to estimate the effect of principal variation in student outcomes. This substitutes the term in equation (4.12) into equation (4.11)

$$E\left[\frac{1}{T}\sum_{t=1}^{T}(\bar{A}_{st}-\bar{A}_{s})^2\right] = \sigma^2_{\theta_s}\left[\frac{1}{T}\sum_{t=1}^{T}q_p\left(1+\frac{1}{T^2}\sum_{k=1}^{J}q_k^2-\frac{2}{T}q_p\right)\right]+\frac{1}{\bar{N}_s}+\sigma^2_{\upsilon}. \tag{6.4}$$

Two identifying assumptions are made in this equation, besides the already discussed construction of the principal turnover parameter. Firstly, the within-school variance in average student quality, $\sigma^2_{\bar{\gamma}_s}$, is taken to be proportional to the inverse of the average number of students enrolled in the school, or the inverse of $\bar{N}_s$. The intuition behind this is simple: if the aim is to measure variation in students' individual qualities, the year-to-year variation will be bigger in schools with fewer students. This requires the further assumption that the variance in student quality at the individual level is the same in all schools.

A crucial identifying assumption to estimate equation (6.4) is that the covariance between principal quality and changes in cohort ability level is zero, that is, $\sigma_{\bar{\gamma}_s\theta_{st}} = 0$. This means that students do not change schools to get specific principals. However, this does not mean students cannot change to continually better schools, but simply that it does not occur based on principal effects.

6.2.2 Within School Variance of Principal Effects and Average Teacher Effects

The second model we propose incorporates teacher effects into the analysis. In this semi-parametric approach, we are able to use individual teacher variation, instead of the teacher compositions employed in the value-added models, because we do not need to track teacher turnover during principal tenures (which complicated our analysis when more than one teacher taught students at a school in a given year), but simply teacher turnovers at the school. It draws from equation (4.32).

$$
\begin{aligned}
E\left[\frac{1}{T}\sum_{t=1}^{T}(\bar{A}_{st}-\bar{A}_s)^2\right] =& \sigma^2_{\theta_s}\left[\frac{1}{T}\sum_{p=1}^{P} q_p\left(1+\frac{1}{T^2}\sum_{k=1}^{P} q_k^2-\frac{2}{T}q_p\right)\right]+\\
&+\sigma^2_{\bar{\pi}_s}\left[\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2+\frac{1}{T^2}\left(\sum_{t=1}^{T}\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2+2\sum_{c=1}^{C}\sum_{t=1}^{T}\sum_{u\neq t}\frac{n_{ct}}{N_t}\frac{n_{cu}}{N_u}\right)-\dots\right]\\
&\left[\dots-\frac{2}{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\left(\sum_{v=1}^{T}\frac{n_{cv}}{N_v}\right)\right]+\frac{1}{\bar{N}_s}+\sigma^2_{\upsilon},
\end{aligned}
\tag{6.5}
$$

in which, as explained in Chapter 4, while the principal term is based on principal turnover alone, the teacher term is based both on teacher turnover and the proportion of students in school s taught by the teacher every year, $\frac{n_{ct}}{N_t}$.

Here, we again make use of both assumptions discussed for the specification in equation (6.4). Nevertheless, additional identifying assumptions to deal with the other two covariance terms in equation (4.32) must be made.

As highlighted in Chapter 3, public school teachers are hired through public tender, a process outside the principal's control. This means that principals do not choose the body of teachers available at the school, which is instead formed out of teacher-led entrance and changes. Such institutional context pivots the assumption made here: that the covariance between principal effects and average principal quality ($\sigma_{\bar{\pi}_s\theta_{st}}$) is zero. This means that there is no systematic sorting of teacher-principal allocations. This is backed by the lack of authority Brazilian principals have in teacher hiring and retention when compared to those in the USA or Europe, which are more contemplated in the literature. What we assume here is that teachers do not choose the school they will work at based on the school's principal and instead focus on other employment characteristics.[10] This is further underscored by the practice of principal elections in Minas Gerais state public schools, as presented in Chapter 3, which creates additional obstacles for principal-teacher sorting among schools.

The trickiest assumption to be made is related to the covariance between average student quality and average teacher quality ($\sigma_{\bar{\gamma}_s\bar{\pi}_s}$). Essentially, what we assume is that the is no systematic sorting of students among public schools based on teacher quality. This isn't equivalent to saying that there is no student sorting in the Brazilian education system, but only that it does not occur among schools in the state public school system.

[10] Relevant characteristics may be correlated with school positions in big cities, with more financial and cultural resources available, as well as higher living and working conditions, as discussed in Chapter 4. Loeb et al. (2010) find that many schools in the USA face high turnover rates due to principals' desire to move, generally not conditioned on pay increases, but rather non-pecuniary benefits. Although there is an interesting space for a big compensating differentials debate between urban and rural (and even intra-city) public schools in Brazil, it is not the objective of this research.

As detailed in Chapter 3, there are both (middle-class and elite) private schools and public technical schools, with heavier workloads. Both of these, private and public technical schools, are the prime target of student movement because they better prepare pupils for university entrance exams. These options, however, do have entry costs, be they tuition in private schools or a disputed entrance exam for technical public schools. A big part of students who may wish to move out of the normal public school system is not able to do so. Our dataset excludes both of these types of schools, considering only normal public schools, and we assume there is no student sorting among these normal public schools. This is a much more reasonable assumption to make than no sorting in all schools, and it still implies zero covariance for necessary identification.

6.3 Management Practice Adoption Association

The last step in our analysis is the association between the estimated principal value-added and the management practice instrument. Both the theoretical and empirical strategies for estimating principal value-added were explained in Chapter 4 and in the first section of Chapter 6, respectively. The estimated principal fixed effects coefficients serve as our value-added measures, and we associate them with the management practice instrument for their respective school. This association is done as shown by equation (6.6).

$$\hat{\theta}_{ps} = \sum_{d=1}^{14} M_{ds}\beta_d + P_p + \varepsilon_{ps}, \tag{6.6}$$

where $\hat{\theta}_{ps}$ is the estimated principal value-added for principal p in school s. M is the management score for domain d in school s where principal p was in charge in 2019, whilst P is a set of principal controls from the SIMAVE surveys. Lastly, ε is the error term. We investigate the relationship between all 14 management domains and principal value-added.

As explained in the first section of this Chapter, we make use of leadership changes in schools to estimate principal value-added. Since the management practice mapping and instrument construction by Henriques et al. (2020) was conducted in 2019, we consider only principals active in a school that participated in the management practice survey in 2019. Table 14 summarizes the intersection between our samples.

Only a fraction of principals in our value-added samples participated in the management practice mapping. Column (3) shows our more restricted sample, used to estimate principal, school and teacher composition value-added, thus requiring both principal changes in schools and teacher composition (both math and Portuguese) in principal tenures. In this sample, only 203 principals were interviewed, and only 139 also had registered SIMAVE questionnaire answers, which supplied us with some control variables.

Table 14 – Management Practice Instrument Sample

Samples	Student Panel (0)	Sample 1 (1)	Sample 2 (2)	Sample 3 (3)
Principals	3112	1681	1895	628
Principals in Management Practice Instrument	1038	381	743	203
Principals in Instrument with SIMAVE Questionnaire	748	262	381	139
Average Principal VA to Mathematics	—	271.3033	273.8760	-0.1272
Average Principal VA to Portuguese	—	259.9220	246.2773	0.0069

Notes: Column (1) references the sample shown in Table 9. Column (2) references the sample shown in Table 11. Column (3) references the sample shown in Table 12. Average mathematics and Portuguese principal value-added data correspond to the results of our preferred specification in each model.

7 Results

In this Chapter, we present the results for the different proposed models. We begin with the three value-added models, showcasing results obtained for each approach in the first section. In the second section, we present results for the within-school variance model, both with and without the proposed teacher extension. Lastly, we show results for the association of management practices to the principal value-added measure estimated in the first section.

7.1 Principal Value-Added Models

7.1.1 School-Principal Model

We begin by showing results for Model 1 (School-Principal), enunciated in equation (6.1). Table 15 has principal fixed effects (value-added) measures estimate statistics for influence on student achievement in mathematics and Portuguese. We present results for several specifications, starting without any controls and estimating separately on our full Sample 1 and our filtered Sample 1; then incrementally include controls on students' demographic and socioeconomic characteristics, peer characteristics, teacher characteristics and school characteristics.[1]

Table 15 – Model 1 standardized value-added estimates distribution statistics

	Mathematics						Portuguese					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Average Principal Value-Added	5.2998	5.3117	5.0328	5.0861	5.0941	4.9638	5.4402	5.4673	4.9330	5.0355	5.0340	5.0089
Principal Value-Added Standard Deviation	0.3695	0.3704	0.3371	0.3358	0.3340	0.3345	0.3386	0.3382	0.2977	0.2954	0.2956	0.3004
Max Principal VA	7.0649	7.1457	7.1512	7.1272	7.1296	7.0225	6.5149	6.5475	6.0230	5.9861	5.9733	5.9605
90^{th} Quantile	5.7557	5.7685	5.4584	5.5035	5.5126	5.3607	5.8709	5.8972	5.2967	5.3972	5.3981	5.3728
75^{th} Quantile	5.5313	5.5449	5.2347	5.2838	5.2906	5.1711	5.6645	5.6878	5.1241	5.2263	5.2205	5.2012
50^{th} Quantile	5.2811	5.2949	5.0126	5.0658	5.0726	4.9492	5.4549	5.4834	4.9448	5.0429	5.0423	5.0170
25^{th} Quantile	5.0511	5.0698	4.8134	4.8695	4.8754	4.7460	5.2155	5.2470	4.7422	4.8525	4.8489	4.8202
10^{th} Quantile	4.8435	4.8567	4.6342	4.6924	4.7037	4.5555	5.0123	5.0375	4.5718	4.6787	4.6780	4.6446
Min Principal VA	4.0255	4.0354	3.8259	3.9833	4.0064	3.7664	3.6577	3.6796	3.3038	3.4312	3.4058	3.2308
Observations	203,080	196,447	196,447	196,447	196,447	196,447	203.080	196,447	196,447	196,447	196,447	196,447
Number of Principals	1683	1681	1681	1681	1681	1681	1683	1681	1681	1681	1681	1681
Number of Principal VA estimated	1683	1681	1681	1681	1681	1681	1683	1681	1681	1681	1681	1681
Number of Schools	848	846	846	846	846	846	848	846	846	846	846	846
Number of School VA estimated	17	17	17	17	17	17	17	17	17	17	17	17
Adjusted R^2	0.1010	0.1008	0.1730	0.1731	0.1733	0.1735	0.0842	0.0830	0.1811	0.1815	0.1817	0.1819
Average PROEB Score	267.6180	268.0661	268.0661	268.0661	268.0661	268.0661	268.9572	269.4916	269.4916	269.4916	269.4916	269.4916
PROEB Score Standard Deviation	50.4002	50.3852	50.3852	50.3852	50.3852	50.3852	49.4696	49.3257	49.3257	49.3257	49.3257	49.3257
Student Controls	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Peer Controls	No	No	No	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes
School Controls	No	No	No	No	No	Yes	No	No	No	No	No	Yes
Teacher Controls	No	No	No	No	Yes	Yes	No	No	No	No	Yes	Yes

Notes: The table presents information on the distribution of standardized fixed effects (FE) estimates, our value-added measure, for specifications used in Model 1 (School-Principal), estimated on Sample 1. Columns (1) through (6) show estimates for mathematics, whilst columns (7) through (12) show estimates for Portuguese. Columns (1) and (7) present FE estimates for the specification without controls on the unfiltered sample. Columns (2) and (8) present FE estimates for the same specification, but on the filtered sample (to ensure comparison with controlled specifications). Columns (3) and (9) present FE estimates for the specification with student controls. Columns (4) and (10) present FE estimates for the specification with student and peer (leave-me-out) controls. Columns (5) and (11) present FE estimates for the specification with student, peer and teacher controls. Finally, columns (6) and (12) present FE estimates for our full and preferred specification, with all controls: student, peer, teacher and school. All value-added measures presented have been standardized based on the sample's PROEB score standard deviation for mathematics and statistics. For information on non-standardized estimates, please refer to Table B.3.

[1] For brevity, we present only the principal value-added estimates distribution statistics. For regression control coefficients concerning the specifications shown in Table 15, please refer to Appendix Table B.4.

Columns (6) and (12) of Table 15 present principal value-added estimates for our preferred specifications for mathematics and Portuguese, respectively. Notice value-added estimates are standardized according to PROEB score standard deviation for mathematics and Portuguese, respectively.[2] The average principal value-added to student achievement is 4.96 standard deviations in mathematics and 5.01 in Portuguese. This amounts to 250 and 247 points, respectively.[3] From these results, it is clear that we were not able to adequately isolate and estimate principals' contribution to student achievement. This is all the more evident when paying attention to the fact that, even though all 1681 principals had their fixed effects estimated, the same could only be said for 17 out of 846 schools.

The results in Table 15 spell a problem we also observe in our two other parametric value-added approaches: the inability to properly isolate principal impact on student achievement using the data available to us. This is a combination of two major flaws that our first design could not accommodate.[4] First, the absence of student longitudinal data disavows proper value-added analysis, meaning we do not have a basis for comparison of achievement growth or development over the analyzed period. Instead, we rely solely on single observations of students, which transforms our analysis into a decomposition exercise among the various factors included, resulting in glaringly big estimates of principal influence. A second problem is the inability to properly disentangle principal from school effects, leading to the exclusion of the majority of school fixed effects from our model, effectively concatenating both effects into the principal estimates. This arises from the absence of principal transitions between schools in our data, meaning principal (and school) comparison groups are much smaller, mostly composed of a single school and two principals. Since estimation requires a factor to be left out, and we prioritize principal fixed effect estimation, we see most school fixed effects excluded from analysis, leaving principal fixed effects to capture both factors' influence.

However, this does not render our parametric study unfruitful. Around 10% of variation in mathematics achievement, and 8% in Portuguese achievement, are explained by a simple regression with principal and school fixed effects, jumping to 17% and 18%, respectively, after controlling for various time-varying characteristics. By decomposing student achievement over the five years and exploring principal changes, our model design still captures school improvement in a state-wide exam. Using a standardized exam means the average is constructed to be the same across observed years but allows for school averages to move up or down, and these are subject to principal influence. This and the problems described in this approach apply also to the two other value-added approaches that we present next.

[2] An identical exposition with non-standardized results for Model 1 can be found in Appendix Table B.3.

[3] Recall that PROEB is an IRT exam consisting of 500 points for each subject, of which the average and standard deviation are constructed to be around 250 and 50, respectively.

[4] These limitations, along with other challenges present in this research, are discussed more thoroughly in Chapter 8.

7.1.2 Teacher-Principal Model

Our second value-added approach is Model 2 (Teacher-Principal), as spelt out in equation (6.2). This is the more unconventional analysis that focuses solely on teacher-body changes during principal administrations and serves as a parsimonious way to introduce the variations analyzed in the last approach. Table 16 presents standardized principal value-added estimate statistics for the specifications that follow this model.[5] Again, results are shown for simple, uncontrolled regressions on the full and filtered Sample 2, and controls are added incrementally: student, peer, school and teacher.[6] Columns (6) and (12) are our full and preferred specifications for Model 2.[7]

Table 16 – Model 2 standardized value-added estimates distribution statistics

	Mathematics						Portuguese					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Average Principal Value-Added	5.1819	5.1909	4.9250	4.8618	5.0488	4.9913	5.3736	5.3994	4.8469	5.0121	5.0116	4.8998
Principal Value-Added Standard Deviation	0.4130	0.4151	0.3799	0.3795	0.3790	0.3804	0.3702	0.3701	0.3259	0.3264	0.3267	0.3263
Max Principal VA	7.6812	7.6412	7.4222	7.5086	7.5747	7.5134	6,7771	6.8597	6.4087	6.7424	6.7000	6.6513
90^{th} Quantile	5.6836	5.6980	5.3752	5.2925	5.4901	5.4319	5.8307	5.8549	5.2329	5.3935	5.4004	5.3832
75^{th} Quantile	5.4203	5.4269	5.1382	5.0689	5.2621	5.2050	5.6209	5.6446	5.0553	5.2112	5.2189	5.1965
50^{th} Quantile	5.1686	5.1816	4.9160	4.8499	5.0387	4.9848	5.3829	5.4081	4.8511	5.0176	5.0262	5.0024
25^{th} Quantile	4.9030	4.9119	4.6741	4.6220	4.8057	4.7421	5.1284	5.1551	4.6507	4.8152	4.8136	4.7903
10^{th} Quantile	4.6849	4.6892	4.4711	4.4149	4.6002	4.5361	5.8953	5.9194	4.4309	4.5989	4.6045	4.5861
Min Principal VA	3.7807	3.7817	3.6925	3.5207	3.5647	3.5084	4.1132	4.1607	3.6875	3.7844	3.6786	3.6355
Observations	540.852	523.664	523.664	523.664	523.664	523.664	540.852	523.664	523.664	523.664	523.664	523.664
Number of Principals	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895
Number of Principal VA estimated	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895
Number of Teacher Compositions	6134	6133	6133	6133	6133	6133	6321	6319	6319	6319	6319	6319
Number of Composition VA estimated	4472	4471	4471	4471	4471	4471	4558	4555	4555	4555	4555	4555
Adjusted R^2	0.1182	0.1179	0.1877	0.1878	0.1878	0.1881	0.1017	0.1006	0.1949	0.1951	0.1951	0.1953
Average PROEB Score	269.8603	270.2740	270.2740	270.2740	270.2740	270.2740	270.9095	271.4228	271.4228	271.4228	271.4228	271.4228
PROEB Score Standard Deviation	51.8326	51.6763	51.6763	51.6763	51.6763	51.6763	49.9206	50.0836	50.0836	50.0836	50.0836	50.0836
Student Controls	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Peer Controls	No	No	No	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes
School Controls	No	No	No	No	Yes	Yes	No	No	No	No	Yes	Yes
Teacher Controls	No	No	No	No	No	Yes	No	No	No	No	No	Yes

Notes: The table presents information on the distribution of standardized fixed effects (FE) estimates, our value-added measure, for specifications used in Model 2 (Teacher-Principal), estimated on Sample 2. Columns (1) through (6) show estimates for mathematics, whilst columns (7) through (12) show estimates for Portuguese. Columns (1) and (7) present FE estimates for the specification without controls on the unfiltered sample. Columns (2) and (8) present FE estimates for the same specification, but on the filtered sample (to ensure comparison with controlled specifications). Columns (3) and (9) present FE estimates for the specification with student controls. Columns (4) and (10) present FE estimates for the specification with student and peer (leave-me-out) controls. Columns (5) and (11) present FE estimates for the specification with student, peer and school controls. Finally, columns (6) and (12) present FE estimates for our full and preferred specification, with all controls: student, peer, teacher and school. All value-added measures presented have been standardized based on the sample's PROEB score standard deviation for mathematics and statistics. For information on non-standardized estimates, please refer to Table B.5.

Following this approach, principals have an impact of 4.99 and 4.89 standard deviations on mathematics and Portuguese scores, respectively. This amounts to 257 and 249 points, respectively. These results exhibit the same challenge observed in Model 1 results, with the caveat that, since no school fixed effects were included, excluded effects are teacher composition effects. In our preferred specifications, 4471 out of 6133 math teacher compositions were estimated, and 4555 out of 6319 Portuguese teacher compositions were estimated, with all principals having their fixed effect estimated. We observe a higher rate of teacher composition fixed effects being estimated due not only to the sheer difference in the number of factors but also due to how comparison groups are structured in this approach. Whereas the standard group in Sample 1 was formed out of two principals and a school,

5 Non-standardized results can be found in Appendix Table B.5.

6 Notice the order of controls is inverted here. We opt to add controls that are already represented in fixed effects last. This means we control for school characteristics in Model 1 only in our last and preferred specification, but that same treatment is bestowed upon teacher characteristics in Model 2.

7 For brevity, we present only the principal value-added estimates distribution statistics. For regression control coefficients concerning the specifications shown in Table 16, please refer to Appendix Table B.6.

in Sample 2 groups are formed of at least one principal and two teacher compositions. This group increases according to the number of different teacher compositions, with the extreme case being a different composition per year for all years studied, rendering a group with one principal and five teacher compositions, of which only one is excluded for estimation.

7.1.3 School-Teacher-Principal Model

Our last value-added approach fuses the strategies used in Models 1 and 2 by exploring both principal changes in schools and teacher composition changes in principal tenures. These identifying conditions structure Sample 3, and Model 3 is defined by equation (6.3). Table 17 presents standardized results for this approach for mathematics and Portuguese scores.[8] First, value-added estimate statistics for regressions without controls are shown, both on the full and filtered Sample 3; then, controls are added incrementally,[9] with columns (7) and (14) representing our full and preferred specifications for mathematics and Portuguese, respectively.[10]

Table 17 – Model 3 standardized value-added estimates distribution statistics

	Mathematics							Portuguese						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Average Principal Value-Added	7.26×10^{-4}	1.1×10^{-3}	1.21×10^{-3}	1.11×10^{-3}	1.09×10^{-3}	7.7×10^{-4}	7.71×10^{-4}	-1.05×10^{-3}	-1.07×10^{-3}	-8.4×10^{-4}	-4.92×10^{-4}	-4.32×10^{-4}	-6.38×10^{-4}	-5.63×10^{-4}
Principal Value-Added Standard Deviation	0.0564	0.0615	0.0602	0.0629	0.0639	0.0647	0.0656	0.0473	0.0465	0.0483	0.0482	0.0484	0.0511	0.0518
Max Principal VA	0.7559	1.0320	1.0325	1.0407	1.0524	1.0374	1.0493	0.4213	0.4269	0.4038	0.3932	0.3799	0.3910	0.4027
90^{th} Quantile	1.23×10^{-1}	2.08×10^{-5}	8.84×10^{-6}	8.05×10^{-6}	1.11×10^{-5}	9.39×10^{-6}	9.6×10^{-6}	3.18×10^{-6}	2.86×10^{-6}	2.64×10^{-6}	2.7×10^{-6}	2.82×10^{-6}	2.52×10^{-6}	2.62×10^{-6}
75^{th} Quantile	6.92×10^{-13}	5.57×10^{-13}	6.53×10^{-13}	4.62×10^{-13}	4.71×10^{-13}	5.27×10^{-13}	4.89×10^{-13}	2.44×10^{-13}	2.16×10^{-13}	1.97×10^{-11}	1.80×10^{-13}	1.84×10^{-13}	1.66×10^{-13}	1.98×10^{-11}
50^{th} Quantile	-1.88×10^{-14}	8.47×10^{-15}	8.98×10^{-15}	-4.52×10^{-14}	2.8×10^{-15}	-8.09×10^{-15}	7.97×10^{-15}	2.17×10^{-15}	4.52×10^{-15}	-8.98×10^{-16}	-1.28×10^{-14}	-8.97×10^{-15}	3.99×10^{-15}	3.7×10^{-15}
25^{th} Quantile	-7.04×10^{-13}	-5.98×10^{-13}	-5.63×10^{-13}	-5.25×10^{-13}	-6.36×10^{-12}	-5.34×10^{-13}	-4.55×10^{-13}	-2.24×10^{-13}	-1.86×10^{-13}	-1.77×10^{-13}	-2.33×10^{-13}	-2.41×10^{-13}	-1.39×10^{-11}	-1.59×10^{-13}
10^{th} Quantile	-9.56×10^{-6}	-1.16×10^{-5}	-1.66×10^{-5}	-6.63×10^{-6}	-7.46×10^{-6}	-1.66×10^{-5}	-1.61×10^{-5}	1.21×10^{-6}	-1.75×10^{-6}	-1.42×10^{-6}	-1.53×10^{-6}	1.51×10^{-5}	-1.46×10^{-6}	-1.46×10^{-6}
Min Principal VA	-0.3393	-0.2854	-0.2947	-0.3162	-0.3250	-0.3167	-0.3293	-0.3023	-0.2764	-0.2837	-0.2546	-0.2512	-0.2588	-0.2666
Observations	137.179	132.970	132.970	132.970	132.970	132.970	132.970	137.179	132.970	132.970	132.970	132.970	132.970	132.970
Number of Principals	689	689	689	689	689	689	689	689	689	689	689	689	689	689
Number of Principal VA estimated	689	689	689	689	689	689	689	689	689	689	689	689	689	689
Number of Schools	628	628	628	628	628	628	628	628	628	628	628	628	628	628
Number of School VA estimated	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Number of Teacher Compositions	1849	1849	1849	1849	1849	1849	1849	1902	1901	1901	1901	1901	1901	1901
Number of Composition VA estimates	1213	1213	1213	1213	1213	1213	1213	1241	1240	1240	1240	1240	1240	1240
Adjusted R^2	0.1073	0.1072	0.1789	0.1791	0.1794	0.1793	0.1796	0.0937	0.0919	0.1898	0.1899	0.1904	0.1901	0.1906
Average PROEB Score	267.3090	267.7094	267.7094	267.7094	267.7094	267.7094	267.7094	268.9381	269.4591	269.4591	269.4591	269.4591	269.4591	269.4591
PROEB Score Standard Deviation	50.6171	50.6069	50.6069	50.6069	50.6069	50.6069	50.6069	49.7669	49.6344	49.6344	49.6344	49.6344	49.6344	49.6344
Student Controls	No	No	Yes	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes	Yes
Peer Controls	No	No	No	Yes	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes	Yes
School Controls	No	No	No	No	No	Yes	Yes	No	No	No	No	No	Yes	Yes
Teacher Controls	No	No	No	No	Yes	No	Yes	No	No	No	No	Yes	No	Yes

Notes: The table presents information on the distribution of standardized fixed effects (FE) estimates, our value-added measure, for specifications used in Model 3 (School-Teacher-Principal), estimated on Sample 3. Columns (1) through (7) show estimates for mathematics, whilst columns (8) through (14) show estimates for Portuguese. Columns (1) and (8) present FE estimates for the specification without controls on the unfiltered sample. Columns (2) and (9) present FE estimates for the same specification, but on the filtered sample (to ensure comparison with controlled specifications). Columns (3) and (10) present FE estimates for the specification with student controls. Columns (4) and (11) present FE estimates for the specification with student and peer (leave-me-out) controls. Columns (5) and (12) present FE estimates for the specification with student, peer and teacher controls. Columns (6) and (13) present FE estimates for the specification with student, peer and school controls. Finally, columns (7) and (14) present FE estimates for our full and preferred specification, with all controls: student, peer, teacher and school. All value-added measures presented have been standardized based on the sample's PROEB score standard deviation for mathematics and statistics. For information on non-standardized estimates, please refer to Table B.7.

A glimpse at the value-added estimates distribution reveals a fairly different scenario from those observed in Models 1 and 2. For our preferred specifications, principals have an average influence of 0.00077 and -0.00056 standard deviations for mathematics and Portuguese, respectively.[11] This amounts to 0.0390 and -0.0279 points in PROEB mathematics and Portuguese exams, respectively. The glaring difference in magnitude is, once again, related to the comparison groups formed in this approach. Notice that, of

8 For non-standardized results, please refer to Appendix Table B.7.

9 Note that we show one specification with student and teacher controls and one with student and school controls. The first was shown in Table 18 referring to Model 1 results, whilst the other was presented in Table 18 concerning Model 2 results. Here, we exhibit both since our identification strategy uses those employed in the two previous models.

10 For brevity, we present only the principal value-added estimates distribution statistics. For regression control coefficients concerning the specifications shown in Table 17, please refer to Appendix Table B.8.

11 Beware that the influence of principals in students' Portuguese achievement refers to a negative influence of 0.00056 standard deviations, which we write as -0.00056 for brevity.

the 628 schools in Sample 3, only 2 have fixed effects estimated, with 1213 of the 1849 math compositions and 1240 of the 1901 Portuguese compositions being estimated. As before, school and teacher composition effects are dropped to allow for full principal effect estimation. However, while principal fixed effects captured school or teacher composition effects in previous models, these are now passed along and concentrated mostly on teacher composition effects in this approach. Hence, we can assume our principal fixed effects come closer to reflecting the isolated principal value-added in this approach.Note that the absence of any basis for comparison persists, meaning these values still reflect a decomposition of student achievement among factors (of which principals now concentrate less influence), than a value-added measure per se.

Table 18 – Standardized value-added estimates distribution statistics across models

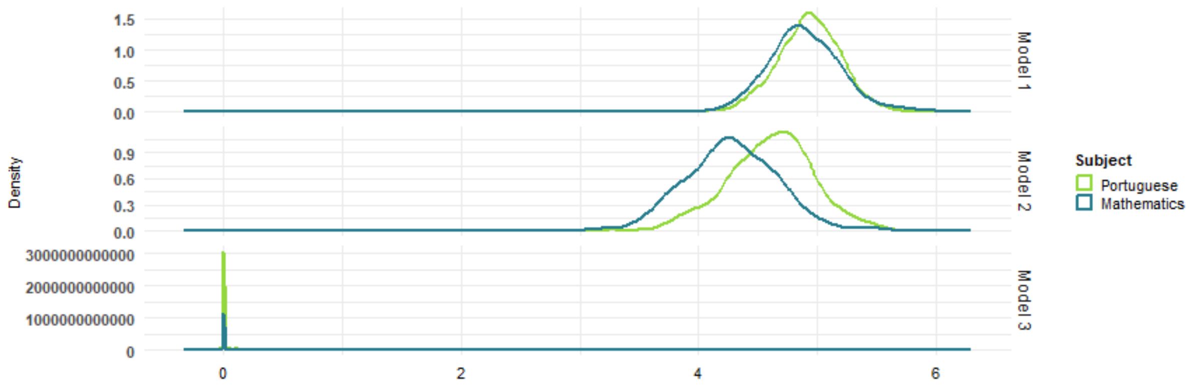
	Mathematics			Portuguese		
	(1a)	(2a)	(3a)	(1b)	(2b)	(3b)
Average Principal Value-Added	4.8905	4.2765	7.71×10^{-4}	4.9279	4.6036	-5.6×10^{-4}
Principal Value-Added Standard Deviation	0.2967	0.3946	0.0656	0.2558	0.3717	0.0518
Max Principal VA	6.0166	6.1300	1.0493	5.6745	6.2971	0.4027
90^{th} Quantile	5.2536	4.7577	9.6×10^{-6}	5.2468	5.0393	2.62×10^{-6}
75^{th} Quantile	5.0747	4.5333	4.9×10^{-13}	5.1038	4.8403	1.98×10^{-11}
50^{th} Quantile	4.8707	4.2748	7.97×10^{-15}	4.9255	4.6308	3.7×10^{-15}
25^{th} Quantile	4.6939	4.0258	-4.55×10^{-13}	4.7627	4.3547	-1.59×10^{-13}
10^{th} Quantile	4.5206	3.7698	-1.6×10^{-5}	4.5937	4.1157	-1.5×10^{-6}
Min Principal VA	4.0889	3.0859	-0.3293	4.0891	3.2561	-0.2666
Observations	132,970	132,970	132,970	132,970	132,970	132,970
Number of Principals	689	689	689	689	689	689
Number of Principal VA estimated	689	689	689	689	689	689
Adjusted R^2	0.1718	0.1835	0.1796	0.1812	0.1945	0.1906
Average PROEB Score	268.0252	268.0252	268.0252	269.8003	269.8003	269.8003
PROEB Score Standard Deviation	50.6595	50.6595	50.6595	46.6139	46.6139	46.6139
Student Controls	Yes	Yes	Yes	Yes	Yes	Yes
Peer Controls	Yes	Yes	Yes	Yes	Yes	Yes
School Controls	Yes	Yes	Yes	Yes	Yes	Yes
Teacher Controls	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The table shows the distribution of principal fixed effects, our value-added measure, for the full specification (all controls) of all three models estimated on Sample 3. Columns "a" refer to mathematics scores, and column "b" refers to Portuguese scores. Column (1) refers to estimates for Model 1 (School-Principal). Column (2) refers to estimates for Model 2 (Teacher-Principal); and column (3) refers to estimates for Model 3 (School-Teacher-Principal). All value-added measures presented have been standardized based on the sample's PROEB score standard deviation for mathematics and statistics. For information on non-standardized estimates, please refer to Table B.9.

These differences between models are evident in Table 18, which presents our full specifications for all models, with Models 1 and 2 restimated in Sample 3, to allow for valid comparisons.[12] Some of the trends observed from Model 1 to Model 2 are not present in Model 3 results: average principal influence was higher in Portuguese, when compared to mathematics, in the two previous models, but is shown to be negative in our third approach. However, the value-added measures' standard deviations remain higher for mathematics scores, with Model 2 having the highest, and Model 3, being the lowest.

[12] In line with previous tables in this chapter, Table 18 presents standardized principal value-added estimates. For non-standardized estimates, please refer to Appendix Table B.9. For control regression coefficients concerning these specifications, please refer to Appendix Table B.10.

Figure 4 – Principal standardized value-added estimates distribution comparison between all models



Model 2 also seems to explain a bit more of the variation in PROEB scores for both subjects, followed by Model 3 and Model 1. This last point highlights the importance of looking beyond school-principal interactions in restrictive institutional settings, with the interaction with teacher bodies being an interesting proposal.

We present a visual comparison of principal value-added estimates distribution in Figure 4 by using a density graph. We caution the reader regarding the different y-axis scale, which was maintained to underscore the difference in estimate distribution between models.[13] Figures 5 and 6 compare principal value-added as estimated on their own samples and on the restricted sample (Sample 3) for Models 1 and 2, respectively. Dashed lines indicate the average principal influence on mathematic and Portuguese scores for each sample estimate. We observe a slight change in average principal value-added in Model 1 reestimation, whilst a big difference in average principal value-added appears in Model 2 reestimation. This is somewhat expected, seeing as the identifying strategy employed in Model 1 (and also in Model 3) is more restrictive than that used in Model 2, as Tables 1 and 10 showed.

7.1.4 Robustness Checks

As mentioned in previous chapters, we conduct several robustness checks. We first investigate if the principal panel transformation from trimestral to annual had any impact by exploring alternative transition filters. We test student panels using different trimesters as reference for the annual panel and evaluate results for Model 1 (Appendix Table B.11), Model 2 (Appendix Table B.12) and Model 3 (Appendix Table B.13). Another evaluation undertaken was if our results were robust to the teacher-to-classroom draw, employed

[13] Recall that in density graphs the area under each curve needs sum 1, which is accomplished in all three graphs. The extreme difference in magnitude in the third graph's y-axis highlights the scale of estimated principal influence.

Figure 5 – Comparison between Model 1 (School-Principal) estimates on Sample 1 and Sample 3

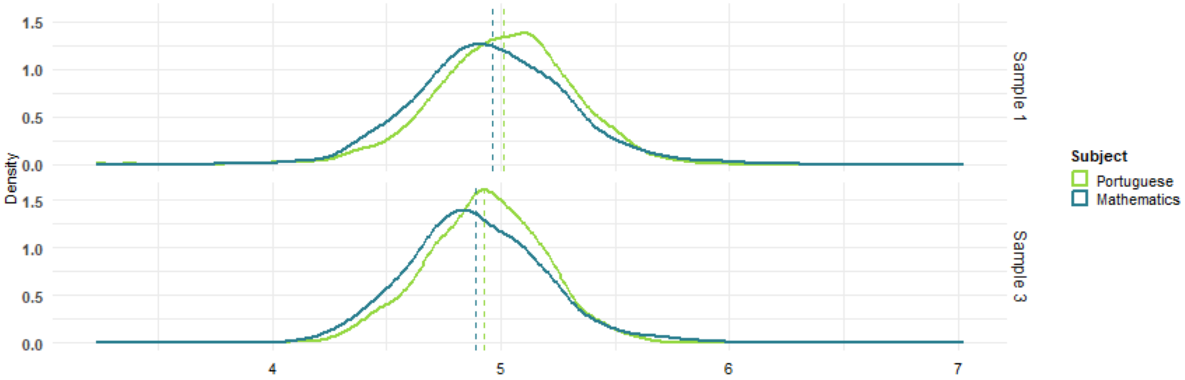
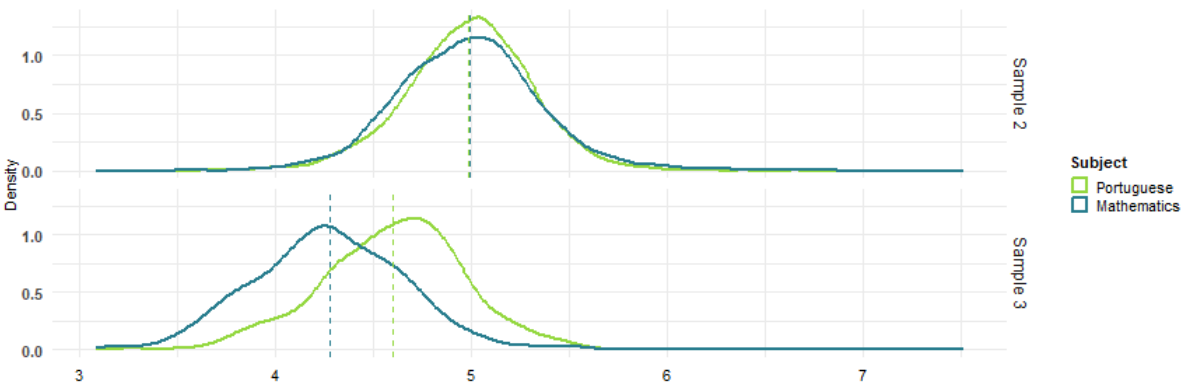


Figure 6 – Comparison between Model 2 (Teacher-Principal) estimates on Sample 2 and Sample 3



to deal with duplicate teachers. We present results for Model 1 (Appendix Table B.14), Model 2 (Appendix Table B.15) and Model 3 (Appendix Table B.16). Results for both these robustness checks are very similar to the ones showcased in our main analysis.

Lastly, we investigate if the overparametrization we encounter in our value-added models can be corrected using a LASSO regression (Least Absolute Shrinkage and Selection Operator). LASSO regressions employ L1-regularization techniques to run regressions in sparse datasets (datasets characterized by many zero inputs) and prevent overfit models, with the main attraction of such method being its feature selection.[14] We find it necessary

[14] LASSO regression is often employed in cases in which the number of features p in bigger than the sample n. To estimate models in this context using normal approaches, econometricians would be required to select which features to include in the model, such that $p' < n$, with $p' \in p$. Much debate can and will be held on the selection criteria used for such features, and this is where LASSO regression holds its appeal. The LASSO technique penalizes feature inclusion in such a way that features can be excluded from the regression entirely, with features that add little information to explain observed variation being more affected by the LASSO penalty. For a review on inference using LASSO regression selection, please refer to Belloni et al. (2014).

to caution the reader that employing a LASSO regression in a fixed effect model is not commonplace and inference in such settings is still an open topic in the literature, with our case being especially sensitive to this approach. Since LASSO estimators penalize variables that contribute little to explaining observed variation, one may find that the fixed effects themselves are excluded from the regression. And since we employ categorical fixed effects (meaning the exclusion of some, but not all fixed effects severely hinders our estimate inference and interpretation) and hold them as our terms of interest, the fixed effect exclusion is fatal to our analysis.[15]

To try and work around these problems the LASSO regression seemingly poses to our models, we use a grouped LASSO (YUAN; LIN, 2006), that essentially considers groups of variables as features, analyzing the grouped contribution to the regression, and excluding the entire group should an exclusion be deemed optimal. In such an approach, we classify each control variable as its own group, and each class of fixed effect as a single group (ie. principal fixed effects are considered a single group). First, we conduct a cross-validation selection for the penalization parameter λ.[16] The selected λ values used in our grouped LASSO are shown in Figures B.1 and B.2 for mathematics and Portuguese, respectively.[17] Appendix Table B.17 presents results for the LASSO regression with feature selection applied to our preferred specifications for Model 3 (School-Teacher-Principal) using the cross-validated λs. Along with some control variables, all fixed effects (principal, school and teacher composition) were excluded from the regression for both mathematics and Portuguese. As such, we did not manage to use a LASSO regression approach in our value-added models to overcome the overparametrization observed in our results. The correct approach for employing LASSO regression in fixed effects centered models remains a subject for future research.

7.2 Within School Variance in Principal Effects

For our semi-parametric value-added model, we investigate the within-school variance in principal and average teacher effects. This is done by estimating equations

[15] According to Belloni & Chernozhukov (2013), feature selection through LASSO regression only to use selected features in another "second stage" regression is incorrect and may lead to biased estimates. This is often referred to as post-LASSO and is not recommended.

[16] In LASSO regressions, λ is the tuning parameter that indicates a more rigorous or more parsimonious penalization of feature selection. The standard procedure is to employ a cross-validation approach to select the λ value to be used in the LASSO regression, with the highest value within one standard deviation from the minimum λ being used. A common procedure is to select λ values through cross-validation, a machine-learning technique that tests various arrangements by dividing the existing sample into groups to be used as training and testing sets. We perform a 5-fold cross-validation of 100 λ values, meaning we divide our sample into 5 groups and test 100 different values for λ.

[17] Note that the penalization parameter cross-validation is sensitive to feature values. Since LASSO regression was created to estimate high-dimension sparse models, we normalize our variables in which values are not restricted to the $[-1; 1]$ interval. A λ selection without such normalization would imply higher values for the penalization parameter, and thus non-optimal feature exclusions.

(6.4) and (6.5).

However, since this is a variance analysis approach, we must control for other influences on student achievement beforehand. This is done in a first stage, in which the dependent variable is effectively constructed by regressing PROEB scores on controls, saving residuals, and creating both the average school achievement residual and the average schoolyear achievement residual. As shown in the left-hand side term of equation (6.5), we subtract the average school achievement residual from the average schoolyear achievement residual, square this difference, and take the mean value over the whole period. This measure is our dependent variable.

Table 19 presents estimate coefficients for the dependent variable construction. We construct four different fits by incrementally incorporating controls: time trends, student demographic characteristics, then socioeconomic characteristics, and finally peer (leave-me-out) characteristics. Fit 4 is our preferred specification, as it considers all controls and has the highest explanation power for mathematics and Portuguese scores.

We present our model's results in Table 20. Besides the four constructed dependent variables, we also consider solely the PROEB scores, without a first-stage construction, both in the full and filtered sample (to allow for comparisons with other specifications). The first part of Table 20 replicates the static model developed by Coelli & Green (2012), considering only the principal term. The second part of the table incorporates our extension for the within-school variance in average teacher effects. Finally, the last part of the table keeps the same specifications as the second part but considers only schools present in our panel during the whole period (all five years). We see that estimates seem to increase for both mathematics and Portuguese the more controlled the dependent variable fit is in all panels.

As mentioned in previous chapters, estimate interpretation in this semi-parametric model is a bit more complex and indirect. Taking our preferred specification for mathematics (column (5a)), when considering only the principal variation term (Panel A), we obtain a coefficient estimate of 32.5373, meaning a one standard deviation of within school principal effect of 5.7041 (ie. the square root of 32.5373). Essentially, the variation in principal effects in schools in this sample is associated with student outcomes 5.7041 points higher in mathematics, or 11.40% of a standard deviation in PROEB scores.[18] Coelli & Green (2012) argue this can also be interpreted as a student who attended a school with a principal that was one standard deviation higher in the "effective" distribution having a 5.7041 points higher mathematics score, on average.. For Portuguese, we find that a principal one standard deviation higher in the "effective" distribution would imply a Portuguese PROEB score 6.7275 points higher, which is equivalent to approximately 13.45% of a

[18] Since our dependent variable is a time average of controlled PROEB scores, we consider the theoretic PROEB score standard deviation of 50 instead of the year-specific standard deviations.

Table 19 – Regression coefficients for dependent variable construction in the semi-parametric within school variance model

	Fit 1		Fit 2		Fit 3		Fit 4	
	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)	(4a)	(4b)
Intercept	272.6357***	274.7622***	291.2931***	281.5084***	258.4922***	245.5448***	251.2954***	233.2986***
	(0.1496)	(0.1446)	(0.1909)	(0.1825)	(0.5225)	(0.4982)	(1.2059)	(1.1540)
Year: 2016	-2.3556***	-0.4893***	-3.6728***	-4.6989***	-1.8174***	-2.4122***	-2.4544***	-3.3584***
	(0.2094)	(0.2024)	(0.2027)	(0.1938)	(0.2047)	(0.1952)	(0.2612)	(0.2499)
Year: 2017	-4.1544***	-3.8232***	-6.0114***	-5.5198***	-5.1969***	-4.3990***	-5.9663***	-5.2724***
	(0.2066)	(0.1998)	(0.2001)	(0.1914)	(0.2007)	(0.1913)	(0.2221)	(0.2126)
Year: 2018	-2.9512***	-1.5735***	-5.2943***	-3.8683***	-4.6281***	-3.0540***	-4.7964***	-3.0621***
	(0.2092)	(0.2022)	(0.2028)	(0.1939)	(0.2015)	(0.1921)	(0.2163)	(0.2070)
Year: 2019	-3.3151***	-8.4278***	-5.2510***	-10.4085***	-5.0120***	-10.0777***	-5.4896***	-10.0064***
	(0.2083)	(0.2014)	(0.2018)	(0.1930)	(0.2005)	(0.1911)	(0.2209)	(0.2114)
Demographic: Female			-6.7304***	12.2935***	-5.8543***	12.9400***	-6.2820***	12.5048***
			(0.1280)	(0.1224)	(0.1264)	(0.1205)	(0.1252)	(0.1198)
Demographic: Non-white			-10.2585***	-8.7005***	-7.7629***	-6.4699***	-5.3148***	-4.9633***
			(0.1344)	(0.1285)	(0.1336)	(0.1273)	(0.1361)	(0.1303)
Demographic: Grade repetition history			-28.0327***	-27.7637***	-25.4288***	-25.0780***	-24.5212***	-24.4454***
			(0.1513)	(0.1447)	(0.1501)	(0.1431)	(0.1488)	(0.1424)
Socioeconomic: Mother middle school degree					-0.2231	0.5767***	-0.4029**	0.3461**
					(0.1790)	(0.1678)	(0.1772)	(0.1696)
Socioeconomic: Mother university degree					9.1778***	7.2550***	8.1070***	6.4205***
					(0.2159)	(0.2047)	(0.2149)	(0.2057)
Socioeconomic: Father middle school degree					0.9217***	0.8269***	0.6474***	0.5493***
					(0.1809)	(0.1694)	(0.1790)	(0.1713)
Socioeconomic: Father university degree					4.4874***	4.128***	3.1233***	2.8208***
					(0.2748)	(0.2583)	(0.2724)	(0.2607)
Socioeconomic: *Bolsa Família*					-6.0588***	-6.2862***	-4.4453***	-4.8214***
					(0.1513)	(0.1442)	(0.1535)	(0.1469)
Socioeconomic: Paved street					0.5181**	2.8259***	-0.3857***	1.7030***
					(0.2144)	(0.2044)	(0.2152)	(0.2059)
Socioeconomic: Garbage collection					2.2908***	2.9330***	0.9051***	1.4258***
					(0.1785)	(0.1701)	(0.1796)	(0.1719)
Socioeconomic: Bathroom					13.6432***	13.8652***	13.2910***	13.6058***
					(0.4267)	(0.4069)	(0.4220)	(0.4038)
Socioeconomic: Washing machine					-7.1456***	-6.2999***	-7.9674***	-7.0430***
					(0.1791)	(0.1708)	(0.1783)	(0.1706)
Socioeconomic: Car					1.5392***	-1.5258***	1.5380***	-2.5175***
					(0.1358)	(0.1295)	(0.1357)	(0.1298)
Socioeconomic: Cellphone					9.8930***	8,9630***	7.7026***	7.7075***
					(0.1461)	(0.1393)	(0.1467)	(0.1404)
Socioeconomic: Computer					14.5246***	16.7681***	13.2910***	15.7526***
					(0.2199)	(0.2097)	(0.2202)	(0.2107)
Socioeconomic: Private school history					2.0820***	0.6470***	0.7835***	-0.7603***
					(0.2146)	(0.2046)	(0.2138)	(0.2045)
Peer: Female							28.1084***	28.5211***
							(0.8429)	(0.8066)
Peer: Non-white							-25.3960***	-13.9798***
							(0.5079)	(0.4860)
Peer: *Bolsa Família*							1.1509	0.3769
							(1.7121)	(0.6814)
Peer: Mother university degree							65.4737***	49.2804***
							(1.2703)	(1.2157)
Peer: Father university degree							8.1240***	15.9847***
							(1.9804)	(1.8952)
Peer: Car							18.3463***	11.2181***
							(0.6815)	(0.6522)
Peer: Cellphone							-1.5151***	5.5275***
							(0.7024)	(0.6722)
Peer: Computer							-5.6679***	-6.9746***
							(0.9624)	(0.9210)
Observations	619,535	319,535	619,535	619,535	619.535	619,535	619,535	619,535
Adjusted R^2	0.0007	0.0032	0.0650	0.0875	0.0998	0.1263	0.1199	0.1397

Notes: The table shows regression coefficients for the dependent variable construction for our semi-parametric within-school variance model (regression residuals are squared and used as the dependent variable in such model). Columns labelled "a" refer to mathematics scores, and columns labelled "b" refer to Portuguese scores. Columns (1) refer to the specification with year tendencies. Columns (2) refer to the specification which adds student demographic controls. Columns (3) refer to the specification which adds student socioeconomic (family and household) controls. Columns (4) refer to the specification which adds peer (leave-me-out) controls. Standard deviations are shown in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

Table 20 – Semi-parametric model estimates for principal effects and average teacher effects within school variance

	Fit 0 - unfiltered		Fit 0 - filtered		Fit 1		Fit 2		Fit 3		Fit 4	
	(0a)	(0b)	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)	(4a)	(4b)	(5a)	(5b)
Panel A: Principal Only												
Princpal term	19.1501	41.3248	18.2194	34.7284**	16.8980	32.6549**	16.7658	35.4821**	22.4778	40.2943***	32.5373	45.2597***
	(22.0643)	(16.7707)	(22.0895)	(16.5541)	(21.7287)	(16.4004)	(21.3736)	(15.1238)	(21.2867)	(14.4734)	(22.4620)	(16.2912)
Adjusted R^2	0.0497	0.0462	0.0529	0.0384	0.0549	0.0425	0.0504	0.0415	0.0521	0.0489	0.0736	0.0618
Observations	2294	2294	2289	2289	2289	2289	2289	2289	2289	2289	2289	2289
Panel B: Principal and Teachers												
Principal term	12.3567	33.2925***	11.8938	26.9092	10.6898	25.0528	11.1691	28.6515*	17.0151	33.5588**	26.1380	37.1964**
	(21.9912)	(16.6472)	(22.0209)	(16.3993)	(21.6617)	(16.2546)	(21.3251)	(14.9978)	(21.2420)	(14.3431)	(22.3934)	(16.1202)
Teacher term	61.3093***	65.3481***	59.5241***	69.5623***	58.4193***	67.6306***	52.6655***	60.7670***	51.4039***	59.2915***	60.2175***	71.7339***
	(12.2442)	(9.5577)	(12.3051)	(9.4517)	(12.1043)	(9.3682)	(11.9162)	(8.6439)	(11.8698)	(8.2666)	(12.5132)	(9.2908)
Adjusted R^2	0.0595	0.0649	0.0621	0.0602	0.0640	0.0634	0.0580	0.0614	0.0594	0.0699	0.0825	0.0852
Observations	2294	2294	2289	2289	2289	2289	2289	2289	2289	2289	2289	2289
Panel C: T = 5 schools												
Principal term			-20.6461	11.1100	-20.0369	10.3131	-24.6815	10.9264	-25.4008	11.9800	-18.8231	16.0637
			(23.8045)	(17.9478)	(23.3461)	(17.5469)	(22.8538)	(15.9161)	(22.5771)	(15.4042)	(23.1237)	(16.1962)
Teacher term			22.1793	64.5028***	21.5350	62.3603***	16.8660	51.5428***	17.7349	49.9496***	20.3237	54.0382***
			(14.6477)	(11.4879)	(14.3656)	(11.2313)	(14.0628)	(10.1874)	(13.8924)	(9.8598)	(14.2288)	(10.3667)
Adjusted R^2			0.0588	0.0642	0.0605	0.0690	0.0549	0.0679	0.0561	0.0729	0.0761	0.0903
Observations			1705	1705	1705	1705	1705	1705	1705	1705	1705	1705

Notes: The table shows estimates for principal and teacher terms in our semi-parametric approach. Specifications vary according to the dependent variable, as described in Table **??**. Panel A presents estimates for principal effects within school variance only; Panel B presents estimates for both principal effects and average teacher effects within school variance; and Panel C present estimates for principal effects and average teacher effects within school variance only for schools present in all years in our panel. Columns labelled "a" refer to mathematics scores, and columns labelled "b" refer to Portuguese scores. Columns (0) refer to estimates conducted without controls in our unfiltered sample. Columns (1) refer to estimates conducted without controls, but on the filtered sample (for comparison with other specifications). Columns (2) refer to estimates using Fit 1 (time tendencies). Columns (3) refer to estimates using Fit 2 (added student demographic controls). Column (4) refers to estimates using Fit 3 (added student socioeconomic controls). Column (5) refers to estimates using Fit 4 (added peer controls). Standard deviations are shown in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

standard deviation.

The second panel in Table 20 presents regression results for the specification considering both the principal and average teacher within school variance effects. Directing once again our attention to our preferred specification (columns (5a) and (5b)), we observe some interesting factors. First, we see a reduction in the magnitude of the estimated within-school variance of principal effects for both mathematics and Portuguese. These figures depart from the ones just discussed to 5.1125 (10.22% of a standard deviation) for mathematics and 6.0988 (12.19% of a standard deviation) for Portuguese. We also observe a loss in statistical significance, given a much smaller reduction in the estimates' standard errors; while mathematics estimates weren't significant to begin with, influence in Portuguese was significant at the 1% level, now being significant at the 5% level. These reductions are explained by the inclusion of within-school variance of average teacher effects, which are successfully separated from within-school variance in principal effects. We find that a one standard deviation increase in average teacher quality[19] would imply an increase in 7.7599 for mathematics (15.51% of a standard deviation) and 8.4695 for Portuguese (16.93% of a standard deviation). These are both significant at the 1% level. Lastly, note that the inclusion of average teacher effect variance within-school in the model increased the portion of the variation in scores explained by our model, as captured by our adjusted R^2 measure, but proportionately more for Portuguese scores, overturning the trend observed in the first panel of Table 20.

These results, taken together, are important to answer our research question. We find that the within-school variance in principal effects has a positive influence on students'

[19] Recall that, in our semi-parametric model, we use individual teachers in our modelling, and not teacher compositions like in the parametric models.

mathematics and Portuguese achievement. Since this within-school variance measure captures principal changes in school administration, we understand that these changes appear to have benefitted students in our sample. An important aspect of this dynamic is the effect teachers have, with the introduction of within-school variance in average teacher effects better explaining variation in data, and at the same time diluting the principal effect influence. This seems to confirm that principals have their own effects on student achievement (that is, not mediated by teachers), but that teacher mediation is a crucial channel of influence.

In the last panel in Table 20, we mirror the specification with both principal and average teacher effects but focus only on schools for which we have student scores during the whole period analyzed (2015-2019). This amounts to 1705 schools (roughly 74.48% of the 2289 schools in the filtered sample). We find very interesting results: estimates appear again diluted, both principal and teacher, with a loss of statistical significance for both mathematics and Portuguese principal within-school variance estimates. More strikingly, we arrive at a negative estimate for the effect of within-school variance in principal effects in mathematics. This may be vexing at first, especially considering the explanation by Coelli & Green (2012), in that the estimates are equivalent to the actual within-school variances. However, their explanation, but not their model, fails to consider the possibility of a higher variance in teacher effects at the same school over time having a negative effect on student achievement, that is, that increased principal turnover could in fact hinder student achievement. What the model actually estimates is the effect of the within-school variance in principal (and average teacher) effects, of which the absolute value of the estimate corresponds to the within-school variance. The literature documents negative changes in student outcomes following principal turnover (LOEB et al., 2010; MILLER, 2013).

This means that what we have in column (5a) in Panel C is not a negative variance, but simply that the within-school variance in principal effects has a negative impact on students' mathematics scores (although not statistically significant). In our reduced sample of schools, higher within-school variance in principal effects is associated with a score of 4.3385 points lower score in mathematics (8.67% of a standard deviation) and a Portuguese score of 4.0079 points higher (8.01% of a standard deviation). However, neither of these estimates is statistically significant at the 10% level. Teacher influence is also reduced, with average teachers being one standard deviation higher implying 4.5081 points (9.01% of a standard deviation) higher score in mathematics and 7.3510 points (14.70% of a standard deviation) in Portuguese. This reduced sample analysis is important because it excludes schools that may be opening or closing, something that may affect students' outcomes due to management and organizational issues and frictions.

The question raised in light of these Panel C results is why such a change in estimates following this subsample delimitation? First, recall that, unlike the previous section, we are not dealing with principal movement sample delimitations in this within-school analysis, and consider all schools available to us. By restricting our gaze to schools with test scores available over the whole period, we guarantee continuous information on principal tenures over this period.[20] Given the rules that specify principal selection covered in Chapter 3, we know that continuous information on principal tenures renders PROEB results on principal change years. Béteille et al. (2012) find evidence that frequent principal turnover has detrimental consequences for schools, influencing student achievement negatively and increasing teacher turnover. Similarly, Miller (2013) finds evidence that students' test scores fall on the first year a new principal takes office (although following a decrease in scores). Such an effect could help explain the change observed in within-school principal effect influence in this subsample analysis.

Overall, results from our semi-parametric, within-school variance approach indicate that principals have moderate effects on student achievement when measured by the impacts of principal changes. Although statistically not significant, these effects persist smaller after including teacher effects. Upon analyzing schools for which PROEB scores were available during our whole sample period, we find that within-school principal effect variance has a positive impact on Portuguese scores, but a negative impact in mathematics (both statistically not significant at 10%). This suggests that incoming principals are able to coordinate school efforts to better support students in Portuguese learning, but potentially not enough to compensate for the negative effect in outcomes following a change in leadership. These impacts are around the magnitude of 8% of a standard deviation in exam scores, which is in line with results in the international literature (GRISSOM et al., 2021) and the principal impacts found in the *Jovem e Futuro* program evaluation by Barros et al. (2019).

7.3 Management Practice Association

We test the association of estimated principal value-added measures from our parametric models to management practice scores. No individual measure of principal efficacy is estimated in our semi-parametric model, and thus is not included in this analysis. As explained in Chapter 6, our approach is simple regression association.

Table 21 presents estimates for the association of Model 1 principal value-added

[20] We stress that this is a focused analysis on schools for which we have PROEB scores available over the whole period, and not on schools functioning over the whole period. Recall 6, in which the lowest number of unique schools registered in a year was 2025, in 2015, much greater than the 1705 unique schools in the Panel C subsample in 20. This means that "missing" schools are in fact missing PROEB test data in the majority of these schools, and that the perceived growth in the number of schools over the period, in this case, is actually a growth in the number of schools for which we have PROEB score data that survived our filters described in Chapter 5 and in this section.

Table 21 – Regression coefficients for management practice association to value-added estimates from Model 1

		Mathematics			Portuguese		
		(1)	(2)	(3)	(4)	(5)	(6)
01	Pedagogical project	0.8508*	0.0026	0.0060	0.0080	-0.0100	-0.0109
		(0.4841)	(0.0118)	(0.0124)	(0.0088)	(0.0111)	(0.0117)
02	Teaching planning process	-0.7792	-0.0105	-0.0086	0.0048	0.0087	0.0130
		(0.5834)	(0.0138)	(0.0139)	(0.0106)	(0.0129)	(0.0132)
03	Teaching and learning customization	0.1886	0.0079	0.0115	0.0076	0.0165	0.0184
		(0.5150)	(0.0125)	(0.0127)	(0.0093)	(0.0117)	(0.0120)
04	New teaching practices	-1.2534**	-0.0206*	-0.0210*	-0.0396***	-0.0370***	-0.0364***
		(0.5016)	(0.0119)	(0.0121)	(0.0091)	(0.0111)	(0.0114)
05	Internal learning assessment	0.8182**	0.0170**	0.0165*	0.0051	0.0092	0.0089
		(0.3400)	(0.0083)	(0.0085)	(0.0061)	(0.0092)	(0.0081)
06	Student flow analysis	0.9453*	0.0185	0.0189	0.0161*	0.0093	0.0082
		(0.4876)	(0.0119)	(0.0119)	(0.0088)	(0.0111)	(0.0112)
07	External learning assessment	-0.7884	-0.0032	-0.0040	-0.0051	0.0072	0.0072
		(0.4977)	(0.0120)	(0.0122)	(0.0090)	(0.0112)	(0.0115)
08	School targets	0.2265	-0.0014	-0.0028	0.0067	0.0017	0.0012
		(0.3745)	(0.0086)	(0.0087)	(0.0068)	(0.0080)	(0.0082)
09	Leadership	-0.0475	-0.0083	-0.0068	-0.0006	-0.0091	-0.0094
		(0.5007)	(0.0121)	(0.0121)	(0.0091)	(0.0113)	(0.0114)
10	Worker evaluation	0.3774	0.0063	0.0061	0.0084*	0.0102*	0.0097
		(0.2783)	(0.0065)	(0.0065)	(0.0050)	(0.0061)	(0.0061)
11	Worker performance management and retention	0.5134	0.0076	0.0068	0.0130	0.0173	0.0167
		(0.5857)	(0.0136)	(0.0138)	(0.0106)	(0.0127)	(0.0130)
12	Infrastructure	-0.1819	0.0044	0.0058	-0.0146*	-0.0086	-0.0035
		(0.4372)	(0.0102)	(0.0107)	(0.0079)	(0.0096)	(0.0101)
13	Financial aspects	0.2924	-0.0059	-0.0034	0.0106	-0.0014	0.0002
		(0.6402)	(0.0150)	(0.0150)	(0.0113)	(0.0140)	(0.0142)
14	School values	0.1396	0.0016	-0.0037	0.0006	0.0038	-0.0001
		(0.4130)	(0.0100)	(0.0105)	(0.0075)	(0.0093)	(0.0100)
Observations		381	261	261	381	261	261
Principal controls		No	No	Yes	No	No	Yes
Adjusted R^2		0.0349	-0.0051	0.0093	0.0516	0.0267	0.0220

Notes: The table shows regression coefficients for the association of principal value-added estimates from Model 1 (School-Principal) to management practice domain scores. Columns (1) through (3) present results for mathematics scores, and columns (4) through (6) present results for Portuguese scores. Columns (1) and (4) refer to the specification without principal controls. Columns (2) and (5) refer to the specification without principal controls but are filtered to include only principals that have control variable markings. Finally, columns (3) and (6) refer to the specification with principal controls. For control regression coefficients, please refer to Table B.18. Standard deviations are shown in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

measures to management practice scores.[21] In this table, but also Tables 22 and 23, concerning Models 2 and 3 value-added measures, respectively, we first consider the whole sample intersection between principals that answered the management practice survey and who had value-added estimates. We then consider a reduced sample to account for control variables' existence, proposing two specifications: with and without principal controls.

In the full 381 principal sample, domains 01 (Pedagogical project), 04 (New teaching practices) and 05 (Internal learning assessment) exhibit statistical significance at the 10% level for mathematics principal value-added measures. For Portuguese principal value-added measures, domains 04 (New teaching practices), 06 (student flow analysis), 10 (Worker evaluation) and 12 (Infrastructure) exhibit correlational statistical significance at the 10% level. However, we do caution readers to take these correlation results as an

[21] For brevity, we present only the regression coefficients for the management practice domains. For control variables coefficients, for this and other model associations, please refer to Appendix Table B.18.

Table 22 – Regression coefficients for management practice association to value-added estimates from Model 2

	Mathematics			Portuguese		
	(1)	(2)	(3)	(4)	(5)	(6)
01 Pedagogical project	-0.0003	-0.0039	-0.0048	0.0014	-0.0062	-0.0072
	(0.0089)	(0.0106)	(0.0108)	(0.0075)	(0.0090)	(0.0092)
02 Teaching planning process	-0.0178*	-0.0242*	-0.0244*	-0.0152*	-0.0187*	-0.0191*
	(0.0104)	(0.0125)	(0.0126)	(0.0088)	(0.0106)	(0.0107)
03 Teaching and learning customization	0.0137	0.0115	0.0094	0.0030	-0.0007	-0.0037
	(0.0092)	(0.0116)	(0.0118)	(0.0078)	(0.0099)	(0.0100)
04 New teaching practices	-0.0159*	-0.0107	-0.0109	-0.0067	-0.0063	-0.0047
	(0.0085)	(0.0098)	(0.0100)	(0.0072)	(0.0084)	(0.0085)
05 Internal learning assessment	0.0046**	0.0041	0.0053	0.0018	0.0077	0.0070
	(0.0062)	(0.0075)	(0.0076)	(0.0053)	(0.0064)	(0.0065)
06 Student flow analysis	0.0137	0.0153	0.0163	-0.0068	-0.0069	-0.0066
	(0.0088)	(0.0104)	(0.0105)	(0.0074)	(0.0089)	(0.0089)
07 External learning assessment	-0.0035	0.0027	0.0018	-0.0008	0.0090	0.0088
	(0.0091)	(0.0111)	(0.0075)	(0.0077)	(0.0094)	(0.0095)
08 School targets	0.0008	0.0038	0.0050	0.0045	0.0030	0.0035
	(0.0063)	(0.0074)	(0.0075)	(0.0054)	(0.0063)	(0.0064)
09 Leadership	0.0008	-0.0088	-0.0067	0.0070	0.0032	0.0067
	(0.0085)	(0.0101)	(0.0102)	(0.0072)	(0.0086)	(0.0087)
10 Worker evaluation	-0.0016	-0.0029	-0.0014	0.0015	0.0011	0.0011
	(0.0047)	(0.0055)	(0.0056)	(0.0040)	(0.0047)	(0.0047)
11 Worker performance management and retention	0.0096	0.0196	0.0174	0.0133	0.0200*	0.0172*
	(0.0100)	(0.0120)	(0.0121)	(0.0085)	(0.0102)	(0.0104)
12 Infrastructure	-0.0041	-0.0151	-0.0182*	-0.0048	-0.0112	-0.0126
	(0.0078)	(0.0092)	(0.0094)	(0.0066)	(0.0079)	(0.0080)
13 Financial aspects	0.0140	0.0195	0.0212	0.0036	0.0086	0.0099
	(0.0108)	(0.0128)	(0.0131)	(0.0091)	(0.0109)	(0.0111)
14 School values	-0.0046	-0.0012	-0.0002	-0.0008	-0.0026	-0.0013
	(0.0072)	(0.0083)	(0.0085)	(0.0061)	(0.0071)	(0.0072)
Observations	743	532	532	743	532	532
Principal controls	No	No	Yes	No	No	Yes
Adjusted R^2	0.0011	0.0027	0.0004	-0.0063	-0.0028	-0.0038

Notes: The table shows regression coefficients for the association of principal value-added estimates from Model 2 (Teacher-Principal) to management practice domain scores. Columns (1) through (3) present results for mathematics scores, and columns (4) through (6) present results for Portuguese scores. Columns (1) and (4) refer to the specification without principal controls. Columns (2) and (5) refer to the specification without principal controls but are filtered to include only principals that have control variable markings. Finally, columns (3) and (6) refer to the specification with principal controls. For control regression coefficients, please refer to Table B.18. Standard deviations are shown in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

exploratory exercise, especially considering the very modest portion of the variation in principal value-added measures for mathematics and Portuguese explained by management practice scores, with an adjusted R^2 of 0.0349 and 0.0516, respectively. When examining our preferred specifications that include controls, we only observe statistical significance for domain 04 in mathematics and Portuguese, both with negative coefficients.

Table 22 shows the same results for the association of principal value-added as estimated by Model 2. In this case, the full intersection of principals with estimated value-added measures and who took part in the survey is greater than that observed in Model 1 association, mainly because Sample 2 includes more teachers, in particular those that did not face any leadership change. Mathematic value-added measures are correlated with management domains 02 (Teaching planning process), 04 (New teaching practices) and 05 (Internal learning assessment) in this full sample, and only with domain 02 (Teaching planning process) when considering Portuguese value-added measures. However, principal

Table 23 – Regression coefficients for management practice association to value-added estimates from Model 3

		Mathematics			Portuguese		
		(1)	(2)	(3)	(4)	(5)	(6)
01	Pedagogical project	-0.0009	0.0007	0.0021	0.0017	0.0030	0.0022
		(0.0021)	(0.0031)	(0.0032)	(0.0018)	(0.0024)	(0.0025)
02	Teaching planning process	0.0016	-0.0007	-0.0026	0.0003	-0.0036	-0.0036
		(0.0026)	(0.0039)	(0.0039)	(0.0022)	(0.0030)	(0.0031)
03	Teaching and learning customization	-0.0006	0.0006	-0.0007	0.0003	0.0013	0.0004
		(0.0022)	(0.0034)	(0.0035)	(0.0018)	(0.0027)	(0.0028)
04	New teaching practices	0.0009	-0.0022	0.0036	-0.0010	-0.0022	-0.0008
		(0.0023)	(0.0033)	(0.0033)	(0.0019)	(0.0025)	(0.0026)
05	Internal learning assessment	0.0023	0.0033	0.0023	-0.0001	0.0019	0.0011
		(0.0015)	(0.0023)	(0.0023)	(0.0012)	(0.0018)	(0.0018)
06	Student flow analysis	-0.0036*	-0.0050	-0.0059*	-0.0012	-0.0022	-0.0031
		(0.0022)	(0.0032)	(0.0031)	(0.0018)	(0.0025)	(0.0025)
07	External learning assessment	-0.0007	-0.0022	-0.0028	0.0024	0.0008	0.0012
		(0.0023)	(0.0033)	(0.0034)	(0.0019)	(0.0017)	(0.0018)
08	School targets	0.0013	-0.0013	0.0020	0.0008	0.0007	0.0012
		(0.0016)	(0.0022)	(0.0023)	(0.0013)	(0.0017)	(0.0018)
09	Leadership	0.0016	0.0037	0.0036	0.0010	0.0006	0.0004
		(0.0023)	(0.0033)	(0.0034)	(0.0019)	(0.0026)	(0.0027)
10	Worker evaluation	0.0000	-0.0005	-0.0005	-0.0016	-0.0013	-0.0012
		(0.0012)	(0.0016)	(0.0016)	(0.0010)	(0.0012)	(0.0012)
11	Worker performance management and retention	0.0021	0.0044	0.0046	-0.0021	0.0017	0.0010
		(0.0027)	(0.0038)	(0.0039)	(0.0022)	(0.0030)	(0.0031)
12	Infrastructure	-0.0012	-0.0027	-0.0042	-0.0003	-0.0002	-0.0003
		(0.0020)	(0.0028)	(0.0029)	(0.0016)	(0.0022)	(0.0023)
13	Financial aspects	0.0003	-0.0001	-0.0004	0.0034	0.0048	0.0053
		(0.0027)	(0.0038)	(0.0038)	(0.0023)	(0.0029)	(0.0030)
14	School values	0.0001	0.0000	-0.0003	-0.0027	-0.0036	-0.0039
		(0.0019)	(0.0028)	(0.0030)	(0.0015)	(0.0021)	(0.0024)
Observations		203	138	138	203	138	138
Principal controls		No	No	Yes	No	No	Yes
Adjusted R^2		-0.1880	-0.0080	0.0053	-0.0056	-0.0234	0.0018

Notes: The table shows regression coefficients for the association of principal value-added estimates from Model 3 (School-Teacher-Principal) to management practice domain scores. Columns (1) through (3) present results for mathematics scores, and columns (4) through (6) present results for Portuguese scores. Columns (1) and (4) refer to the specification without principal controls. Columns (2) and (5) refer to the specification without principal controls but are filtered to include only principals that have control variable markings. Finally, columns (3) and (6) refer to the specification with principal controls. For control regression coefficients, please refer to Table B.18. Standard deviations are shown in parentheses. * p < 0.1 ** p < 0.05 *** p < 0.01.

value-added, as estimated from teacher-principal interactions, seems to fare even worse in this association: adjusted R^2 statistics of 0.0011 and -0.0063 for mathematics and Portuguese, respectively. In our preferred specifications, these statistical significances filter down to domains 02 and 12 (Infrastructure) for mathematics and domains 02 and 11 (Worker performance management and retention) for Portuguese. This highlights the correlation with domain 02, at the same time that it introduces domains 11 and 12 into play.

Lastly, Table 23 presents the association of principal value-added measures as estimated by Model 3 with management practice scores. This includes our more restricted intersection, with only 203 principals having their value-added estimated while also having participated in the management practice survey. Do recall that our value-added estimates from this model were also very close to zero, and this is potentially what drives the results observed. For the full sample, only domain 06 (Student flow analysis) was significant at the 10% level in association with mathematics value-added measures, with no domains being statistically significant in correlation to Portuguese value-added measures. In our preferred specification, with principal controls, the same domain appears as statistically significant.

Table 24 – Regression coefficients for management practice association to value-added estimates from all models

		Mathematics			Portuguese		
		(1a)	(2a)	(3a)	(1b)	(2b)	(3b)
01	Pedagogical project	0.0088	0.0031	0.0021	0.0005	-0.0144	0.0022
		(0.0156)	(0.0205)	(0.0032)	(0.0133)	(0.0229)	(0.0025)
02	Teaching planning process	-0.0320*	-0.0383	-0.0026	-0.0273*	-0.0238	-0.0036
		(0.0191)	(0.0251)	(0.0039)	(0.0163)	(0.0280)	(0.0031)
03	Teaching and learning customization	0.0118	0.0311	-0.0007	0.0155	0.0119	0.0004
		(0.0172)	(0.0226)	(0.0035)	(0.0147)	(0.0252)	(0.0028)
04	New teaching practices	-0.0136	-0.0285	0.0036	-0.0196	-0.0438*	-0.0008
		(0.0165)	(0.0216)	(0.0033)	(0.0140)	(0.0241)	(0.0026)
05	Internal learning assessment	0.0121	0.0090	0.0023	0.0080	0.0041	0.0011
		(0.0115)	(0.0151)	(0.0023)	(0.0098)	(0.0168)	(0.0018)
06	Student flow analysis	0.0183	0.0244	-0.0059*	0.0040	0.0011	-0.0031
		(0.0153)	(0.0202)	(0.0031)	(0.0131)	(0.0225)	(0.0025)
07	External learning assessment	-0.0068	-0.0386*	-0.0028	0.0134	0.0133	0.0012
		(0.0167)	(0.0219)	(0.0034)	(0.0142)	(0.0244)	(0.0018)
08	School targets	-0.0007	-0.0067	0.0020	-0.0011	0.0044	0.0012
		(0.0113)	(0.0148)	(0.0023)	(0.0096)	(0.0165)	(0.0018)
09	Leadership	-0.0294*	-0.0432*	0.0036	-0.0278*	-0.0016	0.0004
		(0.0167)	(0.0219)	(0.0034)	(0.0142)	(0.0244)	(0.0027)
10	Worker evaluation	0.0080	0.0001	-0.0005	0.0113*	0.0215*	-0.0012
		(0.0078)	(0.0103)	(0.0016)	(0.0066)	(0.0114)	(0.0012)
11	Worker performance management and retention	0.0261	0.0619**	0.0046	0.0262	0.0192	0.0010
		(0.0190)	(0.0249)	(0.0039)	(0.0162)	(0.0278)	(0.0031)
12	Infrastructure	-0.0107	-0.0093	-0.0042	-0.0043	0.0103	-0.0003
		(0.0144)	(0.0189)	(0.0029)	(0.0123)	(0.0211)	(0.0023)
13	Financial aspects	0.0125	0.0149	-0.0004	0.0198	0.0219	0.0053
		(0.0189)	(0.0248)	(0.0038)	(0.0161)	(0.0277)	(0.0030)
14	School values	0.0087	0.0166	-0.0003	0.0015	-0.0043	-0.0039
		(0.0147)	(0.0193)	(0.0030)	(0.0125)	(0.0215)	(0.0024)
Observations		138	138	138	138	138	138
Principal controls		Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2		-0.0164	0.0927	0.0053	-0.0103	-0.0508	0.0018

Notes: The table shows regression coefficients for the association of value-added estimates from all models to management practice domain scores. Columns labelled "a" refer to mathematics scores, and columns labelled "b" refer to Portuguese scores. Columns (1) refer to the association with Model 1 (School-Principal) value-added estimates. Columns (2) refer to the association with Model 2 (Teacher-Principal) value-added estimates. Columns (3) refer to the association with Model 3 (School-Teacher-Principal) value-added estimates. All columns refer to associations made with value-added measures estimated on Sample 3, our more restricted sample, and all include principal controls. Standard deviations are shown in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

Following what was done in our parametric model, we conduct a new association of value-added measures estimated in Models 1 and 2 considering only principals observed in our preferred specification in the association with Model 3 results. Table 24 shows such results. Notice first how each model differs significantly from the other, for both mathematics and Portuguese associations. Domains 09 (Leadership) and 11 (Worker performance management and retention) appear as statistically significant at the 10% level for mathematics, and domains 02 (Teaching planning process), 04 (New teaching practices) and 10 (Worker evaluation) appear as statistically significant for Portuguese, albeit none of these for the association of Model 3 value-added estimates. Despite this, Model 3 estimates seem to fare better at explaining variation observed in Portuguese value-added data, although very little. Strikingly, Model 2 estimates exhibit relatively impressive adherence to data for mathematics, breaking the trend observed not only for this very model in Table 22, but for all models in this management practice association exercise.

Recall from Table 7 that the management practice instrument used considered other metrics besides the tally of management practices in each domain. By labelling management practices into different journeys based on their adoption probability, a new, deeper possibility for analysis emerges. Considering the weak association results obtained in this section, a renewed round of association should be carried out once principal value-added measures are better disentangled from other factor effects. As such, we opted not to continue with journey-specific associations to principal value-added in this research. It remains a future step in this research agenda to investigate how different practice adoption probabilities interact with principal effect measures.

8 Research Limitations

Several limitations arise from our research construction and in face of the results obtained. In this section, we discuss these limitations and indicate paths forward to advance this research agenda. We divide these limitations into two main groups: data limitations and modelling limitations.

8.1 Data Limitations

Value-added models are known for their stringent data demands. Compared to other works that used this class of models, we had access to data on a big public school system, but no student longitudinal data was made available. This hindered the results obtained: without being able to specify the whole value gained in terms of learning outcomes for each student due to a lack of previous score comparison basis, the parametric estimation detailed in Chapter 7 resembled more a decomposition of various school factor effects than a value-added estimation, as reviewed in Chapter 2. A solution to this problem would require longitudinal data. Despite PROEB not being applied to 2º EM (the grade that precedes 3º EM), it is applied to 1º EM. This would require a more careful sample delimitation in face of principal changes, as well as a rethinking of the definition of teacher compositions to explore a case with two different grades. Two additional data problems arise: 1º EM evaluations are conducted every two years, which would change our panel structure and require more cross-sections; and CAEd identifiers are not longitudinally linked, which greatly complicates the necessary matching of every students' 1º EM and 3º EM scores.

Throughout this research, we made use of principal changes in school administration to identify principal value-added in our proposed specifications. This is an adaptation of the common identification strategy used in the value-added models literature, which explores principal transitions between schools. Such an adaptation was made given the lack of representative principal transitions in our Minas Gerais data, as detailed in Chapter 6. As can be seen in the results presented in Table 15, in which the model more closely reflects the ones employed in the literature, we were not able to adequately disentangle and distinguish principal and school fixed effects. This is also true for our other value-added models, as can be seen in Tables 16 and 17. Despite all principal fixed effects being estimated in all models, they display an average value close to PROEB average scores for our School-Principal and Teacher-Principal models, and close to zero for our School-Teacher-Principal model. Should these be taken literally, it would imply that principals have an impact of approximately five standard deviations in our first two models, and no impact, in our third model. Both of these results are completely out of touch with the literature, which estimates principal value-added to be around 8% of a standard deviation in scores (GRISSOM et al., 2021). Despite this, some information can be drawn from these models, as explained in Chapter 7.

The failure to disentangle principal effects from other factors' effects is directly connected to the inability to properly estimate a bigger portion of school fixed effects in our models, and teacher composition fixed effects to a lesser extent. As explained in Chapter 7, the use of principal changes as an identification strategy led to small comparison groups compromising of two principals and one school, on average, as seen in Table 8, which characterized our models' overparametrization. That is, the necessity to omit a factor to estimate all other factors in each comparison group (with principals being prioritized over schools and teacher compositions), made our models unable to separate the desired effects. Usual solutions to overparametrization, like the LASSO regression, weren't fruitful.

These results reinforce the necessity for principal transitions as an identification strategy, allowing researchers to observe not only one school under different administrations but also principals in charge of different schools. As Chiang et al. (2016) explain, these are crucial to the linking of various schools and the formation of bigger comparison groups.[1] With bigger groups, fewer factors are omitted to contribute to identification and comparable value-added measures can be estimated. This also suggests that since the overparametrization problem arose from our identification strategy, the LASSO regression approach may not be the ideal path forward in these types of models.

Dealing with this problem is significantly more difficult than with the previous one. This problem arises from our data generating process, namely the manner principals are chosen in Minas Gerais, and not from some sort of data accessibility problem. One possible path forward is to expand the time horizon of our analysis, incorporating more years into our panel in hopes of identifying more principal transitions between schools. However, when coupled with the suggestion for incorporating 1º EM scores into analysis, this seems a quite unlikely solution, with no guarantees of actual principal transitions. More promisingly, a solution might be to look beyond just Minas Gerais in exploring principal value-added in Brazil. This, of course, would require dealing with different subnational institutional contexts, which Simelli et al. (2023) detail having some significant variation in regards to principal eligibility criteria, as well as harmonizing different standardized scores.

Some minor data accessibility points represented setbacks in the face of the full potential our panel analysis presented. The incompatibility of CAEd identifiers longitudinally and, crucially, the absence of fully available teacher information in SEE-MG

[1] Say a principal j was in charge of school A and then moved to school B. Not only j's predecessors in school B can be compared to j and her successor in school A, but in case any of these other principals went to another school C, then these principals in school C are also comparable. Schools are linked in this manner, forming a comparable network of principals.

data proved difficult to circumvent. As detailed in Chapters 6 and 5, both the use of the identified *Censo Escolar* and teacher composition analysis were solutions put forth to bypass the unavailability of fully linked teacher information in SEE-MG data. These alternatives allowed us to conduct our proposed research, but some potential can still be unearthed if these data setbacks are able to be overcome. This will require renewed rounds of interaction with the SEE-MG.

8.2 Model Limitations

Our proposed models share some of the limitations this class of models presents. By departing from the additive education production function, we model principal and school effects independently from one another, instead of a principal-school match, which would explore principals' specific interactions with their allocated schools.[2] Dhuey & Smith (2018) propose this as a step further in understanding principal effects, their variation and interactions with other school components. They find that the majority of principal effects deviation is attributed to this principal match-specific component, reducing the fixed component across schools.

The principal effects' behaviour assumptions described in Chapter 4 are drawn directly from the international literature (COELLI; GREEN, 2012; GRISSOM et al., 2015) and represent common limitations in value-added models. The teacher effects' behaviour assumptions described in the same chapter are formulated mirroring those of principal effects. They figure among common assumptions (and limitations) in teacher value-added models (RIVKIN et al., 2005; HANUSHEK; RIVKIN, 2010).

One very interesting recent work is that of Muñoz & Prem (2022) on school principals in Chile following an educational reform. They also include teacher effects in principal value-added analysis by removing school fixed effects from their model, opting for random effects controls instead. This pivot from school to teacher fixed effects, diverging from the literature and our attempt in this research, seeks to avoid weak identification problems in network data (JOCHMANS; WEIDNER, 2019). Given the similarity of their analysis to our proposal and the problems highlighted in both works, an alternative intermediary model could have coupled principal and teacher fixed effects, just as Muñoz & Prem (2022), only to later reintroduce school effects into the analysis. This was not attempted here due to the requirements in such a model: besides the longitudinal student data available in the authors' Chilean panel, they also had information on various grades (by making use of non-standardized grade scores). Their identification hypothesis relied heavily on this additional data: teacher and student transitions between schools (thus, between principal tenure). At the same time, this eliminates the requirement of principal

[2] Of course, this would require school effects to be estimated practically, something we were not able to achieve in our value-added models for the majority of schools in our sample.

transitions just discussed, allowing simply for principal changes due to the exclusion of school fixed effects, it also places heavy data demands on teacher transitions between schools, which can easily be prohibitive if several grades are not considered simultaneously in the analysis. Still, their model proved a powerful possibility to study the effects we are interested in and has great potential to shape future endeavours.

The identifying assumptions made in our within-school principal and average teacher effect variance model are a different story. As discussed in Chapter 6, some of these are taken from the model presented by Coelli & Green (2012), from which we draw heavily, but a novel set of hypotheses are made specifically to deal with the covariance terms in equation (4.32). Despite our view and justification that these assumptions are attenuated in the Brazilian institutional context described in Chapter 3, they remain very strong. The hypothesis that public tender eliminates covariance between principal effects and average teacher quality does not discuss the characteristics of job opportunities with non-pecuniary benefits (such as in bigger, wealthier cities), as Coelli & Green (2012) themselves note when discussing the related independent principals assumption and as documented in previous research (LOEB et al., 2010; BÉTEILLE et al., 2012); nor does it regard the fact that principals may be selected among teachers in Minas Gerais. The assumption of no student-to-teacher sorting in public schools used to eliminate the covariance between average teacher quality and average student quality slams into the problem that no sorting within normal public schools is still affected by sorting between the private or technical school systems. These assumptions enabled us to carry out our estimations, but their difficulty in adhering to reality motivates a reincursion into modelling principal and teacher variance within schools to advance with this research agenda. Since our estimation aims at identifying principal effects under strict exogeneity, our assumptions made serve as reinforcements for our argument on model selection in a plausibility sense and are ultimately untestable.

One caveat used in international literature to guarantee some school network formation is to explore compulsory principal rotation policies (BRANCH et al., 2012; GRISSOM et al., 2015; CHIANG et al., 2016; DHUEY; SMITH, 2018). These policies ensure principal transitions between schools, securing the formation of comparison groups. These policies also serve researchers for another purpose: guaranteeing principal changes occur exogenously. The absence of such policies in our sample implied an assumption of exogenous principal movement to and from schools. Given the elective nature of the principal role in Minas Gerais, this is somewhat attenuated, seeing as principals need community recognition to achieve leadership position (SIMELLI et al., 2023; MUñOZ et al., 2021; BORGES, 2004). Nevertheless, this isn't sufficient to guarantee exogenous movement.

On this note, Miller (2013) studies student achievement during and around principal transitions, and documents an Ashenfelter dip in achievement starting before principal changes, with student achievement returning to average levels two years after transitions.[3] On the other hand, Andrade et al. (2018) investigate principal turnover in the city of Rio de Janeiro following an increase in accountability measures to principals[4], and find that, under the new measures, student achievement influences the probability of reconduction or reelection, estimated through logistic regressions. This is in line with other studies that also find student achievement influencing principal turnover (DEANGELIS; WHITE, 2011; SHEPPARD, 2010). Loeb et al. (2010) document that principals in the USA actively seek higher-performing schools as future career paths, once again highlighting the endogeneity of student achievement and principal turnover. These works show that a more careful analysis of principal turnover in our sample should be considered.

[3] An Ashenfelter dip is a dip in observed data relative to its average before an event, with improvement after the event. It characterizes the ambiguity that arises from this pattern: is the improvement after the event a simple return to the mean, or is it a causal effect of the event being studied? In the case at hand, is the improvement after principal change a return to the temporal average, or the causal effect of a new principal entering the school?

[4] Recall the debate on the inclusion of more technical selection methods for principals and higher school accountability measures on education goals discussed by Simelli et al. (2023) and Muñoz et al. (2021). The school accountability measures Andrade et al. (2018) discuss are closely related to the nation-wide debate.

9 Conclusion

This research set out to investigate the effects principals have on student learning outcomes in Brazil. We make use of a five-year dataset on Minas Gerais public school system, with mathematics and Portuguese standardized score data for over 600 thousand students in their last basic schooling year in over 2000 schools. In seeking to employ a value-added approach, we managed to pair students with their respective schools, teachers and principals over the 2015-2019 period by linking several datasets from different sources. As reviewed, a value-added approach aims at isolating the specific contribution of factors influencing student learning, in which principal effects are our main interest, and estimated empirically by fixed effects models. We also propose an additional step in the school management literature: to link a management score, as measured by management practices adoption in several management domains, to principal value-added.

Taken together, this research presents several novel efforts. To the best of our knowledge, it is the first estimation of principal value-added using Brazilian data. The reasons became evident throughout the work: stringent data demands, as with any value-added modelling, coupled with the complexity of linking several datasets from different sources while bypassing incompatibilities, proved extremely difficult. This challenge is seldom overcome in developing countries, such as Brazil, rendering studies in developed countries in Western Europe and North America the norm in the literature, which means our analysis extrapolates the Brazilian case and contributes to principal effect research in developing countries more broadly. Insofar as the literature review undertaken, this is also the first attempt at estimating principal, school and teacher effects together. With teachers acting as the main mediation channel for principal influence on students, their joint analysis was both coveted and cautioned by the literature. We regard the added institutional restraints Brazilian principals face in this matter as enablers for this joint estimation. Lastly, this is also the first time school management analysis through management practice adoption scores are associated specifically with principal value-added measures. By estimating these measures, we can pinpoint the specific contribution that principals had to student learning and associate this measure with the management score made available to us.

We conducted our analysis through two different classes of models. First, we proposed three different parametric value-added models. Departing from the model commonly seen in the principal effect literature, which couples principal and school fixed effects identified employing principal transitions, we sought to introduce teacher composition into the analysis. Teacher components in principal value-added models are warily discussed in the literature due to the role principals exert in teacher hiring and retention. As discussed in Chapter 3, Brazilian principals have much less authority in teacher selection, which motivates our inclusion of teacher effects in our value-added models. Detailed in Chapter 6, we first propose a model focused on teacher effects with a similar identifying strategy as the standard model and only then propose a model with all three factors: principals, teachers and schools. In this exercise, since we couldn't link principals and teachers over time individually, we studied teacher compositions: the group of teachers responsible for a subject in a grade in a given year.

Estimation results for these parametric value-added models were presented in the first section of Chapter 7. Neither of the specifications proposed were capable of completely separating principal effects from the others being analyzed, be they school or teacher composition effects. Our model with principal and school effects, standard in the literature, exhibited very high estimates for principal fixed effects, 250 points for mathematics and 247 points for Portuguese in our preferred specification, roughly the PROEB mean, or 5 standard deviations. We begin to comprehend these results by pointing out that only a handful of schools were not excluded and had their fixed effects estimated. Analyzed jointly, we understand that these effects were not successfully separated. The same pattern of results is obtained in the model with teacher composition and school effects. A different pattern is observed in the complete model, with all three factor fixed effects: results are very small in magnitude, averaging 0.0390 points in mathematics and -0.0279 points in Portuguese in our preferred specification. The same logic is at play in these results, in which excluded schools and teacher compositions lead to an unsuccessful separation between factor effects, which are in turn bundled in teacher composition effects, leaving principal fixed effect estimates very close to zero.

The reasons for an unsuccessful separation of principal, school and teacher composition effects in estimation are discussed in Chapters 7 and 8. Two key factors are at the core of this result: the absence of longitudinal student data, which compromised the necessary basis of comparison for growth in students' outcomes; and the scarcity of principal transitions in our Minas Gerais data, which led us to use principal changes, allowing us to observe two principals in charge of a single school, but precluding the other half, that is, principals acting in two different schools. This last point elucidates the reason why comparison groups were too small, which led to an overparametrization of our models and the exclusion of relevant factors in estimation. It also signals that, to expand on this research agenda, focus must be maintained on principal transitions for identification of the relevant fixed effects, and that other surging solutions to overparametrized models, like the LASSO regression, might not prove fruitful.

Despite the unexpected results in principal value-added estimation, we follow up with their association with management scores. This management instrument was described in Chapter 3 and each domain's association with principal value-added were presented in the last section of Chapter 7. These estimate coefficients exhibit reduced explanation power, something that may be inherited from the difficulties in principal value-added estimation just discussed. A renewed round of associations must be considered once principal value-added measures are correctly disentangled from other effects at play. At such a moment, taking management adoption probability into account by exploiting the journey categories in the instrument is a clear path to expand on this research topic.

The second class of models proposed are semi-parametric models exploring within-school variation related to principal value-added. Both model inspiration and proposed expansion are discussed in length in Chapter 6. The central appeal of this model is that it eliminates both problems faced in our parametric estimates, namely the absence of longitudinal data and principal transitions between schools. By looking at principal effect variation in schools due to principal turnover, it requires neither student-specific information nor principal-school matching. However, this comes at the cost of less direct and more complex interpretation of model results. Higher principal effect variance is associated with scores 11.40% of a standard deviation higher in mathematics and 13.45% of a standard deviation higher in Portuguese. When considering variance in average teacher effects in our model, these values drop slightly to 10.22% of a standard deviation higher in mathematics and 12.19% of a standard deviation higher in Portuguese. This reduction in magnitude emphasizes the role teachers play in mediation of principal influences at the same time that it showcases principal effects not related to teachers. Paradoxically, when we restrict our analysis to schools with PROEB grades available for the whole period, we find that schools with a higher principal effect variance are associated with scores 8.67% of a standard deviation lower in mathematics but 8.01% of a standard deviation higher in Portuguese. We argue that this may be observed due to higher principal turnover or lower scores in turnover years, for which there is evidence in the literature. Nevertheless, further study is necessary to better determine this inversion in results.

This research proposed a deep dive into principal effects in Brazilian school, with several novel features proposed. Many data and institutional difficulties were faced in principal value-added estimation, some of which could not be completely overcome, resulting in unsuccessful separation of principal effects and still frail estimates from the standard fixed effects model. These setbacks also hindered the implementation of teacher effects into the value-added models, as well as their association with the management practice adoption instrument. A semi-parametric within-school variation analysis is implemented and better supports findings in the literature, with principal effects averaging from 12% to 8% influence in mathematics and Portuguese standardized testscores. Despite the difficulties faced, this thesis highlights some important findings in principal effects in a developing country and points future research topics in this subject.

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Appendix

APPENDIX A – Within School Variance in Principal Effects Model Derivations

A. Within School Principal Effect Variance Derivation

In this section, we mirror the derivation of the within-school principal effect variance term by Coelli & Green (2012). These passages are available in Appendix B of their paper. However, we expand on its steps here to accompany the derivation of the extension proposed.

We begin by rewriting the first term on the right-hand side of equation (4.11).

$$E[(\theta_{st} - \bar{\theta}_s)^2] = E[\theta_{st}^2 + \bar{\theta}_s^2 - 2\theta_{st}\bar{\theta}_s]. \tag{A.1}$$

Following the demonstration by Coelli & Green (2012), we present here a specific and simpler case and later generalize our results. Suppose there are only two principals in charge of school s during the observed period, T. Let these principal be labelled j and k, with principal effects θ_j and θ_k, respectively. Suppose each principal stays three years in charge of the school ($T = 6$). Let us also consider a year in which principal j is in charge (which is equivalent to say that j indexes st). Developing on equation (A.1), our expectation term then looks:

$$\begin{aligned}
E[\theta_j^2 + \bar{\theta}_s^2 - 2\theta_j\bar{\theta}_s] &= E[\theta_j^2] + E\left[\left(\frac{1}{6}(\theta_j + \theta_j + \theta_j + \theta_k + \theta_k + \theta_k)\right)^2\right] - \\
&\quad - 2E\left[\theta_j\left(\frac{1}{6}(\theta_j + \theta_j + \theta_j + \theta_k + \theta_k + \theta_k)\right)\right] \\
&= E[\theta_j^2] + E\left[\left(\frac{1}{6}(3\theta_j + 3\theta_k)\right)^2\right] - 2E\left[\theta_j\left(\frac{1}{6}(3\theta_j + 3\theta_k)\right)\right] \\
&= E[\theta_j^2] + E\left[\frac{1}{6^2}(9\theta_j + 9\theta_k + 18\theta_j\theta_k)\right] - 2E\left[\theta_j\left(\frac{1}{6}(3\theta_j + 3\theta_k)\right)\right] \\
&= E[\theta_j^2] + \frac{1}{6^2}(3^2)E[\theta_j^2] + \frac{1}{6^2}(3^2)E[\theta_k^2] + \frac{1}{6^2}(18)E[\theta_j\theta_k] - \\
&\quad - \frac{2}{6}(3)E[\theta_j^2] - \frac{2}{6}(3)E[\theta_j\theta_k].
\end{aligned}$$

Under the assumption that each principal is a random draw from the principal distribution, we have that $E[\theta_j\theta_k] = 0$. By definition, $E[\theta_j^2] = \sigma_{\theta_s}^2$.

$$\begin{aligned}
E[\theta_j^2 + \bar{\theta}_s^2 - 2\theta_j\bar{\theta}_s] &= \sigma_{\theta_s}^2 + \frac{1}{6^2}(3^2 + 3^2)\sigma_{\theta_s}^2 - \frac{2}{6}(3)\sigma_{\theta_s}^2 \\
&= \frac{1}{2}\sigma_{\theta_s}^2.
\end{aligned}$$

This means that for the case of two principals, with both staying an equal amount of time in charge of the school, the deterministic term proposed by Coelli & Green (2012) equals one half. Remember that this deterministic term is positive and increasing in principal turnover, which means that in a case in which three principals stay an equal amount of years in charge of a school, this term will equal two-thirds (and not one-third).

For the general case, let J be the total number of principals, and let k index principals, such that $k = 1, \dots, J$. Let q_k be the number of years principal k spends in charge of the school, so that $\sum_{k=1}^{J} q_k = T$. Once again taking a year in which principal j is in charge.

$$\begin{aligned} E[\theta_j^2 + \bar{\theta}_s^2 - 2\theta_j\bar{\theta}_s] &= E[\theta_j^2] + \frac{1}{T^2} E\left[\left(\sum_{t=1}^{T} \theta_{st}\right)^2\right] - 2\frac{1}{T} E\left[\theta_j \sum_{t=1}^{T} \theta_{st}\right] \\ &= E[\theta_j^2] + \frac{1}{T^2} E\left[\left(\sum_{k=1}^{J} q_k\theta_k\right)\right] - \frac{2}{T} E\left[\theta_j \sum_{k=1}^{J} q_k\theta_k\right]. \end{aligned}$$

Again, under the assumption that principals come from a random draw of the principal distribution, $E[\theta_j\theta_k] = 0$.

$$\begin{aligned} E[\theta_j^2 + \bar{\theta}_s^2 - 2\theta_j\bar{\theta}_s] &= E[\theta_j^2] + \frac{1}{T^2} E\left[\sum_{k=1}^{J} q_k^2\theta_k^2\right] - \frac{2}{T} E\left[q_j\theta_j^2\right] \\ &= E[\theta_j^2] + \frac{1}{T^2} \sum_{k=1}^{J} q_k^2 E[\theta_k^2] - \frac{2}{T} q_j E[\theta_j^2]. \end{aligned}$$

By definition $E[\theta_j^2] = \sigma_{\theta_s}^2$.

$$\begin{aligned} E[\theta_j^2 + \bar{\theta}_s^2 - 2\theta_j\bar{\theta}_s] &= \sigma_{\theta_s}^2 + \frac{1}{T^2} \sum_{k=1}^{J} q_k^2\sigma_{\theta_s}^2 - \frac{2}{T} q_j\sigma_{\theta_s}^2 \\ &= \sigma_{\theta_s}^2 \left[1 + \frac{1}{T^2} \sum_{k=1}^{J} q_k^2 - \frac{2}{T} q_j\right]. \end{aligned}$$

Which is the deterministic term developed by Coelli & Green (2012) and described in equation (4.12).

B. Within School Average Teacher Effect Variance Derivation

In this section, we demonstrate how we obtained the deterministic term that accompanies the within-school variance in average teacher quality, expressed in equation (4.30). We follow closely the derivation steps taken by Coelli & Green (2012), adding the necessary assumptions to mirror theirs on principal effect behaviour.

We start from equation (4.21).

$$E[(\bar{\pi}_{st} - \bar{\pi}_s)^2] = E[\bar{\pi}_{st}^2 + \bar{\pi}_s^2 - 2\bar{\pi}_{st}\bar{\pi}_s].$$

Using the fact that $\bar{\pi}_s = \frac{1}{T}\sum_{t=1}^{T}\bar{\pi}_{st}$.

$$E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] = E[\bar{\pi}_{st}^2] + E\left[\left(\frac{1}{T}\sum_{t=1}^{T}\bar{\pi}_{st}\right)^2\right] - 2E\left[\bar{\pi}_{st}\left(\frac{1}{T}\sum_{t=1}^{T}\bar{\pi}_{st}\right)\right].$$

Applying $\bar{\pi}_{st} = \sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}$ from equation (4.17).

$$\begin{aligned} E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] = E\left[\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}\right)^2\right] + E\left[\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}\right)^2\right] - \\ - 2E\left[\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}\right)\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}\right)\right]. \end{aligned} \tag{A.2}$$

Let us start with a simplified case. Consider a school in which two teachers, j and k (with effects π_k and π_k, respectively), each teach a classroom in the grade of interest. We observe this scenario for two years ($T = 2$), where each classroom has n_{j1}, n_{j2}, n_{k1} and n_{k2} students, respectively, such that $n_{j1} + n_{k1} = N_1$ and $n_{j2} + n_{k2} = N_2$. Our expectation term looks like this:

$$\begin{aligned} E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] =& E\left[\left(\frac{n_{j1}\pi_{j1}+n_{k1}\pi_{k1}}{N_1}\right)^2\right] + E\left[\left(\frac{1}{T}\left(\frac{n_{j1}\pi_{j1}+n_{k1}\pi_{k1}}{N_1}+\frac{n_{j2}\pi_{j2}+n_{k2}\pi_{k2}}{N_2}\right)\right)^2\right] - \\ &- 2E\left[\left(\frac{n_{j1}\pi_{j1}+n_{k1}\pi_{k1}}{N_1}\right)\left(\frac{1}{T}\left(\frac{n_{j1}\pi_{j1}+n_{k1}\pi_{k1}}{N_1}+\frac{n_{j2}\pi_{j2}+n_{k2}\pi_{k2}}{N_2}\right)\right)\right]. \end{aligned}$$

Since we assume teacher effects are time-invariant (which also implies that teachers exert the same effect on different classrooms), we have that $E[\pi_{j1}] = E[\pi_{j2}]$ and $E[\pi_{k1}] = E[\pi_{k2}]$. We can then drop the time subscript for teacher effects.

$$\begin{aligned} E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] =& E\left[\left(\frac{n_{j1}\pi_{j}+n_{k1}\pi_{k}}{N_1}\right)^2\right] + E\left[\left(\frac{1}{T}\left(\frac{n_{j1}\pi_{j}+n_{k1}\pi_{k}}{N_1}+\frac{n_{j2}\pi_{j}+n_{k2}\pi_{k}}{N_2}\right)\right)^2\right] - \\ &- 2E\left[\left(\frac{n_{j1}\pi_{j}+n_{k1}\pi_{k}}{N_1}\right)\left(\frac{1}{T}\left(\frac{n_{j1}\pi_{j}+n_{k1}\pi_{k}}{N_1}+\frac{n_{j2}\pi_{j}+n_{k2}\pi_{k}}{N_2}\right)\right)\right]. \end{aligned} \tag{A.3}$$

Let's look at each term individually. Starting with the first term on the right-hand side.

$$\begin{aligned} E\left[\left(\frac{n_{j1}\pi_{j}+n_{k1}\pi_{k}}{N_1}\right)^2\right] =& E\left[\left(\frac{n_{j1}\pi_{j}+n_{k1}\pi_{k}}{N_1}\right)\left(\frac{n_{j1}\pi_{j}+n_{k1}\pi_{k}}{N_1}\right)\right] \\ =& E\left[\frac{n_{j1}^2\pi_j^2+n_{k1}^2\pi_k^2+2n_{j1}n_{k1}\pi_j\pi_k}{N^1}\right] \\ =& \frac{1}{N_1^2}\left(n_{j1}^2E[\pi_j^2]+n_{k1}^2E[\pi_k^2]+2n_{j1}n_{k1}E[\pi_j\pi_k]\right). \end{aligned}$$

Following our third assumption on teacher behaviour, teachers are independent and drawn from a common distribution, $E[\pi_j\pi_k] = 0$.

$$E\left[\left(\frac{n_{j1}\pi_{j}+n_{k1}\pi_{k}}{N_1}\right)^2\right] = \frac{1}{N_1^2}(n_{j1}^2E[\pi_j^2]+n_{k1}^2E[\pi_k^2]).$$

By definition, $E[\pi_j^2] = E[\pi_k^2] = \sigma^2_{\bar{\pi}_s}$.

$$E\left[\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1}\right)^2\right] = \sigma^2_{\bar{\pi}_s}\left[\frac{(n_{j1}^2 + n_{k1}^2)}{N_1^2}\right]. \tag{A.4}$$

Moving on to the second expression on the right-hand side of equation (A.3).

$$E\left[\left(\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2}\right)\right)^2\right] = E\left[\left(\frac{1}{T}\frac{N_2(n_{j1}\pi_j + n_{k1}\pi_k) + N_1(n_{j2}\pi_j + n_{k2}\pi_k)}{N_1N_2}\right)^2\right]$$
$$= \frac{1}{T^2}E\left[\frac{N_2^2(n_{j1}\pi_j + n_{k1}\pi_k)^2 + N_1^2(n_{j2}\pi_j + n_{k2}\pi_k)^2 + 2N_1N_2(n_{j1}\pi_j + n_{k1}\pi_k)(n_{j2}\pi_j + n_{k2}\pi_k)}{N_1^2N_2^2}\right].$$

Once again, using the assumption of teacher effect independence, $E[\pi_j\pi_k] = 0$.

$$E\left[\left(\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2}\right)\right)^2\right] =$$
$$= \frac{1}{T^2}E\left[\frac{N_2^2(n_{j1}^2\pi_j^2 + n_{k1}^2\pi_k^2) + N_1^2(n_{j2}^2\pi_j^2 + n_{k2}^2\pi_k^2) + 2N_1N_2(n_{j1}n_{j2}\pi_j^2 + n_{k1}n_{k2}\pi_k^2)}{N_1^2N_2^2}\right]$$
$$= \frac{1}{T^2}\left[\frac{N_2^2(n_{j1}^2E[\pi_j^2] + n_{k1}^2E[\pi_k^2]) + N_1^2(n_{j2}^2E[\pi_j^2] + n_{k2}^2E[\pi_k^2]) + 2N_1N_2(n_{j1}n_{j2}E[\pi_j^2] + n_{k1}n_{k2}E[\pi_k^2])}{N_1^2N_2^2}\right].$$

And again, by definition, $E[\pi_j^2] = E[\pi_k^2] = \sigma^2_{\bar{\pi}_s}$.

$$\begin{aligned} E\left[\left(\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2}\right)\right)^2\right] &= \\ = \sigma^2_{\bar{\pi}_s}\left[\frac{1}{T^2}\frac{(N_2^2(n_{j1}^2 + n_{k1}^2) + N_1^2(n_{j2}^2 + n_{k2}^2) + 2N_1N_2(n_{j1}n_{j2} + n_{k1}n_{k2})}{N_1^2N_2^2}\right] \\ = \sigma^2_{\bar{\pi}_s}\left[\frac{1}{T^2}\left(\frac{(n_{j1}^2 + n_{k1}^2)}{N_1^2} + \frac{(n_{j2}^2 + n_{k2}^2)}{N_2^2} + \frac{2(n_{j1}n_{j2} + n_{k1}n_{k2})}{N_1N_2}\right)\right]. \end{aligned} \tag{A.5}$$

Finally, tackling the third element on the right-hand side of equation (A.3).

$$2E\left[\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1}\right)\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2}\right)\right] =$$
$$= \frac{2}{T}E\left[\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1}\right)\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2}\right)\right]$$
$$= \frac{2}{T}E\left[\frac{N_2(n_{j1}\pi_j + n_{k1}\pi_k)^2 + N_1(n_{j1}\pi_j + n_{k1}\pi_k)(n_{j2}\pi_j + n_{k2}\pi_k)}{N_1^2N_2}\right].$$

Using the assumption of no teacher effect spillovers one last time, $E[\pi_j\pi_k] = 0$.

$$2E\left[\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1}\right)\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2}\right)\right] =$$
$$= \frac{2}{T}E\left[\frac{N_2(n_{j1}^2\pi_j^2 + n_{k1}^2\pi_k^2) + N_1(n_{j1}n_{j2}\pi_j^2 + n_{k1}n_{k2}\pi_k^2)}{N_1^2N_2}\right]$$
$$= \frac{2}{T}\left[\frac{N_2(n_{j1}^2E[\pi_j^2] + n_{k1}^2E[\pi_k^2]) + N_1(n_{j1}n_{j2}E[\pi_j^2] + n_{k1}n_{k2}E[\pi_k^2])}{N_1^2N_2}\right].$$

By definition, $E[\pi_j^2] = E[\pi_k^2] = \sigma^2_{\bar{\pi}_s}$.

$$\begin{aligned}
2E\left[\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1}\right)\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2}\right)\right] = \\
= \sigma^2_{\bar{\pi}_s}\left[\frac{2}{T}\left(\frac{N_2(n_{j1}^2 + n_{k1}^2) + N_1(n_{j1}n_{j2} + n_{k1}n_{k2})}{N_1^2 N_2}\right)\right] \\
= \sigma^2_{\bar{\pi}_s}\left[\frac{2}{T}\left(\frac{(n_{j1}^2 + n_{k1}^2)}{N_1^2} + \frac{(n_{j1}n_{j2} + n_{k1}n_{k2})}{N_1 N_2}\right)\right].
\end{aligned} \tag{A.6}$$

With this, we can finally gather all the terms. Substituting equations (A.4), (A.5) and (A.6) in (A.3).

$$\begin{aligned}
E[(\bar{\pi}_{st} - \bar{\pi}_s)^2] =& \sigma^2_{\bar{\pi}_s}\left\{\frac{n_{j1}^2 + n_{k1}^2}{N_1^2} + \dots\right\} \\
&\left\{\dots + \left[\frac{1}{T^2}\left(\frac{(n_{j1}^2 + n_{k1}^2)}{N_1^2} + \frac{(n_{j2}^2 + n_{k2}^2)}{N_2^2} + \frac{2(n_{j1}n_{j2} + n_{k1}n_{k2})}{N_1 N_2}\right)\right] - \dots\right\} \\
&\left\{\dots - \left[\frac{2}{T}\left(\frac{(n_{j1}^2 + n_{k1}^2)}{N_1^2} + \frac{(n_{j1}n_{j2} + n_{k1}n_{k2})}{N_1 N_2}\right)\right]\right\}.
\end{aligned} \tag{A.7}$$

Should we consider an additional time period (N_3), how would this expression change? Again, analyzing all right-hand side elements of equation (A.3) individually. Firstly, recall equation (A.4) and notice it does not depend on any time averages, which implies it remains unchanged in our three-period case.[1]

Analyzing the second element.

$$\begin{aligned}
&E\left[\left(\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2} + \frac{n_{j2}\pi_j + n_{k3}\pi_k}{N_3}\right)\right)^2\right] = \\
&= E\left[\left(\frac{1}{T}\frac{N_2N_3(n_{j1}\pi_j + n_{k1}\pi_k) + N_1N_3(n_{j2}\pi_j + n_{k2}\pi_k) + N_1N_2(n_{j2}\pi_j + n_{k3}\pi_k)}{N_1N_2N_3}\right)^2\right] \\
&= \frac{1}{T^2}E\left[\frac{N_2^2N_3^2(n_{j1}\pi_j + n_{k1}\pi_k)^2 + N_1^2N_3^2(n_{j2}\pi_j + n_{k2}\pi_k)^2 + N_1^2N_2^2(n_{j3}\pi_j + n_{k3}\pi_k)^2}{N_1^2N_2^2N_3^2} + \dots\right] \\
&\left[\dots + \frac{2N_1N_2N_3^2(n_{j1}\pi_j + n_{k1}\pi_k)(n_{j2}\pi_j + n_{k2}\pi_k) + 2N_1N_2^2N_3(n_{j1}\pi_j + n_{k1}\pi_k)(n_{j3}\pi_j + n_{k3}\pi_k)}{N_1^2N_2^2N_3^2} + \dots\right] \\
&\left[\dots \frac{2N_1^2N_2N_3(n_{j2}\pi_j + n_{k2}\pi_k)(n_{j3}\pi_j + n_{k3}\pi_k)}{N_1^2N_2^2N_3^2}\right] \\
&= \frac{1}{T^2}E\left[\frac{(n_{j1}\pi_j + n_{k1}\pi_k)^2}{N_1^2} + \frac{(n_{j2}\pi_2 + n_{k2}\pi_k)^2}{N_2^2} + \frac{(n_{j3}\pi_j + n_{k3}\pi_k)^2}{N_3^2} + \dots\right] \\
&\left[\dots + \frac{2(n_{j1}\pi_j + n_{k1}\pi_k)(n_{j2}\pi_j + n_{k2}\pi_k)}{N_1N_2} + \frac{2(n_{j1}\pi_j + n_{k1}\pi_k)(n_{j3}\pi_j + n_{k3}\pi_k)}{N_1N_3} + \dots\right] \\
&\left[\dots + \frac{2(n_{j2}\pi_j + n_{k2}\pi_k)(n_{j3}\pi_j + n_{k3}\pi_k)}{N_2N_3}\right].
\end{aligned}$$

[1] Recall the time subscript in this term refers to the year being analyzed. This also means that differently from the Coelli & Green (2012) application on Canadian schools, our application will make use of a school panel to take teachers into account, instead of a simple school cross-section.

Once again, using the assumption of teacher effect independence, $E[\pi_j\pi_k] = 0$.

$$E\left[\left(\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2} + \frac{n_{j2}\pi_j + n_{k3}\pi_k}{N_3}\right)\right)^2\right] =$$
$$= \frac{1}{T^2}E\left[\frac{(n_{j1}^2\pi_j^2 + n_{k1}^2\pi_k^2)}{N_1^2} + \frac{(n_{j2}^2\pi_2^2 + n_{k2}^2\pi_k^2)}{N_2^2} + \frac{(n_{j3}^2\pi_j^2 + n_{k3}^2\pi_k^2)}{N_3^2} + \frac{2(n_{j1}n_{j2}\pi_j^2 + n_{k1}n_{k2}\pi_k^2)}{N_1N_2} + \ldots\right]$$
$$\left[\ldots + \frac{2(n_{j1}n_{j3}\pi_j^2 + n_{k1}n_{k3}\pi_k^2)}{N_1N_3} + \frac{2(n_{j2}n_{j2}\pi_j^2 + n_{k2}n_{k3}\pi_k^2)}{N_2N_3}\right].$$

Distributing the expectation and using the definition, $E[\pi_j^2] = E[\pi_k^2] = \sigma^2_{\bar{\pi}_s}$.

$$E\left[\left(\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2} + \frac{n_{j2}\pi_j + n_{k3}\pi_k}{N_3}\right)\right)^2\right] =$$
$$= \sigma^2_{\bar{\pi}_s}\frac{1}{T^2}\left[\frac{(n_{j1}^2 + n_{k1}^2)}{N_1^2} + \frac{(n_{j2}^2 + n_{k2}^2)}{N_2^2} + \frac{(n_{j3}^2 + n_{k3}^2)}{N_3^2} + \frac{2(n_{j1}n_{j2} + n_{k1}n_{k2})}{N_1N_2} + \ldots\right] \quad \text{(A.8)}$$
$$\left[\ldots + \frac{2(n_{j1}n_{j3} + n_{k1}n_{k3})}{N_1N_3} + \frac{2(n_{j2}n_{j2} + n_{k2}n_{k3})}{N_2N_3}\right].$$

Lastly, studying the third element of equation (A.3) in this three-period case.

$$2E\left[\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1}\right)\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2} + \frac{n_{j3}\pi_j + |n_{k3}\pi_k}{N_3}\right)\right] =$$
$$= \frac{2}{T}\left[\frac{(n_{j1}\pi_j + n_{k1}\pi_k)^2}{N_1^2} + \frac{(n_{j1}\pi_j + n_{k1}\pi_k)(n_{j2}\pi_j + n_{k2}\pi_k)}{N_1N_2} + \frac{(n_{j1}\pi_j + n_{k1}\pi_k)(n_{j3}\pi_j + n_{k3}\pi_k)}{N_1N_3}\right]. \quad \text{(A.9)}$$

Using the assumption of independent teacher effects, $E[\pi_j\pi_k] = 0$.

$$2E\left[\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1}\right)\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2} + \frac{n_{j3}\pi_j + |n_{k3}\pi_k}{N_3}\right)\right] =$$
$$= \frac{2}{T}\left[\frac{(n_{j1}^2\pi_j^2 + n_{k1}^2\pi_k^2)}{N_1^2} + \frac{(n_{j1}n_{j2}\pi_j^2 + n_{k1}n_{k2}\pi_k^2}{N_1N_2} + \frac{(n_{j1}n_{j3}\pi_j^2 + n_{k1}n_{k3}\pi_k^2)}{N_1N_3}\right]. \quad \text{(A.10)}$$

Finally, distributing the expectation and using the definition $E[\pi_j^2] = E[\pi_k^2] = \sigma^2_{\bar{\pi}_s}$.

$$2E\left[\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1}\right)\frac{1}{T}\left(\frac{n_{j1}\pi_j + n_{k1}\pi_k}{N_1} + \frac{n_{j2}\pi_j + n_{k2}\pi_k}{N_2} + \frac{n_{j3}\pi_j + |n_{k3}\pi_k}{N_3}\right)\right] =$$
$$= \sigma^2_{\bar{\pi}_s}\left[\frac{2}{T}\left(\frac{(n_{j1}^2 + n_{k1}^2)}{N_1^2} + \frac{(n_{j1}n_{j2} + n_{k1}n_{k2})}{N_1N_2} + \frac{(n_{j1}n_{j3} + n_{k1}n_{k3})}{N_1N_3}\right)\right]. \quad \text{(A.11)}$$

Gathering these developed elements and substituting them in equation (A.3).

$$
\begin{aligned}
E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] = \sigma^2_{\bar{\pi}_s}&\left\{\frac{(n^2_{j1}+n^2_{k1})}{N^2_1}+\dots\right\}\\
&\left\{\dots+\frac{1}{T^2}\left[\frac{(n^2_{j1}+n^2_{k1})}{N^2_1}+\frac{(n^2_{j2}+n^2_{k2})}{N^2_2}+\frac{(n^2_{j3}+n^2_{k3})}{N^2_3}+\frac{2(n_{j1}n_{j2}+n_{k1}n_{k2})}{N_1N_2}+\dots\right]\right\}\\
&\left\{\left[\dots+\frac{2(n_{j1}n_{j3}+n_{k1}n_{k3})}{N_1N_3}+\frac{2(n_{j2}n_{j2}+n_{k2}n_{k3})}{N_2N_3}\right]-\dots\right\}\\
&\left\{\dots-\frac{2}{T}\left(\frac{(n^2_{j1}+n^2_{k1})}{N^2_1}+\frac{(n_{j1}n_{j2}+n_{k1}n_{k2})}{N_1N_2}+\frac{(n_{j1}n_{j3}+n_{k1}n_{k3})}{N_1N_3}\right)\right\}.
\end{aligned}
\tag{A.12}
$$

We now generalize the results obtained in (A.7) and (A.12). As defined in equation (4.17), the average teacher effect in school s and cohort t is the sum of all teacher effects in a given grade's classrooms weighted by each classroom's share of students from that grade. Simply put $\bar{\pi}_{st}=\frac{1}{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\bar{\pi}_{stc}$, where C is the total number of classrooms in the grade, such that $c=1,\dots,C$. Since the model deals with student achievement in a single grade, the grade subscripts are dropped for clarity. We depart from equation (A.2) and analyze each element individually for the sake of clarity. Beginning with the first right-hand side element.

$$
E\left[\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}\right)^2\right]=E\left[\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2\pi^2_{stc}\right], \tag{A.13}
$$

in which we make use of the independent teacher effects assumption.

$$
E\left[\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2\pi^2_{stc}\right]=E\left[\pi^2_{stc}\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2\right], \tag{A.14}
$$

in which we make use of the assumption that teacher effects are time and classroom invariant. Notice the summation term after the teacher effect is a deterministic term, so that:

$$
E\left[\pi^2_{stc}\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2\right]=E[\pi^2_{stc}]\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2=\sigma^2_{\bar{\pi}_s}\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2. \tag{A.15}
$$

Moving on to the second element in (A.7).

$$
E\left[\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_ct}{N_t}\pi^2_{stc}\right)^2\right]=E\left[\left(\frac{1}{T}\pi_{stc}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)^2\right],
$$

in which we make use of the teacher effect invariance across time and classroom assumptions.

$$
E\left[\left(\frac{1}{T}\pi_{stc}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)^2\right]=E\left[\frac{1}{T^2}\pi^2_{stc}\left(\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)^2\right],
$$

in which we make use of the independence of teacher effects assumption. Again, the summation after the teacher effect is a deterministic term. We can then rewrite it as:

$$E\left[\frac{1}{T^2}\pi^2_{stc}\left(\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)^2\right] = E[\pi^2_{stc}]\frac{1}{T^2}\left(\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)^2 = \sigma^2_{\bar{\pi}_s}\frac{1}{T^2}\left(\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)^2.$$

We want to further develop the deterministic term in the above expression. The case studies (A.5) and (A.8) are useful illustrations for what follows[2].

$$\frac{1}{T^2}\left(\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)^2 = \frac{1}{T^2}\left[\sum_{t=1}^{T}\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2 + 2\sum_{c=1}^{C}\sum_{t=1}^{T}\sum_{u>t}\frac{n_{ct}}{N_t}\frac{n_{cu}}{N_u}\right]. \tag{A.16}$$

Finally, the third element in expression (A.7) follows the same development path.

$$E\left[\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}\right)\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\pi_{stc}\right)\right] = E\left[\pi_{stc}\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\left(\frac{1}{T}\pi_{stc}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\right],$$

in which we make use of the teacher effect invariance across time and classrooms.

$$E\left[\pi_{stc}\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\left(\frac{1}{T}\pi_{stc}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\right] = E\left[\pi^2_{stc}\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\right],$$

in which we make use of teacher effect independence assumption. The summation elements accompanying the teacher effects are deterministic, so we distribute the expectations.

$$E\left[\pi^2_{stc}\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\right] = E[\pi^2_{stc}]\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right) = \sigma^2_{\bar{\pi}_s}\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right).$$

We further develop the deterministic element in the above expression, once again drawing from the case studies in (A.6) and (A.11).

$$\left(\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right)\left(\frac{1}{T}\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\right) = \frac{1}{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\left(\sum_{t=1}^{T}\frac{n_{cv}}{N_v}\right). \tag{A.17}$$

Therefore, we can gather all developed elements into equation (A.2).

$$E[(\bar{\pi}_{st}-\bar{\pi}_s)^2] = \sigma^2_{\bar{\pi}_s}\left[\sum_{c=1}^{C}\left(\frac{n_{ct}}{N_t}\right)^2 + \frac{1}{T^2}\left(\sum_{t=1}^{T}\sum_{c=1}^{C}\frac{n^2_{ct}}{N^2_t} + 2\sum_{c=1}^{C}\sum_{t=1}^{T}\sum_{u>t}\frac{n_{ct}}{N_t}\frac{n_{cu}}{N_u}\right) - \frac{2}{T}\sum_{c=1}^{C}\frac{n_{ct}}{N_t}\left(\sum_{v=1}^{T}\frac{n_{cv}}{N_v}\right)\right],$$

which is exactly the proposed expression in equation (4.30).

[2] Notice that the inversion of $\sum^T\sum^C$ for $\sum^C\sum^T$ is a direct consequence of applying the teacher effect independence assumption.

APPENDIX B – Figures and Tables

Table B.1 – Classroom name and school code matching name alterations per round

Matching Round	Alterations
1^{st} round	Basic Match
2^{nd} round	"•" or "ª" → "º"
3^{rd} and odd rounds	"º" → ""
4^{th} round	"/" → " "
6^{th} round	"_2023" → "2023"
8^{th} round	"-" → "_"
10^{th} round	"_2023" → "2023"
12^{th} round	"3_" → "3"
14^{th} round	"___" or "__" → "_"
16^{th} round	"T_" → "T"
18^{th} round	"_ANO" → "ANO"
20^{th} round	"2023" → ""
22^{nd} round	"_" → ""
24^{th} round	Add "TURMA_" at beginning

Notes: Table shows the alterations made to classroom names at every round of classroom name and school code matching. The alteration made in the third round also followed every other subsequent alteration. Underscores ("_") signify spaces for clarity. The word "ANO" translates to "YEAR", and the word "TURMA" translates to "CLASS". An explanation of the matching step is available in Chapter 5.

Table B.2 – Regression Control Sample Statistics

	Student Panel	Sample 1		Sample 2		Sample 3	
	(0)	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)
Schools	2294	848	1446	1841	984	628	2197
Panel A: School Controls							
Filtered Water (%)	0,9912	0,9893	0,9923	0,9918	0,9897	0,9904	0,9913
Untreated Sewer (%)	0,1741	0,1969	0,1603	0,1675	0,1961	0,191	0,1735
Internet Access (%)	0,9969	0,9964	0,9972	0,9978	0,9969	0,9984	0,9972
Teachers' Room (%)	0,9553	0,9599	0,9527	0,9586	0,955	0,9665	0,9547
Library (%)	0,9711	0,9716	0,9707	0,976	0,9683	0,9824	0,9707
Gymnasium (%)	0,7795	0,7523	0,7955	0,7965	0,7385	0,7754	0,7766
Television (%)	0,9566	0,9622	0,9534	0,9591	0,953	0,9633	0,9552
Multimedia Equipment (%)	0,9606	0,9705	0,9547	0,9608	0,9652	0,9713	0,9597
Multifunctional Printer (%)	0,7629	0,7594	0,7649	0,7698	0,7589	0,7802	0,7619
Average Rooms	11,89	11,26	12,27	12,36	11,01	11,89	11,89
Average Computers	18,45	17,34	19,11	18,87	17,36	17,75	18,52
Average 3º EM Students	62,43	53,14	67,83	68,12	49,31	59,86	62,44
Average Students per Classroom	56,01	53,58	57,45	57,91	49,95	56,18	54,85
Panel B: Teacher Controls							
Math Teachers	7,863	3,036	5,360	6,854	1,713	2,180	6,514
Female Math Teachers (%)	0,5859	0,5717	0,5942	0,5896	0,5709	0,5701	0,5868
Non-white Math Teachers (%)	0,41	0,4212	0,4035	0,4107	0,4224	0,4232	0,4123
Average Math Teacher Age	41,25	40,66	41,6	41,19	41,08	40,71	41,28
Math Teachers with Higher Education (%)	0,9626	0,9605	0,9638	0,9584	0,9676	0,9546	0,9636
Average Classrooms per Math Teacher	6,26	6,22	6,28	6,3	6,23	6,3	6,27
Portuguese Teachers	8,629	3,295	5,767	7,443	1,891	2,381	7,049
Female Portuguese Teachers (%)	0,8612	0,8531	0,8659	0,8604	0,8638	0,859	0,8623
Non-white Portuguese Teachers (%)	0,4138	0,4279	0,4055	0,4096	0,4325	0,4265	0,415
Average Portuguese Teacher Age	42,43	42,08	42,63	42,33	42,46	42,07	42,46
Portuguese Teachers with Higher Education (%)	0,9747	0,9704	0,9772	0,9744	0,974	0,9711	0,9751
Average Classrooms per Portuguese Teacher	6,34	6,25	6,4	6,37	6,27	6,3	6,35
Panel C: Student Controls							
Female Students (%)	0,5563	0,5559	0,5565	0,5596	0,5523	0,5598	0,5563
Non-white Students (%)	0,6905	0,6893	0,6911	0,6885	0,6936	0,6898	0,6904
Bolsa Família Beneficiary (%)	0,317	0,3338	0,3072	0,3136	0,3311	0,3311	0,3164
Flunking Record (%)	0,2406	0,2439	0,2387	0,2378	0,2448	0,2396	0,2404

Notes: Column (0) presents statistics on controls used for the whole panel, while columns (1) to (3) show the same statistics for the three samples considered in this research. Columns labelled with an *a* show statistics for the respective samples, whilst columns labelled with a "b" pshow the same statistics for each sample's complementary observations. Table panel A presents statistics on school controles; panel B presents statistics on teacher controls; and lastly, panel C provides statistics for student controls.

Non-white status follows the IBGE (*Instituto Brasileiro de Geografia Estatística*) PPI variable, a self-declaration based variable that includes black, brown and indigenous people, here referred to as "non-white", but that excludes Asian descendants. The *Bolsa Família* program is a microcredit policy granted to mothers with children enrolled in school. Families who benefit from the program are part of the *Cadastro Único*, a socioeconomic vulnerability registry in custody of the federal government.

Table B.3 – Model 1 value-added estimates distribution statistics

	Mathematics						Portuguese					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Average Principal Value-Added	267.1105	267.6514	253.6959	256.2822	256.6863	250.1242	269.1262	269.6772	243.3205	248.3782	248.3029	247.0646
Principal Value-Added Standard Deviation	18.6256	18.6646	16.9874	16.9237	16.8299	16.8594	16.7541	16.6851	14.6879	14.5730	14.5800	14.8212
Max Principal VA	356.0743	360.0626	360.3428	359.1324	359.2531	353.8573	322.2898	322.9584	297.0853	295.2677	294.6377	294.0048
90^{th} Quantile	290.0935	290.6674	275.0460	277.3163	277.7751	270.1199	290.4314	290.8842	261.2646	266.2212	266.2640	265.0154
75^{th} Quantile	478.8110	276.4036	263.7720	266.3487	266.5899	260.5688	280.2235	280.5553	252.7482	257.7876	257.5028	256.5511
50^{th} Quantile	266.1730	266.8034	252.5783	255.2604	255.6047	249.3859	269.8545	270.4723	243.9071	248.7425	248.7125	247.4662
25^{th} Quantile	254.5773	255.4620	242.5406	245.3677	245.3677	239.1450	258.0093	258.8122	233.9145	239.3510	239.1748	237.7580
10^{th} Quantile	244.1160	244.7245	233.5141	236.4442	237.0138	229.5479	247.9589	248.4771	225.5056	230.7817	230.7450	229.0979
Min Principal VA	202.8880	203.3419	192.7823	200.7168	201.8779	189.7879	180.9462	181.4987	162.9655	169.2478	167.9925	159.3620
Observations	203,080	196,447	196,447	196,447	196,447	196,447	203.080	196,447	196,447	196,447	196,447	196,447
Number of Principals	1683	1681	1681	1681	1681	1681	1683	1681	1681	1681	1681	1681
Number of Principal VA estimated	1683	1681	1681	1681	1681	1681	1683	1681	1681	1681	1681	1681
Number of Schools	848	846	846	846	846	846	848	846	846	846	846	846
Number of School VA estimated	17	17	17	17	17	17	17	17	17	17	17	17
Adjusted R^2	0.1010	0.1008	0.1730	0.1731	0.1733	0.1735	0.0842	0.0830	0.1811	0.1815	0.1817	0.1819
Average PROEB Score	267.6180	268.0661	268.0661	268.0661	268.0661	268.0661	268.9572	269.4916	269.4916	269.4916	269.4916	269.4916
PROEB Score Standard Deviation	50.4002	50.3852	50.3852	50.3852	50.3852	50.3852	49.4696	49.3257	49.3257	49.3257	49.3257	49.3257
Student Controls	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Peer Controls	No	No	No	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes
School Controls	No	No	No	No	No	Yes	No	No	No	No	No	Yes
Teacher Controls	No	No	No	No	Yes	Yes	No	No	No	No	Yes	Yes

Notes: The table presents information on the distribution of standardized fixed effects (FE) estimates, our value-added measure, for specifications used in Model 1 (School-Principal), estimated on Sample 1. Columns (1) through (6) show estimates for mathematics, whilst columns (7) through (12) show estimates for Portuguese. Columns (1) and (7) present FE estimates for the specification without controls on the unfiltered sample. Columns (2) and (8) present FE estimates for the same specification, but on the filtered sample (to ensure comparison with controlled specifications). Columns (3) and (9) present FE estimates for the specification with student controls. Columns (4) and (10) present FE estimates for the specification with student and peer (leave-me-out) controls. Columns (5) and (11) present FE estimates for the specification with student, peer and teacher controls. Finally, columns (6) and (12) present FE estimates for our full and preferred specification, with all controls: student, peer, teacher and school. For standardized information on FE estimates, please refer to Table 15.

Figure B.1 – Cross-validation for selecting LASSO regression penalization parameter (λ) for mathematics

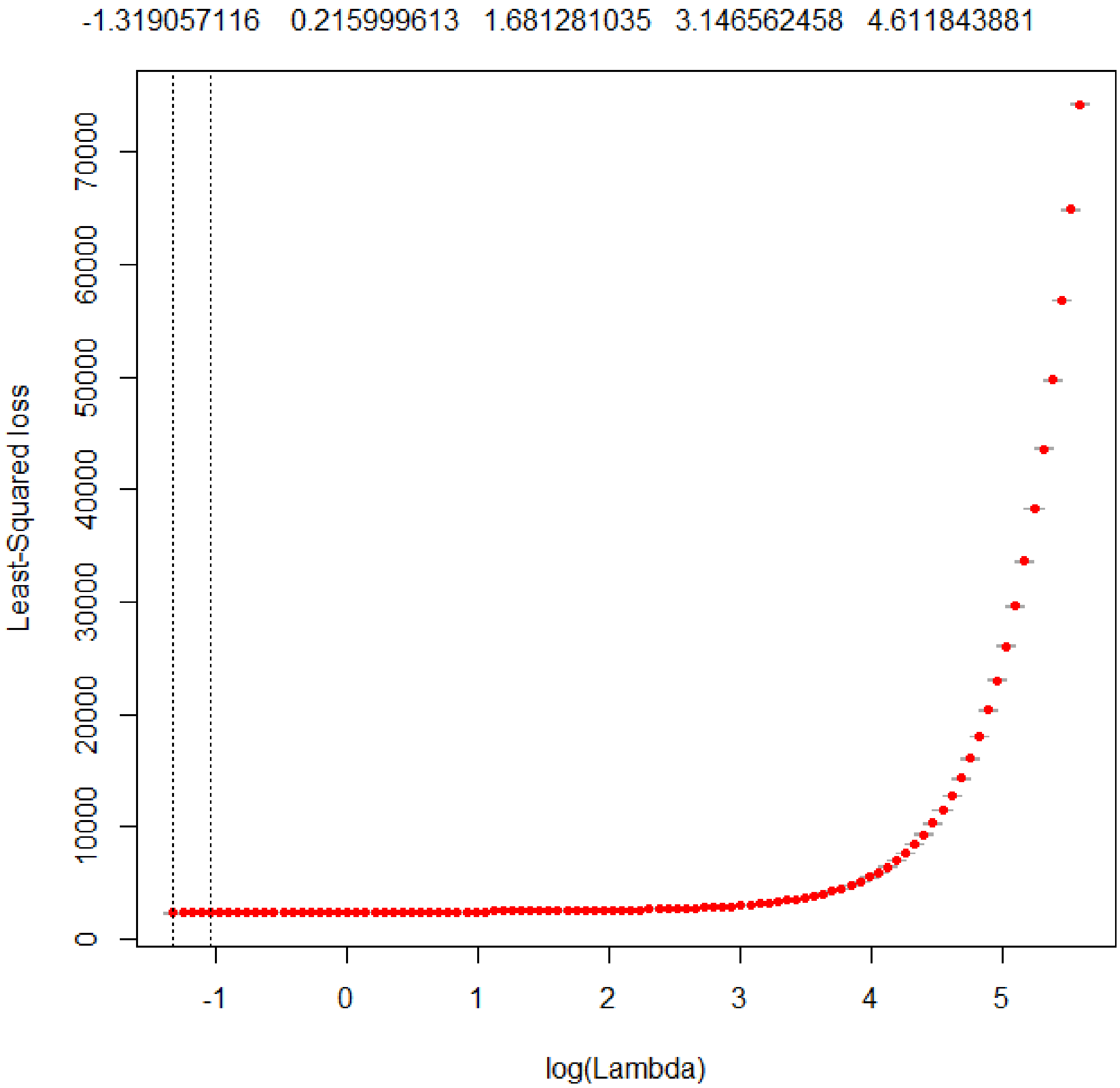

Table B.4 – Model 1 regression coefficients

	Mathematics				Portuguese			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Student: Female	-5.7867***	-5.7583***	-5.7653***	-5.7746***	12.5375***	12.6168***	12.5993***	12.5862***
	(0.2345)	(0.2368)	(0.2369)	(0.2366)	(0.2270)	(0.2291)	(0.2289)	(0.2288)
Student: Non-white	-4.3064***	-4.2859***	-4.2867***	-4.2957***	-4.4520***	-4.4407***	-4.4476***	-4.4572***
	(0.2452)	(0.2472)	(0.2472)	(0.2472)	(0.2372)	(0.2396)	(0.2395)	(0.2395)
Student: Mother middle school degree	2.1569***	2.1488***	2.1445***	2.1531***	2.6782***	2.6656***	2.6650***	2.6719***
	(0.3181)	(0.3183)	(0.3184)	(0.3182)	(0.3111)	(0.3113)	(0.3112)	(0.3110)
Student: Mother high school degree	5.6818***	5.6743***	5.6673***	5.6613***	5.4978***	5.4910***	5.4884***	5.4822***
	(0.2916)	(0.2916)	(0.2915)	(0.2915)	(0.2859)	(0.2858)	(0.2857)	(0.2856)
Student: Mother university degree	8.7877***	8.7973***	8.7833***	8.7805***	7.7142***	7.7027***	7.6961***	7.6912***
	(0.4322)	(0.4363)	(0.4364)	(0.4364)	(0.4154)	(0.4197)	(0.4196)	(0.4196)
Student: Father middle school degree	1.3369***	1.3413***	1.3348***	1.3393***	1.4529***	1.4601***	1.4623***	1.4704***
	(0.3230)	(0.3230)	(0.3229)	(0.3230)	(0.3114)	(0.3113)	(0.3114)	(0.3114)
Student: Father high school degree	3.7717***	3.7680***	3.7664***	3.7632***	4.7389***	4.7334***	4.7338***	4.7324***
	(0.3205)	(0.3203)	(0.3203)	(0.3202)	(0.3016)	(0.3013)	(0.3014)	(0.3012)
Student: Father university degree	3.6389***	3.6589***	3.6458***	3.6487***	2.9491***	2.9277***	2.9137***	2.9174***
	(0.5550)	(0.5599)	(0.5598)	(0.5602)	(0.5184)	(0.5229)	(0.5229)	(0.5231)
Student: Private school history	-0.3352	-0.3348	-0.3437	-0.3432	-1.8571***	-1.8509***	-1.8527***	-1.8505***
	(0.4366)	(0.4364)	(0.4365)	(0.4363)	(0.4294)	(0.4291)	(0.4288)	(0.4289)
Student: Grade repetition history	-23.7440	-23.7341***	-23.7074***	-23.7025***	-24.0191***	-24.0042***	-23.9687***	-23.9667***
	(0.2641)	(0.2643)	(0.2642)	(0.2641)	(0.2668)	(0.2667)	(0.2668)	(0.2666)
Household: *Bolsa Família*	-3.1357***	-3.1555***	-3.1488***	-3.1483***	-3.4743***	-3.4868***	-3.4768***	-3.4826***
	(0.2524)	(0.2548)	(0.2548)	(0.2547)	(0.2535)	(0.2561)	(0.2562)	(0.2563)
Household: Paved street	1.2127***	1.1988***	1.2029***	1.1987 ***	1.5136***	1.4925***	1.4894***	1.4856***
	(0.3135)	(0.3135)	(0.3135)	(0.3135)	(0.3113)	(0.3112)	(0.3111)	(0.3111)
Household: Garbage collection	-0.2045	-0.2001	-0.1869	-0.2002	1.6485***	1.6597***	1.6658***	1.6527***
	(0.3691)	(0.3691)	(0.3692)	(0.3691)	(0.3703)	(0.3704)	(0.3705)	(0.3705)
Household: Bathroom	11.5602***	11.5324***	11.5338***	11.5492***	11.3957***	11.3436***	11.3364***	11.3440***
	(0.6760)	(0.6758)	(0.6757)	(0.6752)	(0.6913)	(0.6910)	(0.6913)	(0.6908)
Household: Car	-1.3698***	-1.3677***	-1.3652***	-1.3627***	-3.7526***	-3.7307***	-3.7386***	-3.7440***
	(0.2272)	(0.2296)	(0.2297)	(0.2296)	(0.2222)	(0.2241)	(0.2240)	(0.2240)
Household: Cellphone	11.5828***	11.6575***	11.6510***	11.6936***	14.1711***	14.2997***	14.2779***	14.3168***
	(0.3532)	(0.3516)	(0.3515)	(0.3519)	(0.3527)	(0.3513)	(0.3514)	(0.3517)
Household: Computer	6.1563***	6.1868***	6.1780***	6.1730***	5.9773***	6.0206***	6.0121***	6.0071***
	(0.2469)	(0.2492)	(0.2493)	(0.2492)	(0.2434)	(0.2455)	(0.2456)	(0.2456)
Peer: Female		3.3752	3.3048	2.8512		10.2550***	10.1418***	9.4672***
		(2.6320)	(2.6344)	(2.6210)		(2.5955)	(2.6030)	(2.5679)
Peer: Non-white		2.0427	1.7634	1.2827		-0.0706	-0.5916	-1.0673
		(2.9250)	(2.9209)	(2.9301)		(2.8778)	(2.8716)	(2.8829)
Peer: *Bolsa Família*		-2.9950	-2.8414	-2.8076		-2.5795	-2.4315	-2.6567
		(2.8454)	(2.8470)	(2.8600)		(2.8382)	(2.8444)	(2.8616)
Peer: Mother university degree		3.9191	3.6088	3.3454		1.4786	1.8115	1.5713
		(5.5146)	(5.5363)	(5.5435)		(5.3483)	(5.3403)	(5.3448)
Peer: Father university degree		4.1477	4.2528	4.3049		-2.7079	-3.1685	-3.0994
		(7.6685)	(7.6773)	(7.7139)		(7.4737)	(7.4692)	(7.4862)
Peer: Car		-1.6626	-1.4006	-1.1852		-0.7647	-0.7150	-0.8875
		(2.7986)	(2.7994)	(2.8011)		(2.7534)	(2.7451)	(2.7456)
Peer: Cellphone		-8.3657***	-8.3140***	-6.5707**		-15.4111***	-16.0074***	-14.4393***
		(2.7708)	(2.7717)	(2.8193)		(2.6700)	(2.6782)	(2.7285)
Peer: Computer		5.9072 **	5.8975**	5.5072*		8.4695***	8.2380***	7.8484***
		(2.9109)	(2.9135)	(2.9054)		(2.9468)	(2.9462)	(2.9332)
Teacher: Age			-0.0033	-0.0034			0.0106	0.0106
			(0.0268)	(0.0268)			(0.0274)	(0.0273)
Teacher: Female			1.3150**	1.3007**			0.9429	0.9943
			(0.5121)	(0.5122)			(0.7304)	(0.7293)
Teacher: Non-white			-0.2932	-0.3285			-0.0257	-0.0512
			(0.5269)	(0.5271)			(0.5342)	(0.5346)
Teacher: University degree			0.3719	0.4416			1.8385	1.9981
			(1.0934)	(1.0942)			(1.5488)	(1.5468)
Teacher: Number of classrooms			-0.2014**	-0.1954**			-0.2979***	-0.2911***
			(0.0954)	(0.0949)			(0.0744)	(0.0745)
School: Filtered water				12.0344**				9.4093**
				(4.7975)				(4.7198)
School: Absence of sewage system				-1.8197				-0.7307
				(1.3439)				(1.3618)
School: Internet				-7.0153				-5.9018
				(4.8444)				(5.0690)
School: Teacher's room				5.0593*				3.1647
				(3.0361)				(2.7721)
School: Library				1.2865				0.9985
				(2.3583)				(2.1920)
School: Sports court				-2.3118				-1.1732
				(1.7177)				(1.7741)
School: TV equipment				0.0078				-0.9284
				(1.6761)				(1.6428)
School: Multimedia equipment				1.8500				-1.0263
				(1.6358)				(1.9860)
School: Multifunction Printer				-0.6915				-0.7677
				(0.6505)				(0.6669)
School: Number of rooms				0.0133				0.1584
				(0.1571)				(0.1716)
School: Number of computers				0.0576				0.0313
				(0.0395)				(0.0375)
School: Number of students				-0.0104***				-0.0104***
				(0.0031)				(0.0032)
School: Student-classroom ratio				0.0045				0.0117*
				(0.0060)				(0.0063)
Observations	196,447	196,447	196,447	196,447	196,447	196,447	196,447	196,447
Adjusted R^2	0.1730	0.1731	0.1733	0.1735	0.1811	0.1815	0.1817	0.1819

Notes: The table presents the regression coefficients for the specifications used in Model 1 (School-Principal) estimation, estimated on Sample 1. Information on fixed effects estimates is available in Table 15. Columns (1) through (4) present results for mathematics, and columns (5) through (8) present results for Portuguese. Columns (1) and (5) refer to the specification in which only student controls were included. Columns (2) and (6) refer to the specification with student and peer (leave-me-out) controls included. Columns (3) and (7) refer to the specification in which student, peer and teacher controls were included. Finally, columns (4) and (8) refer to the specification with all controls: student, peer, teacher and school; and are deemed our preferred specification. Standard errors are shown in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

Table B.5 – Model 2 standardized value-added estimates distribution statistics

	Mathematics						Portuguese					
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Average Principal Value-Added	267.7851	268.1742	254.4410	251.1709	260.8360	257.8627	269.1303	269.6568	242.0663	250.3159	250.2915	249.1572
Principal Value-Added Standard Deviation	21.3424	21.4480	19.6301	19.6268	19.5817	19.6521	18.5414	18.4845	16.2794	16.3029	16.3183	16.2968
Max Principal VA	396.9367	394.7634	383.4479	387.9140	391.3279	388.1602	339.4244	342.5906	320.0639	336.7299	334.6122	332.1785
90^{th} Quantile	293.7102	294.3712	277.6951	273.4269	283.6317	280.6276	292.0267	292.4048	261.3418	269.3661	269.7096	268.8523
75^{th} Quantile	280.1023	280.3697	265.4547	261.8743	271.8547	268.9048	281.5160	281.9039	252.4722	260.2604	260.6422	259.8263
50^{th} Quantile	267.0940	267.6944	253.9743	250.5568	260.3153	257.5262	269.5959	270.0915	242.2776	250.5892	251.0198	249.8297
25^{th} Quantile	253.3721	253.7612	241.4775	238.7851	248.2771	244.9924	256.8508	257.4576	232.2650	240.4817	240.4003	239.2384
10^{th} Quantile	242.0995	242.2546	230.9872	228.0845	237.6603	234.3467	245.1762	245.6863	221.2913	229.6784	229.9614	229.0392
Min Principal VA	195.3729	195.3729	190.7633	161.8875	184.1620	181.2527	206.0062	207.7932	184.1632	189.0006	183.7163	181.5671
Observations	540.852	523.664	523.664	523.664	523.664	523.664	540.852	523.664	523.664	523.664	523.664	523.664
Number of Principals	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895
Number of Principal VA estimated	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895	1895
Number of Teacher Compositions	6134	6133	6133	6133	6133	6133	6321	6319	6319	6319	6319	6319
Number of Composition VA estimated	4472	4471	4471	4471	4471	4471	4558	4555	4555	4555	4555	4555
Adjusted R^2	0.1182	0.1179	0.1877	0.1878	0.1878	0.1881	0.1017	0.1006	0.1949	0.1951	0.1951	0.1953
Average PROEB Score	269.8603	270.2740	270.2740	270.2740	270.2740	270.2740	270.9095	271.4228	271.4228	271.4228	271.4228	271.4228
PROEB Score Standard Deviation	51.8326	51.6763	51.6763	51.6763	51.6763	51.6763	49.9206	50.0836	50.0836	50.0836	50.0836	50.0836
Student Controls	No	No	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes
Peer Controls	No	No	No	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes
School Controls	No	No	No	No	Yes	Yes	No	No	No	No	Yes	Yes
Teacher Controls	No	No	No	No	No	Yes	No	No	No	No	No	Yes

Notes: The table presents information on the distribution of standardized fixed effects (FE) estimates, our value-added measure, for specifications used in Model 2 (Teacher-Principal), estimated on Sample 2. Columns (1) through (6) show estimates for mathematics, whilst columns (7) through (12) show estimates for Portuguese. Columns (1) and (7) present FE estimates for the specification without controls on the unfiltered sample. Columns (2) and (8) present FE estimates for the same specification, but on the filtered sample (to ensure comparison with controlled specifications). Columns (3) and (9) present FE estimates for the specification with student controls. Columns (4) and (10) present FE estimates for the specification with student and peer (leave-me-out) controls. Columns (5) and (11) present FE estimates for the specification with student, peer and school controls. Finally, columns (6) and (12) present FE estimates for our full and preferred specification, with all controls: student, peer, teacher and school. For standardized information on FE estimates, please refer to Table 16.

Table B.6 – Model 2 regression coefficients

	Mathematics				Portuguese			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Student: Female	-6.1770***	-6.1564***	-6.1566***	-6.1741***	12.6788***	12.7724***	12.7670***	12.7542***
	(0.1458)	(0.1512)	(0.1513)	(0.1513)	(0.1405)	(0.1456)	(0.1456)	(0.1455)
Student: Non-white	-4.7091***	-4.7212***	-4.7154***	-4.7045***	-4.5149***	-4.5517***	-4.5528***	-4.5520***
	(0.1519)	(0.1574)	(0.1574)	(0.1573)	(0.1424)	(0.1487)	(0.1486)	(0.1486)
Student: Mother middle school degree	2.3367***	2.3376***	2.3396***	2.3346***	2.7894***	2.7809***	2.7836***	2.7816***
	(0.1995)	(0.1995)	(0.1995)	(0.1994)	(0.1938)	(0.1938)	(0.1938)	(0.1937)
Student: Mother high school degree	5.6686***	5.6683***	5.6671***	5.6640***	5.3208***	5.3124***	5.3124***	5.3070***
	(0.1806)	(0.1806)	(0.1806)	(0.1806)	(0.1740)	(0.1740)	(0.1740)	(0.1739)
Student: Mother university degree	9.3943***	9.4469***	9.4557***	9.4351***	7.6537***	7.5299***	7.5442***	7.5346***
	(0.2643)	(0.2730)	(0.2730)	(0.2729)	(0.2471)	(0.2555)	(0.2555)	(0.2556)
Student: Father middle school degree	1.4404***	1.4397***	1.4412***	1.4344***	1.4988***	1.5000***	1.5028***	1.5029***
	(0.1982)	(0.1982)	(0.1982)	(0.1982)	(0.1883)	(0.1883)	(0.1883)	(0.1883)
Student: Father high school degree	3.7713***	3.7691***	3.7694***	3.7611***	4.3270***	4.3252***	4.3263***	4.3212***
	(0.1931)	(0.1930)	(0.1930)	(0.1929)	(0.1806)	(0.1805)	(0.1805)	(0.1805)
Student: Father university degree	3.6630***	3.7321***	3.7055***	3.6979***	3.6138***	3.6829***	3.6712***	3.6523***
	(0.3247)	(0.3374)	(0.3373)	(0.3373)	(0.3003)	(0.3124)	(0.3127)	(0.3127)
Student: Private school history	0.8334	0.8263	0.8262***	0.8179***	-1.2517***	-1.2487***	-1.2499***	-1.2465***
	(0.2523)	(0.2522)	(0.2522)	(0.2521)	(0.2418)	(0.2418)	(0.2418)	(0.2416)
Student: Grade repetition history	-23.9888	-23.9828***	-23.9846***	-23.9108***	-23.8556***	-23.8497***	-23.8508***	-23.8007***
	(0.1715)	(0.1716)	(0.1716)	(0.1716)	(0.1689)	(0.1688)	(0.1688)	(0.1685)
Household: *Bolsa Família*	-3.7814***	-3.8374***	-3.8327***	-3.8284***	-4.1562***	-4.2406***	-4.2505***	-4.2486***
	(0.1625)	(0.1690)	(0.1690)	(0.1689)	(0.1605)	(0.1674)	(0.1674)	(0.1673)
Household: Paved street	1.2019***	1.2063***	1.2062***	1.2049***	1.8102***	1.8071***	1.8067***	1.8056***
	(0.2027)	(0.2027)	(0.2027)	(0.2026)	(0.1947)	(0.1947)	(0.1947)	(0.1946)
Household: Garbage collection	0.5842**	0.5800	0.5788	0.5746***	2.6857***	2.6809***	2.6816***	2.6808***
	(0.2454)	(0.2454)	(0.2454)	(0.2453)	(0.2387)	(0.2386)	(0.2386)	(0.2386)
Household: Bathroom	10.2569***	10.2485***	10.2490***	10.2460***	10.7323***	10.7131***	10.7119***	10.7064***
	(0.4428)	(0.4428)	(0.4427)	(0.4427)	(0.4479)	(0.4478)	(0.4477)	(0.4478)
Household: Car	-1.2444***	-1.2657***	-1.2703***	-1.2774***	-3.7349***	-3.7488***	-3.7508***	-3.7560***
	(0.1414)	(0.1475)	(0.1475)	(0.1475)	(0.1382)	(0.1443)	(0.1445)	(0.1445)
Household: Cellphone	11.9980***	11.9779***	12.0055***	11.9904***	14.5830***	14.4349***	14.4271***	14.4268***
	(0.2288)	(0.2319)	(0.2324)	(0.2324)	(0.2279)	(0.2317)	(0.2320)	(0.2320)
Household: Computer	6.1689***	6.3004***	6.3002***	6.2783***	5.1974***	6.3417***	6.3462***	6.3415***
	(0.1551)	(0.1609)	(0.1607)	(0.1608)	(0.1522)	(0.1586)	(0.1586)	(0.1586)
Peer: Female		1.9543	1.9591	1.6694		8.2815***	7.9433***	7.9490***
		(2.9227)	(2.9400)	(2.9420)		(2.8576)	(2.8668)	(2.8626)
Peer: Non-white		-1.2375	-0.8582	-0.4814		-3.5859	-3.6642	-3.7121
		(3.2502)	(3.2409)	(3.2466)		(3.3211)	(3.3085)	(3.3040)
Peer: *Bolsa Família*		-4.4109	-4.1055	-4.2314		-6.7317**	-7.3022	-7.4651**
		(3.1708)	(3.1645)	(3.1580)		(3.2291)	(3.2224)	(3.2209)
Peer: Mother university degree		5.7143	6.4046	6.1198		-11.1487**	10.0781*	10.0932*
		(6.0149)	(5.9863)	(5.9861)		(5.6134)	(5.6005)	(5.5854)
Peer: Father university degree		7.6886	5.3663	6.0574		7.3509	6.2622	5.7071
		(8.1487)	(8.1585)	(8.1638)		(7.9971)	(8.0272)	(8.0125)
Peer: Car		-1.9716	-2.2709	-2.3698		-1.3146	-1.3938	-1.5667
		(3.1363)	(3.1384)	(3.1341)		(3.0274)	(3.0622)	(3.0636)
Peer: Cellphone		-2.2684	-0.7639***	-1.2300**		-15.0314***	-15.4302***	-15.5307***
		(3.1248)	(3.2401)	(3.2320)		(3.1835)	(2.2693)	(3.2659)
Peer: Computer		11.5089**	11.4966***	11.3882***		12.2854***	12.5632***	12.7213***
		(3.1599)	(3.1437)	(3.1415)		(3.1835)	(3.2453)	(3.2432)
Teacher: Age				0.0885***				0.0395*
				(0.0221)				(0.0227)
Teacher: Female				1.4882***				1.7975***
				(0.4288)				(0.5388)
Teacher: Non-white				0.3988				-0.7710
				(0.4551)				(0.4183)
Teacher: University degree				1.2851				0.3249
				(1.0618)				(1.2931)
Teacher: Number of classrooms				-0.4454***				-0.2642***
				(0.0725)				(0.0557)
School: Filtered water			-8.3470	-8.7202**			7.5329	7.3233
			(9.7152)	(9.7583)			(4.6188)	(4.6195)
School: Absence of sewage system			-2.7442*	-2.6614			1.1842	1.2486
			(1.5602)	(1.5580)			(1.5476)	(1.5493)
School: Internet			-3.5621	-3.4961			-2.8357	-2.7581
			(3.3955)	(3.3970)			(4.1228)	(4.0941)
School: Teacher's room			7.4381**	7.4703**			1.3146	1.2413
			(3.5705)	(3.5746)			(3.0193)	(3.0347)
School: Library			0.4614	0.5080			2.4463	2.4744
			(3.5740)	(3.5764)			(2.8657)	(2.8570)
School: Sports court			-0.3551	-0.0903			-3.4545*	-3.5215*
			(2.3372)	(2.3312)			(1.9836)	(1.9883)
School: TV equipment			-1.5112	-1.3087			0.7658	0.9545
			(1.6769)	(1.6797)			(2.0240)	(2.0149)
School: Multimedia equipment			1.4797	1.2506			0.3192	0.2691
			(1.5395)	(1.5312)			(2.0508)	(2.0453)
School: Multifunction Printer			0.4436	0.4316			0.8633	0.9438
			(0.7563)	(0.7562)			(0.7226)	(0.7225)
School: Number of rooms			-0.0471	-0.0373			-0.1785	-0.1804
			(0.2034)	(0.2001)			(0.2186)	(0.2178)
School: Number of computers			-0.0056	-0.0082			0.0110	0.0104
			(0.0417)	(0.0416)			(0.0391)	(0.0390)
School: Number of students			-0.0081**	-0.0077**			-0.0063	-0.0064
			(0.0035)	(0.0035)			(0.0039)	(0.0039)
School: Student-classroom ratio			0.0017	0.0022			0.0114	0.0111
			(0.0061)	(0.0060)			(0.0070)	(0.0068)
Observations	523.664	523.664	523.664	523.664	523.664	523.664	523.664	523.664
Adjusted R^2	0.1877	0.1878	0.1878	0.1881	0.1949	0.1951	0.1951	0.1953

Notes: The table presents the regression coefficients for the specifications used in Model 2 (Teacher-Principal) estimation, estimated on Sample 2. Information on fixed effects estimates is available in Table 16. Columns (1) through (4) present results for mathematics, and columns (5) through (8) present results for Portuguese. Columns (1) and (5) refer to the specification in which only student controls were included. Columns (2) and (6) refer to the specification with student and peer (leave-me-out) controls included. Columns (3) and (7) refer to the specification in which student, peer and school controls were included. Finally, columns (4) and (8) refer to the specification with all controls: student, peer, teacher and school; and are deemed our preferred specification. Standard errors are shown in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

Table B.7 – Model 3 standardized value-added estimates distribution statistics

	Mathematics							Portuguese						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Average Principal Value-Added	0.0367	0.0558	0.0615	0.0564	0.0556	0.0390	0.0390	-0.0525	-0.0531	-0.0421	-0.0244	-0.0214	-0.0316	-0.0279
Principal Value-Added Standard Deviation	2.8558	3.1132	3.0480	3.1847	3.2388	3.2753	3.3207	2.3561	2.3118	2.3977	2.3933	2.4055	2.5390	2.5710
Max Principal VA	38.2627	52.2364	52.2619	52.6746	53.2664	52.5071	53.1079	20.9700	21.1936	20.0453	19.5182	18.8566	19.4103	19.9890
90th Quantile	6.24×10^{-5}	1.05×10^{-3}	4.45×10^{-4}	4.09×10^{-4}	5.66×10^{-4}	$4,75 \times 10^{-4}$	4.86×10^{-4}	1.58×10^{-4}	1.42×10^{-4}	1.31×10^{-4}	1.34×10^{-4}	1.4×10^{-4}	1.25×10^{-4}	1.3×10^{-4}
75th Quantile	3.50×10^{-12}	2.82×10^{-11}	3.3×10^{-11}	2.33×10^{-11}	2.38×10^{-11}	2.66×10^{-11}	2.48×10^{-11}	1.21×10^{-11}	1.07×10^{-11}	9.77×10^{-12}	8.95×10^{-12}	9.17×10^{-12}	8.24×10^{-12}	9.82×10^{-12}
50th Quantile	-9.54×10^{-13}	4.29×10^{-13}	4.54×10^{-13}	-2.29×10^{-12}	1.42×10^{-13}	-4.09×10^{-13}	4.03×10^{-13}	1.08×10^{-13}	2.24×10^{-13}	-4.46×10^{-14}	-6.4×10^{-13}	-4.45×10^{-13}	1.98×10^{-13}	1.83×10^{-13}
25th Quantile	-3.56×10^{-11}	-3.07×10^{-11}	-2.85×10^{-11}	-2.65×10^{-11}	-3.22×10^{-11}	-2.7×10^{-11}	-2.3×10^{-11}	-1.11×10^{-11}	-9.27×10^{-12}	-8.79×10^{-12}	-1.15×10^{-11}	-1.2×10^{-11}	-6.94×10^{-12}	-7.9×10^{-12}
10th Quantile	-4.84×10^{-4}	-5.91×10^{-4}	-8.43×10^{-4}	-3.35×10^{-4}	-3.77×10^{-4}	-8.44×10^{-4}	-8.16×10^{-4}	6.02×10^{-5}	-8.66×10^{-5}	-7.2×10^{-5}	-7.6×10^{-5}	7.5×10^{-5}	-7.27×10^{-5}	-7.27×10^{-5}
Min Principal VA	-0.3393	-0.2854	-0.2947	-0.3162	-0.3250	-0.3167	-0.3293	-0.3023	-0.2764	-0.2837	-0.2546	-0.2512	-0.2588	-0.2666
Observations	137.179	132.970	132.970	132.970	132.970	132.970	132.970	137.179	132.970	132.970	132.970	132.970	132.970	132.970
Number of Principals	689	689	689	689	689	689	689	689	689	689	689	689	689	689
Number of Principal VA estimated	689	689	689	689	689	689	689	689	689	689	689	689	689	689
Number of Schools	628	628	628	628	628	628	628	628	628	628	628	628	628	628
Number of School VA estimated	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Number of Teacher Compositions	1849	1849	1849	1849	1849	1849	1849	1902	1901	1901	1901	1901	1901	1901
Number of Composition VA estimates	1213	1213	1213	1213	1213	1213	1213	1241	1240	1240	1240	1240	1240	1240
Adjusted R^2	0.1073	0.1072	0.1789	0.1791	0.1794	0.1793	0.1796	0.0937	0.0919	0.1898	0.1899	0.1904	0.1901	0.1906
Average PROEB Score	267.3090	267.7094	267.7094	267.7094	267.7094	267.7094	267.7094	268.9381	269.4591	269.4591	269.4591	269.4591	269.4591	269.4591
PROEB Score Standard Deviation	50.6171	50.6069	50.6069	50.6069	50.6069	50.6069	50.6069	49.7669	49.6344	49.6344	49.6344	49.6344	49.6344	49.6344
Student Controls	No	No	Yes	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes	Yes
Peer Controls	No	No	No	Yes	Yes	Yes	Yes	No	No	No	Yes	Yes	Yes	Yes
School Controls	No	No	No	No	No	Yes	Yes	No	No	No	No	No	Yes	Yes
Teacher Controls	No	No	No	No	Yes	No	Yes	No	No	No	No	Yes	No	Yes

Notes: The table presents information on the distribution of standardized fixed effects (FE) estimates, our value-added measure, for specifications used in Model 3 (School-Teacher-Principal), estimated on Sample 3. Columns (1) through (7) show estimates for mathematics, whilst columns (8) through (14) show estimates for Portuguese. Columns (1) and (8) present FE estimates for the specification without controls on the unfiltered sample. Columns (2) and (9) present FE estimates for the same specification, but on the filtered sample (to ensure comparison with controlled specifications). Columns (3) and (10) present FE estimates for the specification with student controls. Columns (4) and (11) present FE estimates for the specification with student and peer (leave-me-out) controls. Columns (5) and (12) present FE estimates for the specification with student, peer and teacher controls. Columns (6) and (13) present FE estimates for the specification with student, peer and school controls. Finally, columns (7) and (14) present FE estimates for our full and preferred specification, with all controls: student, peer, teacher and school. For standardized information on FE estimates, please refer to Table ??.

Figure B.2 – Cross-validation for selecting LASSO regression penalization parameter (λ) for portuguese

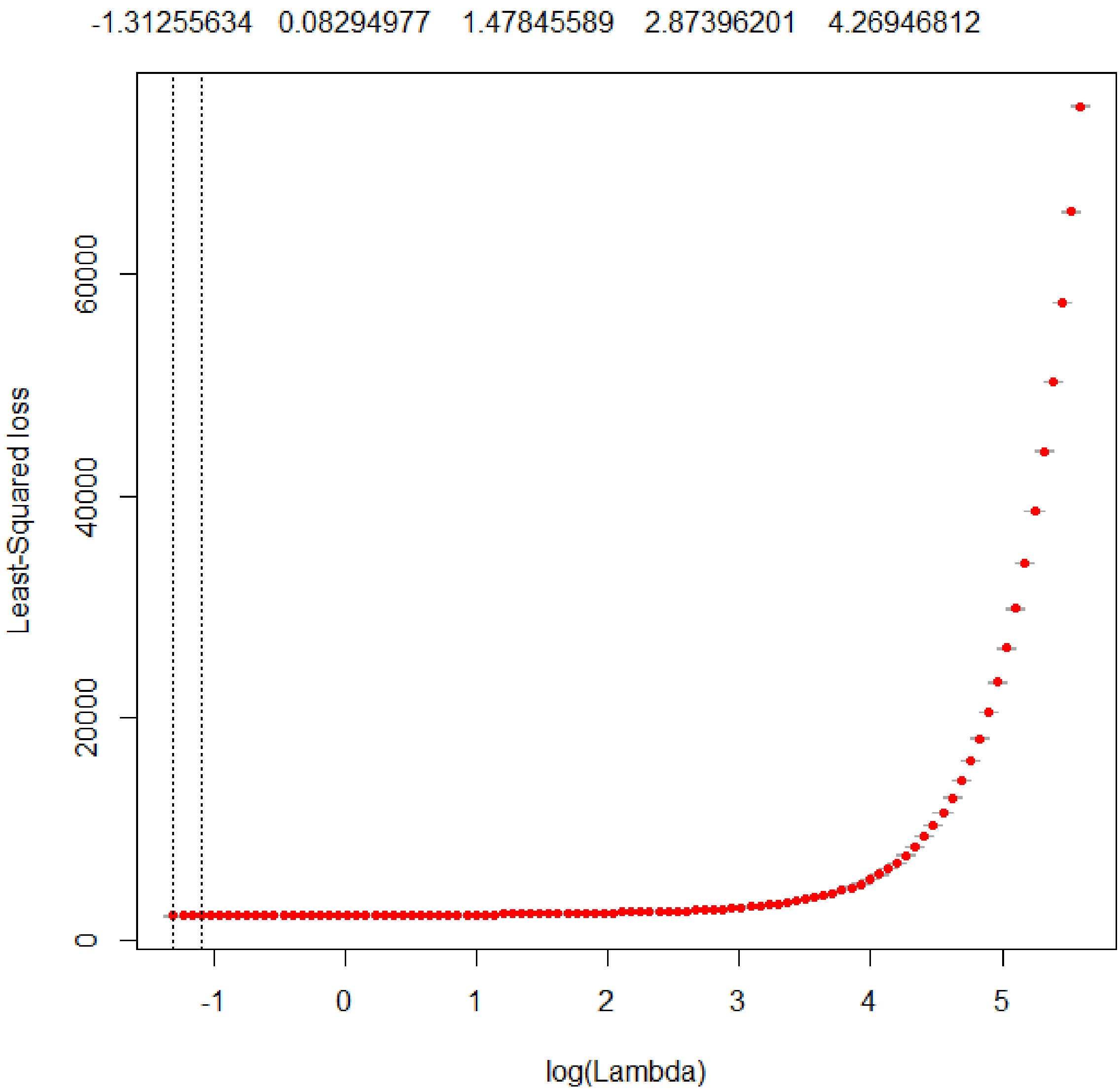

Table B.8 – Model 3 regression coefficients

	Mathematics					Portuguese				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Student: Female	-5.7162***	-5.9319***	-5.9423***	-5.9041***	-5.9164***	12.5983***	12.6501***	12.6184***	12.6207***	12.5893***
	(0.2844)	(0.2989)	(0.2990)	(0.2987)	(0.2988)	(0.2754)	(0.2899)	(0.2894)	(0.2898)	(0.2893)
Student: Non-white	-4.0604***	-3.9294***	-3.9180***	-3.8820***	-3.8722***	-4.3132***	-4.2929***	-4.2887***	-4.2867***	-4.2801***
	(0.3020)	(0.3164)	(0.3164)	(0.3169)	(0.3169)	(0.2897)	(0.3023)	(0.3021)	(0.3030)	(0.3029)
Student: Mother middle school degree	2.2595***	2.2686***	2.2575***	2.2742***	2.2638***	2.8497***	2.8448***	2.8436***	2.8487***	2.8467***
	(0.3911)	(0.3910)	(0.3911)	(0.3911)	(0.3911)	(0.3800)	(0.3800)	(0.3799)	(0.3802)	(0.3801)
Student: Mother high school degree	5.7892***	5.7976***	5.7894***	5.7929***	5.7852***	5.3484***	5.3448***	5.3269***	5.3530***	5.3350***
	(0.3544)	(0.3544)	(0.3542)	(0.3543)	(0.3542)	(0.3481)	(0.3482)	(0.3480)	(0.3482)	(0.3480)
Student: Mother university degree	8.5378***	8.6512***	8.6511***	8.5811***	8.5883***	7.4450***	7.4645***	7.4408***	7.5295***	7.5094***
	(0.5192)	(0.5450)	(0.5449)	(0.5471)	(0.5469)	(0.5017)	(0.5314)	(0.5310)	(0.5322)	(0.5318)
Student: Father middle school degree	1.7754***	1.1803***	1.1643***	1.1680***	1.1529***	1.1165***	1.1169***	1.1111***	1.1210***	1.1150***
	(0.3879)	(0.3879)	(0.3879)	(0.3881)	(0.3880)	(0.3738)	(0.3739)	(0.3738)	(0.3739)	(0.3738)
Student: Father high school degree	3.2239***	3.2256***	3.2166***	3.2207***	3.2119***	4.3461***	4.3413***	4.3318***	4.3429***	4.3333***
	(0.3871)	(0.3868)	(0.3879)	(0.3868)	(0.3867)	(0.3595)	(0.3594)	(0.3595)	(0.3594)	(0.3595)
Student: Father university degree	3.6728***	3.7329***	3.6898***	3.8008***	3.7576***	2.7197***	2.9863***	2.9804***	2.9715***	2.9615***
	(0.6625)	(0.7079)	(0.7093)	(0.7075)	(0.7076)	(0.6114)	(0.6395)	(0.6391)	(0.6401)	(0.6399)
Student: Private school history	0.2776	0.2752	0.2512	0.2733	0.2506	-1.8611***	-1.8671***	-1.8507***	-1.8554***	-1.8403***
	(0.5147)	(0.5145)	(0.5143)	(0.5142)	(0.5140)	(0.5037)	(0.5038)	(0.5036)	(0.5037)	(0.5034)
Student: Grade repetition history	-23.7675***	-23.7497***	-23.6874***	-23.7323***	-23.6742***	-24.0048***	-24.0013***	-23.9279***	-24.0032***	-23.9286***
	(0.3230)	(0.3229)	(0.3232)	(0.3229)	(0.3232)	(0.3337)	(0.3337)	(0.3331)	(0.3334)	(0.3329)
Household: *Bolsa Família*	-3.6345***	-3.7541***	-3.7354***	-3.6851***	-3.6705***	-3.6228***	-3.8116***	-3.8133***	-3.7965***	-3.7980***
	(0.3116)	(0.3271)	(0.3266)	(0.3271)	(0.3268)	(0.3114)	(0.3294)	(0.3295)	(0.3302)	(0.3303)
Household: Paved street	1.1448***	1.1654***	1.1825***	1.1762***	1.1919***	1.6511***	1.6469***	1.6379***	1.6454***	1.6366***
	(0.3917)	(0.3916)	(0.3915)	(0.3913)	(0.3912)	(0.3870)	(0.3871)	(0.3870)	(0.3870)	(0.3868)
Household: Garbage collection	-0.2214	-0.2239	-0.2040	-0.2416	-0.2215	2.1436***	2.1496***	2.1416***	2.1543***	2.1471***
	(0.4576)	(0.4574)	(0.5675)	(0.4568)	(0.4570)	(0.4560)	(0.4562)	(0.4564)	(0.4561)	(0.4563)
Household: Bathroom	11.9935***	11.9868***	11.9807***	11.9392***	11.9361***	11.5712***	11.5403***	11.5096***	11.5141***	11.4821***
	(0.8192)	(0.8182)	(0.8176)	(0.8184)	(0.8180)	(0.8416)	(0.8414)	(0.8421)	(0.8415)	(0.8422)
Household: Car	-1.4130***	-1.6105***	-1.6114***	-1.5884***	-1.5915***	-3.9039***	-3.9203***	-3.9446***	-3.9053***	-3.9295***
	(0.2757)	(0.2922)	(0.2922)	(0.2917)	(0.2917)	(0.2707)	(0.2880)	(0.2877)	(0.2885)	(0.2883)
Household: Cellphone	11.8544***	11.9457***	11.9342***	12.0889***	12.0721***	14.7965***	14.6157***	14.5893***	14.6570***	14.6281***
	(0.4077)	(0.4149)	(0.4145)	(0.4162)	(0.4158)	(0.4113)	(0.4223)	(0.4221)	(0.4251)	(0.4249)
Household: Computer	6.1860***	6.3984***	6.3804***	6.3793***	6.3622***	5.9933***	6.1098***	6.1020***	6.1225***	6.1168***
	(0.2946)	(0.3104)	(0.3103)	(0.3113)	(0.3112)	(0.2943)	(0.3126)	(0.3125)	(0.3131)	(0.3131)
Peer: Female		-15.6820***	-15.8856***	-14.1260**	-14.4490**		3.7945	3.9662	2.0697	2.2681
		(5.9044)	(5.9022)	(5.8686)	(5.8739)		(5.8576)	(5.8006)	(5.8475)	(5.7878)
Peer: Non-white		10.1371	10.8856*	12.9992**	13.2232**		0.6804	0.7373	0.9940	1.1918
		(6.2452)	(6.2241)	(6.2642)	(6.2484)		(6.1213)	(6.0973)	(6.1809)	(6.1560)
Peer: *Bolsa Família*		-8.3227	-7.9176	-4.6672	-4.4317		-12.1923*	-12.7659**	-11.4355*	-11.9915*
		(5.7941)	(5.7646)	(5.8089)	(5.7952)		(6.4750)	(6.4559)	(6.6165)	(6.5948)
Peer: Mother university degree		8.7842	9.3551	3.8409	4.9008		2.0051	2.2617	6.2871	6.8160
		(12.4281)	(12.4349)	(12.9150)	(12.9211)		(12.1279)	(12.0198)	(12.1501)	(12.0573)
Peer: Father university degree		5.9748	4.4854	11.1467	9.5936		23.7519	25.3030	23.0043	24.2395
		(20.5754)	(20.4526)	(19.9019)	(19.8648)		(16.5934)	(16.4896)	(17.0345)	(16.9587)
Peer: Car		-14.0352**	-13.7204**	-12.8279**	-12.6566**		-1.2946	-1.6251	-0.3273	-0.6505
		(6.2325)	(6.2301)	(6.2261)	(6.2328)		(6.3964)	(6.3750)	(6.4528)	(6.4255)
Peer: Cellphone		6.0712	5.9311	12.8643**	12.4177*		13.6868**	-14.2938**	-11.8893*	-12.6081*
		(5.9961)	(5.9842)	(6.3680)	(6.3581)		(6.1330)	(6.1062)	(6.4938)	(6.4492)
Peer: Computer		15.5111**	15.3943**	14.3211**	14.2252**		8.3400	8.5619	9.0685	9.4101
		(6.2031)	(6.1934)		(6.3116)		(6.6178)	(6.5904)	(6.6538)	(6.6341)
Teacher: Age			0.0494		0.0487			0.0333		0.0315
			(0.0433)		(0.0434)			(0.0442)		(0.0442)
Teacher: Female			1.8579**		1.8593**			1.1775		1.2048
			(0.8130)		(0.8112)			(1.1252)		(1.2232)
Teacher: Non-white			-0.2962		-0.3153			-2.1053***		-2.1283***
			(0.8249)		(0.8219)			(0.7903)		(0.7911)
Teacher: University degree			-0.0134		0.1283			-3.6765		-3.3954
			(1.7564)		(1.7652)			(2.4176)		(2.4261)
Teacher: Number of classrooms			-0.4619***		-0.4299***			-0.4972***		-0.5025***
			(0.1441)		(0.1434)			(0.1174)		(0.1170)
School: Filtered water				6.4651	6.4198				34.8738*	33.2926
				(4.1789)	(4.1746)				(19.3234)	(21.2098)
School: Absence of sewage system				-6.3915**	-6.1221*				0.0536	-0.2186
				(3.6400)	(3.6348)				(3.5796)	(3.5551)
School: Internet				-2.3357	-2.7371				-15.3310	-17.4814
				(5.2547)	(5.2649)				(11.1600)	(11.1862)
School: Teacher's room				6.4980	6.3164				7.8709	9.0051
				(4.6189)	(4.5894)				(5.6689)	(5.6823)
School: Library				1.0614	0.7803				9.2429*	8.8385*
				(4.9158)	(4.9070)				(5.0151)	(4.9172)
School: Sports court				4.4459	4.3566				-5.3385*	-6.0576*
				(5.2166)	(5.2694)				(3.1681)	(3.1914)
School: TV equipment				-2.0092	-1.8567				1.0232	1.5946
				(3.8653)	(3.8856)				(4.2239)	(4.4343)
School: Multimedia equipment				4.9540	4.8853				8.9012*	9.0334*
				(3.5344)	(3.5721)				(4.9291)	(4.8019)
School: Multifunction Printer				-0.2004	-0.3707				0.8978	1.1536
				(1.6906)	(1.6959)				(1.7801)	(1.7777)
School: Number of rooms				0.7961	0.7920				0.2822	0.2753
				(0.6557)	(0.6560)				(0.5342)	(0.5355)
School: Number of computers				0.2602*	0.2422*				-0.0638	-0.0625
				(0.1366)	(0.1361)				(0.1114)	(0.1107)
School: Number of students				-0.0153**	-0.0145*				-0.0249***	-0.0243***
				(0.0078)	(0.0078)				(0.0091)	(0.0090)
School: Student-classroom ratio				0.0097	0.0098				0.0156	0.0153
				(0.0121)	(0.0120)				(0.0213)	(0.0213)
Observations	132.970	132.970	132.970	132.970	132.970	132.970	132.970	132.970	132.970	132.970
Adjusted R^2	0.1789	0.1791	0.1794	0.1793	0.1796	0.1898	0.1899	0.1904	0.1901	0.1906

Notes: The table presents the regression coefficients for the specifications used in Model 3 (School-Teacher-Principal) estimation, estimated on Sample 3. Information on fixed effects estimates is available in Table **??**. Columns (1) through (5) present results for mathematics, and columns (6) through (10) present results for Portuguese. Columns (1) and (6) refer to the specification in which only student controls were included. Columns (2) and (7) refer to the specification with student and peer (leave-me-out) controls were included. Columns (3) and (8) refer to the specification in which student, peer and teacher controls were included. Columns (4) and (9) refer to the specifications in which student, peer and school controls were included. Finally, columns (5) and (10) refer to the specification with all controls: student, peer, teacher and school; and are deemed our preferred specification. Standard errors are shown in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

Table B.9 – Value-added estimates distribution statistics across models

	Mathematics			Portuguese		
	(1a)	(2a)	(3a)	(1b)	(2b)	(3b)
Average Principal Value-Added	247.5229	216.4474	0.0390	244.5922	228.4980	-0.0279
Principal Value-Added Standard Deviation	15.0198	19.9752	3.3207	12.6984	18.4486	2.5710
Max Principal VA	304.3512	310.2555	53.1079	281.6509	312.5531	19.9890
90^{th} Quantile	265.9023	240.8014	4.86×10^{-4}	260.4239	250.1239	1.3×10^{-4}
75^{th} Quantile	256.8470	229.4451	2.48×10^{11}	253.3249	240.2469	9.83×10^{-12}
50^{th} Quantile	246.5195	216.3608	4.04×10^{-13}	244.4760	229.8503	1.84×10^{-13}
25^{th} Quantile	237.5747	203.7560	-2.3×10^{-11}	236.3963	216.1417	-7.9×10^{-12}
10^{th} Quantile	228.8010	190.8026	-8.2×10^{-4}	228.0041	204.2833	-7.3×10^{-5}
Min Principal VA	206.9506	156.1871	-16.6682	202.9641	161.6144	-13.2322
Observations	132,970	132,970	132,970	132,970	132,970	132,970
Number of Principals	689	689	689	689	689	689
Number of Principal VA estimated	689	689	689	689	689	689
Number of Schools	628	628	628	628	628	628
Number of School VA estimated	2	—	2	2	—	2
Number of Teacher Compositions	1849	1849	1849	1901	1901	1901
Number of Composition VA estimated	—	1215	1213	—	1242	1240
Adjusted R^2	0.1718	0.1835	0.1796	0.1812	0.1945	0.1906
Average PROEB Score	268.0252	268.0252	268.0252	269.8003	269.8003	269.8003
PROEB Score Standard Deviation	50.6595	50.6595	50.6595	46.6139	46.6139	46.6139
Student Controls	Yes	Yes	Yes	Yes	Yes	Yes
Peer Controls	Yes	Yes	Yes	Yes	Yes	Yes
School Controls	Yes	Yes	Yes	Yes	Yes	Yes
Teacher Controls	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The table shows the distribution of principal fixed effects, our value-added measure, for the full specification (all controls) of all three models estimated on Sample 3. Columns "a" refer to mathematics scores, and column "b" refers to Portuguese scores. Column (1) refers to estimates for Model 1 (School-Principal). Column (2) refers to estimates for Model 2 (Teacher-Principal); and column (3) refers to estimates for Model 3 (School-Teacher-Principal). For information on standardized value-added measures, please refer to Table 18.

Table B.10 – Comparison of models' regression coefficients

	Mathematics			Portuguese		
	(1a)	(2a)	(3a)	(1b)	(2b)	(3b)
Student: Female	-5.6179***	-5.9164***	-5.9164***	12.7078***	12.5893***	12.5893***
	(0.2869)	(0.2981)	(0.2988)	(0.2758)	(0.2986)	(0.2893)
Student: Non-white	-4.0824***	-3.8722***	-3.8722***	-4.3425***	-4.2801***	-4.2801***
	(0.3022)	(0.3161)	(0.3169)	(0.2924)	(0.3021)	(0.3029)
Student: Mother middle school degree	2.2762***	2.2638***	2.2638***	2.9006***	2.8467***	2.8467***
	(0.3907)	(0.3902)	(0.3911)	(0.3813)	(0.3792)	(0.3801)
Student: Mother high school degree	5.7233***	5.7852***	5.7852***	5.3141***	5.3350***	5.3350***
	(0.3539)	(0.3534)	(0.3542)	(0.3479)	(0.3472)	(0.3480)
Student: Mother university degree	8.5375***	8.5883***	8.5883***	7.4033***	7.5094***	7.5094***
	(0.5235)	(0.5456)	(0.5469)	(0.5063)	(0.5305)	(0.5318)
Student: Father middle school degree	1.0124***	1.1529***	1.1529***	1.0480***	1.1150***	1.1150***
	(0.3885)	(0.3871)	(0.3880)	(0.3748)	(0.3792)	(0.3738)
Student: Father high school degree	3.2963***	3.2119***	3.2119***	3.3979***	4.3333***	4.3333***
	(0.3870)	(0.3858)	(0.3867)	(0.3610)	(0.3586)	(0.3595)
Student: Father university degree	3.6710***	3.7576***	3.7576***	2.6054***	2.9615***	2.9615***
	(0.6668)	(0.7059)	(0.7076)	(0.6164)	(0.6384)	(0.6399)
Student: Private school history	0.1302	0.2506	0.2506	-2.0.09***	-1.8403***	-1.8403***
	(0.5161)	(0.5127)	(0.5140)	(0.5089)	(0.5022)	(0.5034)
Student: Grade repetition history	-23.8882***	-23.6742***	-23.6742***	-24.0898***	-23.9286***	-23.9286***
	(0.3261)	(0.3225)	(0.3232)	(0.3347)	(0.3321)	(0.3329)
Household: *Bolsa Família*	-3.6312***	-3.6705***	-3.6705***	-3.6033***	-3.7980***	-3.7980***
	(0.3111)	(0.3260)	(0.3268)	(0.3115)	(0.3295)	(0.3303)
Household: Paved street	1.1799***	1.1919***	1.1919***	1.6788***	1.6366***	1.6366***
	(0.3910)	(0.3903)	(0.3912)	(0.3874)	(0.3859)	(0.3868)
Household: Garbage collection	-0.0563	-0.2215	-0.2215	2.3159***	2.1471***	2.1471***
	(0.4593)	(0.4559)	(0.4570)	(0.4594)	(0.4552)	(0.4563)
Household: Bathroom	12.2532***	11.9361***	11.9361***	12.0869***	11.4821***	11.4821***
	(0.8158)	(0.8160)	(0.8180)	(0.8484)	(0.8401)	(0.8422)
Household: Car	-1.3762***	-1.5915***	-1.5915***	-3.9248***	-3.9295***	-3.9295***
	(0.2782)	(0.2910)	(0.2917)	(0.2710)	(0.2876)	(0.2883)
Household: Cellphone	11.6481***	12.0721***	12.0721***	14.5565***	14.6281***	14.6281***
	(0.4090)	(0.4148)	(0.4158)	(0.4103)	(0.4239)	(0.4249)
Household: Computer	6.2436***	6.3622***	6.3622***	6.1247***	6.1168***	6.1168***
	(0.2980)	(0.3104)	(0.3112)	(0.2967)	(0.3123)	(0.3131)
Peer: Female	5.0228	-14.4490**	-14.4490**	9.9736***	2.2681	2.2681
	(3.1729)	(5.8597)	(5.8739)	(3.1648)	(5.7739)	(5.7878)
Peer: Non-white	0.3680	13.2232**	13.2232**	-1.5038	1.1918	1.1918
	(3.5632)	(6.2334)	(6.2484)	(3.5429)	(6.1412)	(6.1560)
Peer: *Bolsa Família*	-3.4936	-4.4317	-4.4317	-2.8892	-11.9915*	-11.9915*
	(3.4222)	(5.7813)	(5.7952)	(3.4529)	(5.5789)	(6.5948)
Peer: Mother university degree	3.6498	4.9008	4.9008	-1.9074	6.8160	6.8160
	(6.6058)	(12.8900)	(12.9211)	(6.3417)	(12.0283)	(6.8160)
Peer: Father university degree	3.3234	9.5936	9.5936	-2.8597	24.2395	24.2395
	(9.0816)	(19.8169)	(19.8648)	(8.7244)	(16.9179)	(16.9587)
Peer: Car	-1.4956	-12.6566**	-12.6566**	-1.6576	-0.6505	-0.6505
	(3.4100)	(6.2178)	(6.2328)	(3.3032)	(6.4101)	(6.4255)
Peer: Cellphone	-6.3206*	12.4177*	12.4177*	-13.9365***	-12.6081*	-12.6081*
	(3.2310)	(6.3428)	(6.3581)	(3.1562)	(6.4337)	(6.4492)
Peer: Computer	7.7167**	14.2252**	14.2252**	10.0041***	9.4101	9.4101
	(3.5042)	(6.2964)	(6.3116)	(3.5384)	(6.6181)	(6.6341)
Teacher: Age	-0.0187	0.0487	0.0487	-0.0040	0.0315	0.0315
	(0.0299)	(0.4331)	(0.0434)	(0.0309)	(0.0441)	(0.0442)
Teacher: Female	1.4427**	1.8593**	1.8593**	1.2188	1.2048***	1.2048
	(0.5732)	(0.8092)	(0.8112)	(0.8374)	(1.1196)	(1.1223)
Teacher: Non-white	-0.1573	-0.3153	-0.3153	-0.3351	-2.1283	-2.1283***
	(0.5892)	(0.8199)	(0.8219)	(0.6003)	(0.7892)	(0.7911)
Teacher: University degree	-0.0473	0.1283	0.1283	1.7342	-3.3954	-3.3954
	(1.1757)	(1.7610)	(1.7652)	(1.8128)	(2.4202)	(2.4261)
Teacher: Number of classrooms	-0.1853*	-0.4299***	-0.4299***	-0.2702***	-0.5025***	-0.5025***
	(0.1103)	(0.1430)	(0.1434)	(0.0865)	(0.1167)	(0.1170)
School: Filtered water	4.1212	6.4198	6.4198	0.8266	33.2967	33.2926
	(6.1413)	(4.1646)	(4.1746)	(6.3191)	(21.1588)	(21.2098)
School: Absence of sewage system	-2.0844	-6.1221*	-6.1221*	-0.8780	-0.2186	-0.2186
	(1.5498)	(3.6260)	(3.6348)	(1.5835)	(3.5466)	(3.5551)
School: Internet	-2.5910	-2.7371	-2.7371	0.2483	-17.4814	-17.4814
	(5.2936)	(5.2567)	(5.2649)	(4.8452)	(11.1593)	(11.1862)
School: Teacher's room	7.0646**	6.3164	6.3164	4.5659	9.0051	9.0051
	(3.5495)	(4.5784)	(4.5894)	(3.1121)	(5.6686)	(5.6823)
School: Library	1.6445	0.7803	0.7803	0.4443	8.8385*	8.8385*
	(2.8986)	(4.8952)	(4.9079)	(2.5153)	(4.9054)	(4.9172)
School: Sports court	-1.2830	4.3566	4.3566	-0.5523	-6.0576*	-6.0576*
	(1.9807)	(5.2567)	(5.2694)	(2.2114)	(3.1837)	(3.1914)
School: TV equipment	-0.6696	-1.8567	-1.8567	-1.0274	1.5946	1.5946
	(1.9315)	(3.8763)	(3.8856)	(1.9123)	(4.4236)	(4.4343)
School: Multimedia equipment	2.8053	4.8853	4.8853	0.8591	9.0334	9.0334*
	(1.8622)	(3.5635)	(3.5721)	(2.2534)	(4.7903)	(4.8019)
School: Multifunction Printer	-0.4115	-0.3707	-0.3707	-0.5628	1.1536	1.1536
	(0.7536)	(1.6919)	(1.6959)	(0.7705)	(1.7734)	(1.7777)
School: Number of rooms	0.2147	0.7920	0.7920	0.2945	0.2753	0.2753
	(0.1834)	(0.6545)	(0.6560)	(0.1951)	(0.5342)	(0.5355)
School: Number of computers	0.0655	0.2422*	0.2422*	0.0395*	-0.0625	-0.0625
	(0.0444)	(0.1358)	(0.1361)	(0.0417)	(0.1104)	(0.1107)
School: Number of students	-0.0121***	-0.0145*	-0.0145*	-0.0128***	-0.0243***	-0.0243***
	(0.0033)	(0.0078)	(0.0078)	(0.0034)	(0.0094)	(0.0090)
School: Student-classroom ratio	0.0061	0.0098	0.0098	0.0139**	0.0153	0.0153
	(0.0064)	(0.0120)	(0.0120)	(0.0067)	(0.0213)	(0.0213)
Observations	132,970	132,970	132,970	132,970	132,970	132,970
Adjusted R^2	0.1718	0.1835	0.1796	0.1812	0.1945	0.1906

Notes: The table shows regression coefficients for the full specification of all three models estimated on Sample 3. Columns labelled "a" refer to mathematics scores, and columns "b" refer to Portuguese scores. Column (1) refers to estimates for Model 1 (School-Principal); column (2) refers to estimates for Model 2 (Teacher-Principal); and column (3) refers to estimates for Model 3 (School-Teacher-Principal). Standard errors are shown in parentheses. * $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$.

Table B.11 – Model 1 standardized principal value-added estimates robustness check for principal panel construction

	Mathematics				Portuguese			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Average Principal Value-Added	4.9638	4.7990	5.1359	4.9646	5.0089	5.0857	5.0212	5.0077
Principal Value-Added Standard Deviation	0.3345	0.3350	0.3335	0.3346	0.3004	0.3001	0.2987	0.3005
Max Principal VA	7.0225	6.8470	7.2144	7.0225	5.9605	6.0201	6.0954	5.9593
90^{th} Quantile	5.3607	5.1971	5.5407	5.3613	5.3728	5.4451	5.3826	5.3711
75^{th} Quantile	5.1711	5.0090	5.3452	5.1722	5.2012	5.2798	5.2165	5.2006
50^{th} Quantile	4.9492	4.7852	5.1200	4.9502	5.0170	5.0993	5.0287	5.0166
25^{th} Quantile	4.7460	4.5795	4.9211	4.7469	4.8202	4.8994	4.8372	4.8192
10^{th} Quantile	4.5555	4.3840	4.7269	4.5565	4.6446	4.7090	4.6594	4.6432
Min Principal VA	3.7664	3.6066	3.9439	3.7665	3.2308	3.3074	3.2696	3.2314
Observations	196,447	196,447	196,447	196,447	196,447	196,447	196,447	196,447
Number of Principals	1681	1666	1648	1681	1681	1666	1648	1681
Number of Principal VA estimated	1681	1666	1648	1681	1681	1666	1648	1681
Number of Schools	846	846	846	846	846	846	846	846
Number of School VA estimated	17	16	16	17	17	16	16	16
Adjusted R^2	0.1735	0.1737	0.1730	0.1736	0.1819	0.1821	0.1814	0.1819
Average PROEB Score	268.0661	268.0713	268.0699	268.0699	269.4916	269.4961	269.4950	269.4950
PROEB Score Standard Deviation	50.3852	50.3870	50.3870	50.3870	49.3257	49.3236	49.3235	49.3235
Student Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Peer Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
School Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Teacher Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The table shows standardized principal value-added estimates for Model 1 (School-Principal). Columns (1) through (4) refer to mathematics scores, and columns (5) through (8) refer to Portuguese scores. Columns (1) and (5) are our standard estimates, present in Table 15, which consider principals in charge during the second trimesters every year. Columns (2) and (6) consider principals in charge during the first trimester. Columns (3) and (7) consider principals in charge during the third trimester. Columns (4) and (8) consider principals in charge during the fourth trimester. All value-added estimates are standardized using each sample's PROEB scores standard deviation. All columns also present value-added estimates for our preferred specification for Model 1, which considers all controls.

Table B.12 – Model 2 standardized principal value-added estimates robustness check for principal panel construction

	Mathematics				Portuguese			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Average Principal Value-Added	4.9913	5.0180	5.0018	4.9855	4.8998	4.9774	5.0000	4.9909
Principal Value-Added Standard Deviation	0.3804	0.3847	0.3828	0.3824	0.3263	0.3255	0.3240	0.3259
Max Principal VA	7.5134	7.5282	7.5167	7.5090	6.6513	6.6539	6.4087	6.6526
90^{th} Quantile	5.4319	5.4718	5.4461	5.4262	5.3832	5.3728	5.3909	5.3854
75^{th} Quantile	5.2050	5.2359	5.2170	5.2004	5.1965	5.1875	5.2110	5.1999
50^{th} Quantile	4.9848	5.0064	4.9936	4.9786	5.0024	4.9909	5.0156	5.0048
25^{th} Quantile	4.7421	4.7679	4.7521	4.7356	4.7903	4.7803	4.8006	4.7923
10^{th} Quantile	4.5361	4.5649	4.5466	4.5301	4.5861	4.5728	4.6051	4.5880
Min Principal VA	3.5084	3.5403	3.5416	3.5004	3.6355	3.6309	3.3535	3.6366
Observations	523,664	522,633	525,689	524,042	523,664	522,633	525,689	524,042
Number of Principals	1895	1918	1904	1902	1895	1918	1904	1902
Number of Principal VA estimated	1895	1918	1904	1902	1895	1918	1904	1902
Number of Teacher Compositions	6133	6144	6160	6142	6319	6325	6355	6328
Number of Composition VA estimated	4471	4466	4495	4473	4555	4545	4582	6328
Adjusted R^2	0.1881	0.1918	0.1877	0.1881	0.1953	0.1946	0.1953	0.1953
Average PROEB Score	269.8603	270.2500	270.3205	270.2792	270.9095	271.3992	271.4570	271.4237
PROEB Score Standard Deviation	51.6763	51.6496	51.6550	51.6634	50.0836	49.9182	49.9251	49.9421
Student Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Peer Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
School Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Teacher Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The table shows standardized principal value-added estimates for Model 2 (Teacher-Principal). Columns (1) through (4) refer to mathematics scores, and columns (5) through (8) refer to Portuguese scores. Columns (1) and (5) are our standard estimates, present in Table 16, which consider principals in charge during the second trimesters every year. Columns (2) and (6) consider principals in charge during the first trimester. Columns (3) and (7) consider principals in charge during the third trimester. Columns (4) and (8) consider principals in charge during the fourth trimester. All value-added estimates are standardized using each sample's PROEB scores standard deviation. All columns also present value-added estimates for our preferred specification for Model 2, which considers all controls.

Table B.13 – Model 3 standardized principal value-added estimates robustness check for principal panel construction

	Mathematics				Portuguese			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Average Principal Value-Added	$7.71 \cdot 10^{-4}$	$1.01 \cdot 10^{-3}$	$4.10 \cdot 10^{-4}$	$7.60 \cdot 10^{-4}$	$-5.60 \cdot 10^{-4}$	$-9.63 \cdot 10^{-4}$	$-2.03 \cdot 10^{-3}$	$-5.93 \cdot 10^{-4}$
Principal Value-Added Standard Deviation	0.0656	0.0707	0.0583	0.0655	0.0518	0.0619	0.0442	0.0515
Max Principal VA	1.0493	1.0228	1.0625	1.0463	0.4027	0.4106	0.4047	0.4035
90^{th} Quantile	$9.60 \cdot 10^{-6}$	$2.16 \cdot 10^{-6}$	$2.41 \cdot 10^{-6}$	$1.08 \cdot 10^{-5}$	$2.62 \cdot 10^{-6}$	$1.67 \cdot 10^{-6}$	$5.37 \cdot 10^{-6}$	$2.51 \cdot 10^{-6}$
75^{th} Quantile	$4.90 \cdot 10^{-13}$	$4.00 \cdot 10^{-13}$	$4.08 \cdot 10^{-13}$	$4.46 \cdot 10^{-13}$	$1.98 \cdot 10^{-13}$	$1.66 \cdot 10^{-13}$	$1.73 \cdot 10^{-13}$	$1.72 \cdot 10^{-13}$
50^{th} Quantile	$7.97 \cdot 10^{-15}$	$-1.71 \cdot 10^{-15}$	$-6.20 \cdot 10^{-15}$	$-1.29 \cdot 10^{-15}$	$3.70 \cdot 10^{-15}$	$9.80 \cdot 10^{-15}$	$2.51 \cdot 10^{-15}$	$3.15 \cdot 10^{-15}$
25^{th} Quantile	$-4.55 \cdot 10^{-13}$	$-3.71 \cdot 10^{-13}$	$-4.35 \cdot 10^{-13}$	$-4.64 \cdot 10^{-13}$	$-1.59 \cdot 10^{-13}$	$-1.56 \cdot 10^{-13}$	$-1.47 \cdot 10^{-13}$	$-1.57 \cdot 10^{-13}$
10^{th} Quantile	$-1.60 \cdot 10^{-5}$	$-2.02 \cdot 10^{-5}$	$-4.53 \cdot 10^{-6}$	$-1.34 \cdot 10^{-5}$	$-1.50 \cdot 10^{-5}$	$-1.32 \cdot 10^{-5}$	$-8.78 \cdot 10^{-6}$	$-1.26 \cdot 10^{-6}$
Min Principal VA	$3.29 \cdot 10^{-1}$	$3.20 \cdot 10^{-1}$	$-3.3 \cdot 10^{-1}$	$-3.29 \cdot 10^{-1}$	$-2.66 \cdot 10^{-1}$	$-4.47 \cdot 10^{-1}$	$-2.67 \cdot 10^{-1}$	$-2.67 \cdot 10^{-1}$
Observations	132,970	134,399	135,898	133,065	132,970	134,399	135,898	133,065
Number of Principals	689	708	695	692	689	708	695	692
Number of Principal VA estimated	689	708	695	692	689	708	695	692
Number of Schools	628	630	637	631	628	630	637	631
Number of School VA estimated	2	3	2	2	2	3	2	2
Number of Teacher Compositions	1849	1870	1889	1851	1901	1926	1946	1905
Number of Composition VA estimated	1213	1224	1247	1212	1240	1249	1278	1241
Adjusted R^2	0.1796	0.1798	0.1780	0.1794	0.1953	0.1900	0.1907	0.1908
Average PROEB Score	267.7094	267.9336	267.8665	267.7121	270.9095	269.6282	269.5458	269.4598
PROEB Score Standard Deviation	50.6069	50.6735	50.6103	50.6174	50.0836	49.5928	49.6340	49.6397
Student Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Peer Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
School Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Teacher Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The table shows standardized principal value-added estimates for Model 3 (School-Teacher-Principal). Columns (1) through (4) refer to mathematics scores, and columns (5) through (8) refer to Portuguese scores. Columns (1) and (5) are our standard estimates, present in Table **??**, which consider principals in charge during the second trimesters every year. Columns (2) and (6) consider principals in charge during the first trimester. Columns (3) and (7) consider principals in charge during the third trimester. Columns (4) and (8) consider principals in charge during the fourth trimester. All value-added estimates are standardized using each sample's PROEB scores standard deviation. All columns also present value-added estimates for our preferred specification for Model 3, which considers all controls.

Table B.14 – Model 1 standardized principal value-added estimates robustness check for teacher-classroom allocation

	Mathematics					Portuguese				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Average Principal Value-Added	4.9638	4.9643	4.9564	4.9587	4.9629	5.0089	5.0076	5.0054	5.0226	5.0082
Principal Value-Added Standard Deviation	0.3345	0.3346	0.3346	0.3346	0.3346	0.3004	0.3004	0.3005	0.3005	0.3005
Max Principal VA	7.0225	7.0238	7.0154	7.0151	7.0231	5.9605	5.9576	5.9555	5.9746	5.9579
90^{th} Quantile	5.3607	5.3605	5.3533	5.3564	5.3595	5.3728	5.3704	5.3684	5.3874	5.3717
75^{th} Quantile	5.1711	5.1716	5.1629	5.1665	5.1700	5.2012	5.2006	5.1983	5.2150	5.2007
50^{th} Quantile	4.9492	4.9495	4.9403	4.9431	4.9476	5.0170	5.0166	5.0145	5.0322	5.0171
25^{th} Quantile	4.7460	4.7473	4.7382	4.7406	4.7452	4.8202	4.8188	4.8169	4.8332	4.8197
10^{th} Quantile	4.5555	4.5560	4.5470	4.5492	4.5547	4.6446	4.6439	4.6407	4.6567	4.6432
Min Principal VA	3.7664	3.7670	3.7582	3.7602	3.7654	3.2308	3.2312	3.2279	3.2461	3.2311
Observations	196,447	196,447	196,447	196,447	196,447	196,447	196,447	196,447	196,447	196,447
Number of Principals	1681	1681	1681	1681	1681	1681	1681	1681	1681	1681
Number of Principal VA estimated	1681	1681	1681	1681	1681	1681	1681	1681	1681	1681
Number of Schools	846	846	846	846	846	846	846	846	846	846
Number of School VA estimated	17	17	17	17	17	17	17	17	17	17
Adjusted R^2	0.1735	0.1736	0.1736	0.1736	0.1736	0.1819	0.1819	0.1819	0.1819	0.1819
Average PROEB Score	268.0661	268.0699	268.0699	268.0699	268.0699	269.4916	269.4950	269.4950	269.4950	269.4950
PROEB Score Standard Deviation	50.3852	50.3870	50.3870	50.3870	50.3870	49.3257	49.3235	49.3235	49.3235	49.3235
Student Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Peer Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
School Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Teacher Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The table shows standardized principal value-added estimates for Model 1 (School-Principal). Columns (1) through (5) refer to mathematics scores, and columns (6) through (10) refer to Portuguese scores. Columns (1) and (6) are our standard estimates, present in Table 15, which consider the teacher-to-classroom allocation used in our main analysis. Columns (2) and (7) consider the second teacher allocation draw, and so on. All value-added estimates are standardized using each sample's PROEB scores standard deviation. All columns also present value-added estimates for our preferred specification for Model 3, which considers all controls.

Table B.15 – Model 2 standardized principal value-added estimates robustness check for teacher-classroom allocation

	Mathematics					Portuguese				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Average Principal Value-Added	4.9913	4.9810	5.1197	5.1029	5.1163	4.8998	5.0072	5.0584	5.1531	5.0540
Principal Value-Added Standard Deviation	0.3804	0.3831	0.3903	0.3911	0.3901	0.3263	0.3282	0.3294	0.3317	0.3284
Max Principal VA	7.5134	7.5087	7.6788	7.6623	7.6728	6.6513	6.6447	6.7367	6.8145	6.7374
90^{th} Quantile	5.4319	5.4224	5.5720	5.5557	5.5692	5.3832	5.3973	5.4523	5.5483	5.4496
75^{th} Quantile	5.2050	5.1967	5.3415	5.3244	5.3377	5.1965	5.2209	5.2707	5.3679	5.2662
50^{th} Quantile	4.9848	4.9729	5.1121	4.0938	5.1079	5.0024	5.0206	5.0733	5.1677	5.0687
25^{th} Quantile	4.7421	4.7319	4.8702	4.8524	4.8669	4.7903	4.8065	4.8561	4.9489	4.8535
10^{th} Quantile	4.5361	4.5255	4.6606	4.6438	4.6579	4.5861	4.6050	4.6552	4.7464	4.6509
Min Principal VA	3.5084	3.4919	3.6314	3.6106	3.6256	3.6355	3.6394	3.6759	3.7647	3.6850
Observations	523,664	523,800	523,971	523,984	524,029	523,664	523,800	523,971	523,984	524,029
Number of Principals	1895	1900	1900	1901	1901	1895	1900	1900	1901	1901
Number of Principal VA estimated	1895	1900	1900	1901	1901	1895	1900	1900	1901	1901
Number of Teacher Compositions	6133	6142	6142	6147	6148	6319	6323	6324	6321	6325
Number of Composition VA estimated	4471	4477	4474	4481	4479	4555	4553	4553	4551	4553
Adjusted R^2	0.1881	0.1882	0.1881	0.1882	0.1881	0.1953	0.1953	0.1954	0.1952	0.1954
Average PROEB Score	270.2740	270.2760	270.2774	270.2791	270.2775	271.4228	271.4205	271.4231	271.4237	271.4321
PROEB Score Standard Deviation	51.6596	51.6642	51.6630	51.6640	51.6624	49.9426	49.9432	49.9427	49.9424	49.9425
Student Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Peer Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
School Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Teacher Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The table shows standardized principal value-added estimates for Model 2 (Teacher-Principal). Columns (1) through (5) refer to mathematics scores, and columns (6) through (10) refer to Portuguese scores. Columns (1) and (6) are our standard estimates, present in Table 16, which consider the teacher-to-classroom allocation used in our main analysis. Columns (2) and (7) consider the second teacher allocation draw, and so on. All value-added estimates are standardized using each sample's PROEB scores standard deviation. All columns also present value-added estimates for our preferred specification for Model 3, which considers all controls.

Table B.16 – Model 3 standardized principal value-added estimates robustness check for principal panel construction

	Mathematics					Portuguese				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Average Principal Value-Added	$7.71 \cdot 10^{-4}$	$6.56 \cdot 10^{-4}$	$-4.16 \cdot 10^{-4}$	$7.17 \cdot 10^{-4}$	$-3.19 \cdot 10^{-4}$	$-5.60 \cdot 10^{-4}$	$-3.59 \cdot 10^{-4}$	$-4.10 \cdot 10^{-4}$	$-4.07 \cdot 10^{-4}$	$-3.96 \cdot 10^{-4}$
Principal Value-Added Standard Deviation	0.0656	0.0652	0.0511	0.0659	0.0512	0.0518	0.0528	0.0510	0.0530	0.0510
Max Principal VA	1.0493	1.0405	0.5333	1.0511	0.5334	0.4027	0.4310	0.4009	0.4324	0.4053
90^{th} Quantile	$9.60 \cdot 10^{-6}$	$6.67 \cdot 10^{-6}$	$2.32 \cdot 10^{-6}$	$1.06 \cdot 10^{-4}$	$2.16 \cdot 10^{-4}$	$2.62 \cdot 10^{-6}$	$2.40 \cdot 10^{-6}$	$1.41 \cdot 10^{-4}$	$3.22 \cdot 10^{-4}$	$1.13 \cdot 10^{-4}$
75^{th} Quantile	$4.90 \cdot 10^{-13}$	$4.37 \cdot 10^{-13}$	$4.58 \cdot 10^{-13}$	$4.82 \cdot 10^{-13}$	$4.40 \cdot 10^{-13}$	$1.98 \cdot 10^{-13}$	$1.80 \cdot 10^{-13}$	$1.97 \cdot 10^{-13}$	$1.80 \cdot 10^{-13}$	$1.89 \cdot 10^{-13}$
50^{th} Quantile	$7.97 \cdot 10^{-15}$	$-1.18 \cdot 10^{-15}$	$4.39 \cdot 10^{-15}$	$-6.27 \cdot 10^{-15}$	$-1.99 \cdot 10^{-15}$	$3.70 \cdot 10^{-15}$	$1.35 \cdot 10^{-15}$	$-3.57 \cdot 10^{15}$	$7.36 \cdot 10^{-16}$	$8.92 \cdot 10^{-16}$
25^{th} Quantile	$-4.55 \cdot 10^{-13}$	$-5.02 \cdot 10^{-13}$	$-5.13 \cdot 10^{-13}$	$-4.89 \cdot 10^{-13}$	$-5.10 \cdot 10^{-13}$	$-1.59 \cdot 10^{-13}$	$-1.55 \cdot 10^{-13}$	$-1.89 \cdot 10^{-13}$	$-1.56 \cdot 10^{-13}$	$-1.68 \cdot 10^{-13}$
10^{th} Quantile	$-1.60 \cdot 10^{-5}$	$-1.29 \cdot 10^{-5}$	$-1.06 \cdot 10^{-5}$	$-2.01 \cdot 10^{-5}$	$-1.40 \cdot 10^{-5}$	$-1.50 \cdot 10^{-5}$	$-5.72 \cdot 10^{-5}$	$-4.11 \cdot 10^{-5}$	$-1.75 \cdot 10^{-5}$	$-2.83 \cdot 10^{-5}$
Min Principal VA	$3.29 \cdot 10^{-1}$	$-3.31 \cdot 10^{-1}$	$-3.31 \cdot 10^{-1}$	$-3.31 \cdot 10^{-1}$	$-3.31 \cdot 10^{-1}$	$-2.66 \cdot 10^{-1}$	$-2.85 \cdot 10^{-1}$	$-2.65 \cdot 10^{-1}$	$-2.86 \cdot 10^{-1}$	$-2.58 \cdot 10^{-4}$
Observations	132,970	132,823	132,994	133,007	133,052	132,970	132,823	132,994	133,007	133,052
Number of Principals	689	690	690	691	690	689	690	690	690	690
Number of Principal VA estimated	689	690	690	691	690	689	690	690	690	690
Number of Schools	628	630	630	630	630	628	630	630	630	630
Number of School VA estimated	2	2	2	2	2	2	2	2	2	2
Number of Teacher Compositions	1849	1849	1848	1853	1854	1901	1898	1899	1898	1902
Number of Composition VA estimated	1213	1212	1210	1215	1215	1240	1235	1226	1235	1238
Adjusted R^2	0.1796	0.1796	0.1794	0.1795	0.1793	0.1953	0.1906	0.1908	0.1905	0.1908
Average PROEB Score	267.7094	267.6946	267.7036	267.7105	267.7052	270.9095	269.4436	269.4564	269.4588	269.4573
PROEB Score Standard Deviation	50.6069	50.6181	50.6148	50.6194	50.6128	49.3235	49.6430	49.6420	49.6407	49.6410
Student Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Peer Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
School Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Teacher Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Notes: The table shows standardized principal value-added estimates for Model 3 (School-Teacher-Principal). Columns (1) through (5) refer to mathematics scores, and columns (6) through (10) refer to Portuguese scores. Columns (1) and (6) are our standard estimates, present in Table 17, which consider the teacher-to-classroom allocation used in our main analysis. Columns (2) and (7) consider the second teacher allocation draw, and so on. All value-added estimates are standardized using each sample's PROEB scores standard deviation. All columns also present value-added estimates for our preferred specification for Model 3, which considers all controls.

Table B.17 – Model 3 LASSO regression coefficients and statistics after feature selection

	Mathematics	Portuguese
Student: Female	-3.5877	11.9964
Student: Non-white	-4.3737	-4.0077
Student: Mother middle school degree	0	0
Student: Mother high school degree	3.8651	3.0652
Student: Mother university degree	7.2230	4.5866
Student: Father middle school degree	0	0
Student: Father high school degree	1.8015	2.9813
Student: Father university degree	0	0
Student: Private school history	0	0
Student: Grade repetition history	-23.0960	-23.8014
Household: *Bolsa Família*	-3.3474	-3.4362
Household: Paved street	0	1.6138
Household: Garbage collection	0	0.0156
Household: Bathroom	0	0.7932
Household: Car	0	-0.4634
Household: Cellphone	10.1698	12.9245
Household: Computer	7.2411	6.6408
Peer: Female	0	0
Peer: Non-white	-6.2838	0
Peer: *Bolsa Família*	0	0
Peer: Mother university degree	0	0
Peer: Father university degree	0	0
Peer: Car	3.8263	0
Peer: Cellphone	0	0
Peer: Computer	6.5224	9.6260
Teacher: Age	0	-0.1766
Teacher: Female	0.6731	0
Teacher: Non-white	0	-0.4412
Teacher: University degree	0	0
Teacher: Number of classrooms	-0.2987	-0.1730
School: Filtered water	0	0
School: Absence of sewage system	0	0
School: Internet	0	0
School: Teacher's room	0	0
School: Library	0	0
School: Sports court	-4.4197	-3.2746
School: TV equipment	0	0
School: Multimedia equipment	0	0
School: Multifunction Printer	0	0
School: Number of rooms	0	0.0227
School: Number of computers	1.3258	1.4771
School: Number of students	0	0
School: Student-classroom ratio	0	0
Observations	132,970	132,970
Number of Principals	689	689
Number of Principal FE estimated	0	0
Number of Schools	628	628
Number of School FE estimated	0	0
Number of Teacher Compositions	1849	1901
Number of Teacher Composition FE estimated	0	0
Penalization parameter (λ)	0.353470	0.331797

Notes: This table presents coefficients for the LASSO regression with feature selection in our preferred specification of Model 3. The results shown are for mathematics and Portuguese. We display control variables coefficients and the number of estimated FE for each factor at the bottom. Note that all fixed effects are excluded from the model, both for mathematics and Portuguese.

Table B.18 – Regression coefficients for management practice association to value-added estimates from all models

	Mathematics					Portuguese				
	(1)	(2)	(3)	(4)	(5)	(6) (7)	(8) (9) (10)			
Principal: Female	-0.0300	-0.0511	-0.0163	-0.0999	-0.0195**	0.0229	0.0010	-0.0162	-0.0325	-0.0149*
	(0.0384)	(0.0473)	(0.0329)	(0.0627)	(0.0097)	(0.0363)	(0.0404)	(0.0208)	(0.0693)	(0.0077)
Principal: Non-white	-0.0729**	-0.0458	-0.0150	-0.0033	-0.0013	-0.0502	-0.0213	0.0100	0.0385	0.0013
	(0.0359)	(0.0476)	(0.0319)	(0.0626)	(0.0098)	(0.0340)	(0.0406)	(0.0272)	(0.0697)	(0.0077)
Principal: Univeristy degree	-0.1510*	-0.1123	-0.1531**	-0.1147	-0.0094	-0.1484*	-0.1177	-0.1107**	-0.2370	-0.0185
	(0.0836)	(0.1040)	(0.0660)	(0.1366)	(0.0213)	(0.0790)	(0.0887)	(0.0563)	(0.1523)	(0.0169)
Principal: Specialization	-0.1029	-0.0599	-0.0989	-0.0215	0.0072	-0.1607**	-0.1083	-0.0412	-0.1349	-0.0056
	(0.0836)	(0.1013)	(0.0621)	(0.1331)	(0.0208)	(0.0790)	(0.0864)	(0.0529)	(0.1483)	(0.0165)
Principal: Experience > 5 years	-0.0664	-0.1287*	-0.352	-0.2591***	-0.0219	-0.0394	-0.0938	0.0395	-0.0803	-0.0065
	(0.0562)	(0.0708)	(0.0376)	(0.0931)	(0.0145)	(0.0531)	(0.0604)	(0.0320)	(0.1038)	(0.0115)
Principal: Experience other school	-0.0284	0.0224	0.0184	0.1664**	-0.0156	0.0079	0.0496	-0.0295	0.0808	0.0151*
	(0.0412)	(0.0515)	(0.0384)	(0.0677)	(0.0106)	(0.0386)	(0.0440)	(0.0327)	(0.0755)	(0.0084)
Principal: Another job	0.0209	0.0835	-0.0015	-0.1488	-0.0028	0.0183	0.0092	-0.0208	-0.1575	-0.0441**
	(0.1001)	(0.1257)	(0.0960)	(0.1651)	(0.0258)	(0.0946)	(0.1072)	(0.0818)	(0.1840)	(0.0205)
Principal: Excessive workload	0.0154	0.0349	0.0131	0.0219	-0.0001	0.0093	0.0298	0.0221	0.0784	-0.0007
	(0.0399)	(0.0523)	(0.0354)	(0.0688)	(0.0107)	(0.0377)	(0.0447)	(0.0302)	(0.0767)	(0.0085)
Principal: Training	-0.0200	0.1016	0.0664	0.0644	0.0414**	-0.0538	-0.0604	-0.0162	-0.1641	0.0001
	(0.0656)	(0.0992)	(0.0642)	(0.1304)	(0.0204)	(0.0620)	(0.0847)	(0.0547)	(0.1454)	(0.0162)
Principal: Elected	-0.0137	-0.0144	-0.0392	-0.1435*	0.0076	0.0076	0.0460	-0.0011	0.0590	-0.0050
	(0.0465)	(0.0596)	(0.0460)	(0.0783)	(0.0122)	(0.0440)	(0.0508)	(0.0392)	(0.0873)	(0.0097)
Observations	261	138	532	138	138	261	138	532	138	138
Principal controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Adjusted R^2	0.0093	-0.0164	0.0004	-0.0508	0.0530	0.0220	-0.0103	-0.0038	-0.0508	0.0018

Notes: The table shows regression coefficients for control variables in the association between principal value-added estimates from all models to management practices domain scores. Columns (1) and through (5) refer to mathematics scores, and columns (6) through (10) refer to Portuguese scores. Columns (1) and (6) refer to Model 1 (School-Principal) value-added estimates on Sample 1, while columns (2) and (7) refer to estimates on Sample 3. Columns (3) and (8) refer to Model 2 (Teacher-Principal) value-added estimates on Sample 2, while columns (4) and (9) refer to estimates on Sample 3. Columns (5) and (10) refer to Model 3 (School-Teacher-Principal) value-added estimates. Excessive workload is considered a weekly journey (accounting for all jobs) of over 40 hours. Standard deviations are shown in parentheses. * p < 0.1 ** p < 0.05 *** 0 < 0.01.