Persing Junior Cardenas Vivanco

**Nonlinear modeling and parameter identification for AUV
dynamics**

Revised Version

(The original version is in the unit hosting of
the Postgraduate Program)

**Thesis presented to the Pos-graduate
program in Mechanical Engineering at the
Polytechnic School of the São Paulo
University, to obtain the degree of Doctor of
Science.**

**Concentration area: Control Engineering
and Mechanical Automation**

Advisor:

Prof. Dr. Ettore Apolônio de Barros

São Paulo

2019

Autorizo a reprodução e divulgação total ou parcial deste trabalho, por qualquer meio convencional ou eletrônico, para fins de estudo e pesquisa, desde que citada a fonte.

Este exemplar foi revisado e corrigido em relação à versão original, sob responsabilidade única do autor e com a anuência de seu orientador.

São Paulo, _____ de _____ de _____

Assinatura do autor: _____

Assinatura do orientador: _____

Catálogo-na-publicação

CARDENAS VIVANCO, Persing Junior

Nonlinear modeling and parameter identification for AUV dynamics / P. J. CARDENAS VIVANCO -- versão corr. -- São Paulo, 2019.
107 p.

Tese (Doutorado) - Escola Politécnica da Universidade de São Paulo.
Departamento de Engenharia Mecatrônica e de Sistemas Mecânicos.

1.AUV 2.Hydrodynamic Coefficients 3.Model Parameter Identification
4.Nonlinear dynamic systems 5.Dynamics of underwater vehicles
I.Universidade de São Paulo. Escola Politécnica. Departamento de
Engenharia Mecatrônica e de Sistemas Mecânicos II.t.

Name: Persing Junior Cardenas Vivanco

Title: Nonlinear modeling and parameter identification for AUV dynamics

Thesis presented to the Pos-graduate program in Mechanical Engineering at the Polytechnic School of the São Paulo University, to obtain the degree of Doctor of Science.

Date of approval: 11/11/2019

Examination Board

Prof. Dr. Ettore Apolonio de Barros

Institution: EP - USP

Judgment: Aprovado

Prof. Dr. Agenor de Toledo Fleury

Institution: EP - USP

Judgment: Aprovado

Prof. Dr. Luiz Carlos Sandoval Góes

Institution: ITA - Externo

Judgment: Aprovado

Prof. Dr. Paulo de Tarso Themistocles Esperança

Institution: UFRJ - Externo

Judgment: Aprovado

Prof. Dr. Hélio Koiti Kuga

Institution: INPE - Externo

Judgment: Aprovado

Dedication

I dedicate this work to my parents Alejandrina Lidia Vivanco Sánchez and Romulo Persing Cárdenas Ruiz for the affection and support that they always had to me.

Acknowledgments

I thank first of all my parents, my sister Lydia D. Cardenas Vivanco and Ligia M. Tonon Parra for the support and motivation given to me during the realization of this work.

Thanks also to CAPES agency for sponsoring my Doctorate project and to Professor Dr. Ettore Apolônio de Barros for the guidance during my research.

Modelagem Não linear e Identificação de Parâmetros para a Dinâmica de um AUV

Resumo

A dinâmica de um veículo submarino é intrinsecamente não linear. Sendo assim, quando for desejável a reprodução de vários tipos de manobras, tais como “linha reta”, “zig-zag” e “giros”, um modelo matemático não linear é o mais adequado. Usualmente, para veículos tais como um AUV, veículo autônomo submarino, a dinâmica é modelada como um sistema não linear desacoplado, o qual admite que o veículo manobre no plano horizontal ou no plano vertical. Este trabalho propõe um novo método para a identificação da dinâmica de AUVs, combinando uma abordagem analítica e semi-empírica, ASE, com uma abordagem de identificação de sistemas para estimar os coeficientes hidrodinâmicos do modelo do veículo manobrando no plano horizontal. O método ASE é usado na inicialização do processo de estimativa de parâmetros, admitindo uma geometria simplificada do AUV. A fase de identificação de sistemas adota estimadores não lineares, tais como o Filtro Estendido de Kalman (EKF) e o Filtro de Kalman “Unscented”, estimando os coeficientes hidrodinâmicos de acordo com o tipo de manobra em que estes são relevantes. Os dados experimentais foram obtidos com os sensores do AUV Pirajuba durante testes no mar. O modelo identificado é utilizado para simular as manobras do veículo e as respectivas variáveis de movimento são comparadas com os dados experimentais, validando assim, o modelo identificado. O método é aplicado para a estimativa de todos os coeficientes hidrodinâmicos presentes nas equações de movimento.

Palavras chave: Coeficientes hidrodinâmicos. Dinâmica de veículos submersíveis. Identificação de parâmetros. Sistemas dinâmicos não lineares. Veículo submarino autônomo.

Nonlinear Modeling and Parameter Identification for AUV Dynamics

Abstract

The dynamics of an underwater vehicle is intrinsically nonlinear. Hence, when it is desirable to reproduce various types of maneuvers of an underwater vehicle, such as straight line, zigzag and turning in circles; a nonlinear mathematical model is required. Usually, for vehicles such as an autonomous underwater vehicle (AUV), the dynamics is modeled as a decoupled-nonlinear system which considers that the vehicle is maneuvering in the horizontal plane or in the vertical plane. This work proposes a new method for identification of the AUV Dynamics combining an analytical and semi-empirical (ASE) approach and a system identification approach to estimate the hydrodynamic coefficients related to the horizontal maneuvering of the vehicle. The ASE method is used to initialize the parameter estimation process, assuming a simplified geometry of the AUV. The system identification phase adopted nonlinear estimators, such as the Extended Kalman Filter (EKF) and the Unscented Kalman Filter (UKF), to estimate the hydrodynamic coefficients according to the type of maneuver in which they are relevant. The experimental data were obtained with the sensors of the Pirajuba AUV during sea-trials. The identified model is used to simulate the vehicle maneuver and the movement variables are compared to the experimental data, thus validating the identified model. The method is applied to the estimation of the complete set of hydrodynamic coefficients included in the equations of motion.

Keywords: AUV; Hydrodynamic coefficients; Model parameter identification; Nonlinear dynamic systems; Dynamics of underwater vehicles.

Symbols List

$\hat{x}, \hat{y}, \hat{z}$	Unitary vectors whose form the vector base of the vehicle reference frame.
x, y, z	Cartesians coordinates of a point in the vehicle and relative to the vehicle frame.
u, v, w	Components on $\hat{x}, \hat{y}$ and $\hat{z}$ of the absolute velocity vector of the vehicle frame center.
ϕ, θ, ψ	Euler angles of the vehicle frame relative to an inertial reference frame.
p, q, r	Components on $\hat{x}, \hat{y}$ and $\hat{z}$ of the angular velocity vector of the vehicle and relative to an inertial reference frame
X, Y, Z	Components on $\hat{x}, \hat{y}$ and $\hat{z}$ of the resultant of the external forces acting on the vehicle.
K, M, N	Components on $\hat{x}, \hat{y}$ and $\hat{z}$ of the resultant moment of the external forces acting on the vehicle.
v_o	Absolute velocity vector of the vehicle frame center.
a_o	The absolute acceleration vector of vehicle frame center.
ω_o	Angular velocity vector of the vehicle relative to an inertial reference frame.
$p_{A/o}$	Position of a point “A” fixed on the vehicle and relative to the center of the vehicle reference frame.
v_A	Absolute velocity vector of a point “A” fixed on the vehicle.
a_A	Absolute acceleration vector of a point “A”, fixed on the vehicle.
a_g	Absolute acceleration vector of the center of gravity of the vehicle.
m	Submerged vehicle mass.
g	Gravitational constant.
F_{HD}	Resultant of the hydrodynamic forces over the vehicle
F_{GB}	Resultant of the buoyancy and gravitational forces;
F_{TH}	Thrust force produced by the propeller.
$X_{\#}$	Hydrodynamic coefficient of a force in the $\hat{x}$ direction.

$Y_{\#}$	Hydrodynamic coefficient of a force in the $\hat{\mathbf{y}}$ direction.
$Z_{\#}$	Hydrodynamic coefficient of a force in the $\hat{\mathbf{z}}$ direction.
$M_{\#}$	Hydrodynamic coefficient of a moment in the $\hat{\mathbf{y}}$ direction;
$N_{\#}$	Hydrodynamic coefficient of a moment in the $\hat{\mathbf{z}}$ direction.
η	Rotation rate of the propeller.
δ_R, δ_E	Rudder and elevator angles, respectively.
V^{hull}	Volume of the vehicle hull.
a	Axial semi-edge longitude of the equivalent ellipsoid.
b	Radial semi-edge longitude of the maximal cross-section of the equivalent ellipsoid
e	Eccentricity of the equivalent ellipsoid.
ρ	Density of the vehicle surrounding fluid.
$c_{(x)}^{ad}$	Added-mass coefficient of the vehicle per unit length.
$r_{(x)}$	Radio of the hull cross-section as a function of the longitudinal position “ x ”.
S_{rudd}	Semi-span of the vehicle rudders, calculated as the sum of the vehicle radio and the exposed semi-span of the rudders.
S_{sail}	Semi-span of the vehicle sail calculated as the sum of the bare hull radius and the exposed semi-span of the sail.
$F_{Munk}^{hull}, M_{Munk}^{hull}$	Munk cross force acting on the vehicle hull and its moment.
$\frac{dF_{Munk}^{hull}}{dx}$	Munk cross force per unit length.
$k_2 - k_1$	Munk apparent-mass factor.
A_{cs}^{hull}	Cross section area of the vehicle hull.
U_O	Velocity of the vehicle relative to the fluid.
α	Angle of attack, it is defined between the vehicle velocity vector relative to the fluid ($\mathbf{U}_O$) and the longitudinal axis of the vehicle.
$F_{pot}^{hull}, M_{pot}^{hull}$	Potential force due to the potential effects acting on the vehicle hull and its moment.

x^*	Axial position of the "station" of the body, at which the flow can no longer be considered as potential.
A^*	Cross-section area in the point x^* .
V^*	Hull's volume from the nose tip to x^* .
L	Longitudinal length of the vehicle hull.
L_{ref}	Reference length of the vehicle hull.
A_{ref}	Reference area of the vehicle hull.
$c_{pot}^{hull}, c_{mpot}^{hull}$	Dimensionless coefficients of the potential force acting on the hull and its moment, respectively
X_{pot}^{hull}	Axial component of the potential force acting on the vehicle hull.
F_{d0}^{hull}	Axial drag force acting on the vehicle hull, it is due to viscous effects.
F_{dc}^{hull}	Cross-flow drag force acting on the vehicle hull, it is due to viscous effects.
$c_{d0}^{hull}, c_{dc}^{hull}$	Dimensionless coefficients corresponding to the axial and cross-flow drag forces respectively.
R	Radio of the maximal cross-section of the vehicle hull.
D	Maximal diameter of the vehicle hull.
f	Fineness ratio of the vehicle hull ($f = L/D$).
A_{wett}	Wetted surface area, in the case of a submersible vehicle it is equal to the total surface of the hull.
c_{sf}^{hull}	Skin friction drag coefficient of the hull.
Re	Reynolds number.
A_p	Plan-form area of the hull in the $\hat{x}$ - $\hat{y}$ plane.
$c_{dc}^{cylinder}, \eta_{dc}$	Cross-flow drag coefficient, in an infinitely long cylinder, and its correction factor for a finite cylinder respectively.
M_n	Mach number
ν	kinematic viscosity of the seawater
M_{drag}^{hull}	The moment produced by the cross flow on the vehicle, due to the viscous-drag forces on the hull.

x_p	Longitudinal position of the plan-form-geometric center of the hull.
$X^{hull}, Y^{hull}, Z^{hull}$	Components on $\hat{x}$, $\hat{y}$ and $\hat{z}$ of the resultant of the external forces acting on the vehicle hull.
$K^{hull}, M^{hull}, N^{hull}$	Components on $\hat{x}$, $\hat{y}$ and $\hat{z}$ of the resultant moment of the external forces acting on the vehicle hull.
β	Sideslip angle
F_L^{fin}, F_D^{fin}	Lift and drag component of the resultant hydrodynamic forces that act on a fin.
C_L^{fin}, C_D^{fin}	lift and drag coefficients corresponding to the Lift and drag forces, respectively.
U_F	Velocity of the fin's hydrodynamic center relative to the fluid.
A^{fin}	Area of the fin's plan-form.
α^{fin}	Attack angle of the fin relative to the flow around to the fin.
a^{fin}	Geometrical aspect ratio of the fin.
c_l^{fin}	linear part of the lift coefficient, it is the curve's slope of the lift coefficient as a function of the angle of attack when the angle of attack is zero.
$c_{d0}^{fin}, c_{dc}^{fin}$	Axial and cross-flow dimensionless-drag coefficients of the fin.
h^{fin}	Height of the fin.
ε	Efficiency factor obtained from experimental observations.
$\Lambda_{c/4}^{fin}$	Sweep Angle for a quarter chord in the fin.
c_{id}^{fin}	Induced drag coefficient of the fin, it is due to potential effects.
α_F^{fin}	Angle between the velocity vector, of the fin' hydrodynamic center (F), and the root chord of the fin.
$X^{fin}, Y^{fin}, Z^{fin}$	Components on $\hat{x}$, $\hat{y}$ and $\hat{z}$ of lift and drag forces acting on a fin.
$K^{fin}, M^{fin}, N^{fin}$	Components on $\hat{x}$, $\hat{y}$ and $\hat{z}$ of the resultant moment of the lift and drag forces acting on a fin.
u_F, v_F, w_F	Components on $\hat{x}$, $\hat{y}$ and $\hat{z}$ of the velocity vector of the fin' hydrodynamic center (F).
$K_{f(h)}, K_{h(f)}$	Interference factors of the fin due to the hull and hull due to the fin,

respectively.

X^{th}	Thrust force acting on the vehicle due to the propeller.
C_{th}	Propeller's thrust coefficient.
J	Advance rate of the vehicle.
D_{sc}	Screw diameter.
η	Screw rotation rate.
Θ	Parameters vector.
x, y, u	Simple state, measurement and inputs variable vectors.
f, h	Process and measurements nonlinear functions.
P	Covariance matrix of the state vector.
n_f, n_y	Process and measurement noise vectors, respectively.
Q, R	Covariance matrices of the process and measurements noise vectors, respectively.
σ	Standard deviation of a state variable.
F, H	Jacobians matrices of the nonlinear functions f and h respectively.
K	Gain matrix of the Kalman Filter.
T	Period of iteration in a discrete system.
$m_{11}, m_{21}, m_{22}, m_{31}, m_{32}$	Changed variables of the added mass coefficients.
I_{zz}	Moment of inertia in "z" of the vehicle.
l_n, l_t, l_c	Length of the vehicle's nose, tail and cylindrical section; respectively.
h_R, h_S	Height of the rudders and sail, respectively.

Summary

1. Introduction.....	13
1.1. Justification.....	13
1.2. State of the Art.....	14
1.3. Objectives.....	18
1.4. Manuscript Structure.....	19
2. Mathematical Modeling of the AUV Motion.....	20
2.1. Kinematics of an Underwater Vehicle.....	20
2.2. Dynamics of an Underwater Vehicle.....	22
2.3. Measurements Model for an AUV.....	27
3. ASE Estimations of the Hydrodynamic Coefficients.....	29
3.1. Added-Mass Coefficients.....	29
3.2. Hydrodynamic Coefficients Related to the Bare Hull.....	31
3.3. Hydrodynamic Coefficients Related to the Vehicles Fins.....	40
3.4. Hydrodynamic Coefficients of the Fins-Hull Combinations.....	54
3.5. Propeller's Thrust Coefficients.....	57
4. Model Parameters Identification of an AUV.....	59
4.1. Structure of a Model Parameters Estimator.....	59
4.2. The Extended Kalman Filter.....	64
4.3. The Unscented Kalman Filter.....	68
4.4. A New Method for the Hydrodynamic Coefficients Identification.....	70
5. Experimental Validation of the ASE and Parameters Identification Methods....	75
5.1. Sea-Trial Data.....	75
5.2. Estimated Hydrodynamic Coefficients.....	78
5.3. Identified Dynamic Models Results.....	87
5.4. Discussions of the Results.....	96
6. Conclusions and Future Works.....	98
7. Bibliography References.....	100
8. Appendices.....	103

1. Introduction

1.1. Justification

Autonomous vehicles, whether land, air or water; are finding more and more applications in the military as well as civil areas. One of the advantages of this kind of vehicle is that it can be sent for inspection or intervention missions in dangerous zones or difficult access places for humans. That is especially important in the case of autonomous underwater vehicles (AUV) that can carry out missions such as: searching for aircraft or vessels sunk in the sea, supporting on off-shore oil and gas extraction, oceanographic mapping, etc. A conventional AUV has a torpedo shape (see Figure 1), a principal propeller for thrust, a dorsal sail, four cruciform fins and standard navigation sensors, such as an inertial measurement unit (IMU), a Doppler velocimeter (DVL), a depth sensor and a magnetic heading sensor.

Figure 1.1.1- The Pirajuba AUV; this image was provided by the Unmanned Vehicles Laboratory of the São Paulo University.

A mathematical model of the AUV dynamics is necessary in order to predict the vehicle motion characteristics using computational simulations that can be applied to autopilot design, test based on hardware in the loop, navigation systems aided by model, etc.

If it is desirable to reproduce various kinds of maneuvers, such as straight line, turning in circles, zigzag, lawn mowing; a nonlinear mathematical model is required. In these typical maneuvers the motion equations can be decoupled into vertical and horizontal planes. Therefore, in this work the dynamics of an autonomous underwater vehicle (AUV) is modeled as a decoupled-nonlinear system which considers that the vehicle is maneuvering in the horizontal plane or in the vertical plane and they will be independently analyzed.

1.2. State of the Art

Mathematical models of underwater vehicles have been rigorously studied for military applications, since the past century to nowadays. In the literature it is possible to observe theoretical and experimental studies in unclassified technical reports, such as: Imlay (1961); Gertler and Hagen (1967); Humphreys (1976); Humphreys and Watkinson (1978); Feldman (1979); Mackay (2003). Using these studies as a background, some nonlinear mathematical models were derived to the case of AUVs, such as: Sen (2000) and Prestero(2001). Once a mathematical model is defined, its hydrodynamic coefficients can be estimated using different types of methods, for instance: analytic and semi-empirical methods (ASE), computational methods (CFD) or experimental method using captive or free model tests.

The model parameter identification method can be classified as an experimental method type due to the use of free model trials data. The advantage of this kind of approach is that even the effects of the vehicle's geometric imperfections and appendices like antennas, acoustic transducers or other sensors that affect the hull geometry can be taken into account. These effects are difficult to be included in ASE methods. Flow numerical simulation approaches, such as the CFD methods can be used, but are computationally and time intensive, particularly for dynamic efforts. The application of system identification methods requires an equipped AUV prototype that can execute basic maneuvers in sea-trials and acquire experimental data. They do not require expensive towing tank facilities, such as the

case of captive model tests in towing tanks. Moreover, unlike captive tests or CFD methods, system identification methods can handle easily the estimation of a new model when equipment such as a camera, sonar or another sensor are included.

Linear filters such as *ARX*, *ARMAX*, *ARIMAX* and *Box-Jenkins* are widely used to identify AUV single input and output models (SISO) as rudder-yaw Nomoto models. However, these linear models cannot reproduce the vehicle's intrinsic nonlinear dynamic or variables coupling that are present in the different types of maneuvers (DANTAS et al., 2013). State space identification methods can cope with multiple input-output (MIMO) models, and provide an explicit estimation of hydrodynamic coefficients that have explicit physical meaning, and can have an important role in the dynamical analysis and design of an underwater vehicle. Moreover, they can be applied to both linear and non-linear models.

Linear models may satisfactorily represent the AUV dynamics in a specific type of maneuver and cruising speed. For these models, classical linear estimation methods have been applied to identify the model parameters, such as the *Least-Squares*; (SANDMAN; KELLY, 1974); (ETTORE A. DE BARROS; S. MIYAJIMA; H. MAEDA, 1992) and (AVILA; PABLO, 2008); the *Nelder-Mead Simplex method* (RENTSCHLER; HOVER; CHRYSSOSTOMIDIS, 2006); and the conventional *Kalman Filter* (TIANO et al., 2007) and (SHAHINFAR; BOZORG; BIDOKY, 2010). Non-linear models may accurately reproduce the AUV maneuvering in a wider range of operation states (cruising speed, rudder angle, and sideslip angle). For those models, the *Extended Kalman Filter* (EKF) is the most commonly used for nonlinear systems. Applications of EKF in the identification of AUV hydrodynamic coefficients can be seen in Kim et al. (2002), Luque, Donha and Barros (2009) and Sabet et al. (2014). Other nonlinear methods applied to AUVs can be found in the literature. In Kim et al. (2002) a *Sliding Mode Observer* was used for estimating the coefficients of an AUV, but showed a lower accuracy than the EKF. In Xu et al. (2013) and Hegrenaes, Hallingstad and

Jalving (2007), a change of variables was performed in order to apply the *Least-Squares method* on the identification of a nonlinear maneuvering model, which showed a good agreement with the measurements. The *Maximum Likelihood Algorithm* (MLA) was applied to AUVs in Liang et al. (2010) and Luque and Donha (2011). In the last work, the MLA is compared to the EKF, showing that both could accurately estimate the model parameters, although the MLA estimator exhibited a faster convergence than EKF. In Sabet et al. (2014) and Sabet et al. (2018) the *Unscented Kalman Filter* (UKF) was compared with the EKF, and it was found that the UKF provided more accurate results than the EKF, which showed biased estimations. Other identification algorithms applied to AUV models are *Multi-variable regression* (KIM; CHOI, 2007), Fuzzy Modeling (HASSANEIN; ANAVATTI; RAY, 2011) and algorithms of optimization using an evolution strategy (KARRAS et al., 2013) and (TAUBERT et al., 2014).

One of the common problems that occur when applying system identification techniques to the estimation of a rich set of hydrodynamic coefficients is the over parameterization problem (ROBERTS; SUTTON, 2006) and (ABKOWITZ, 1980). In this case, the estimates of two or more parameters may diverge, while the respective efforts cancel each other. In order to counteract the over-parameterization problem, Abkowitz proposed two identification techniques “parallel processing” and “under and over estimations”. In the first one, the experimental sensor data of two or more maneuvers are simultaneously processed, setting the model parameters as common in the state vector to all the maneuvers processed. In the second identification technique (under and over estimations), the mutual cancelation of two parameters is avoided by the manipulation of their initial values. One of the parameters is initiated with an overestimated value, while the other is initialized with an underestimated value. Therefore, when one estimated parameter increases the other decreases until both of them converge to a single value. This technique is effective to the estimation of two

parameters at a time; for that reason, its application is usually restricted to the identification of lift and drag coefficients. In both approaches, there is no concern about the initialization of the estimates and the convergence performance of the method.

Another current practice in the modeling and identification of underwater vehicles is to rely on analytical methods that assume the ideal flow hypothesis for estimating the added mass coefficients (KIM et al., 2002); (XU et al., 2013), (SABET; POURIA SARHADI; ZARINI, 2014); (SABET et al., 2018). The propeller effects are also approximated by results from towing tank tests where only the steady-state values of thrust and moment are measured. The analytical estimation of added mass coefficients is usually adopted to simplify the EKF estimation phase, letting the identification be focused on the damping coefficients only. However, it is important to emphasize the inaccuracies of the analytical approach to the added mass estimations (D. E. HUMPHREYS; K. W. WATKINSON, 1978). The bare hull shape is commonly approximated by ellipsoid geometries; whose added mass terms are well known in the literature. The combination with hydroplanes may be taken into account by the empirical formula. Alternatively, another approach is to consider the bare hull and the hull fin combination by the strip method, where an added mass per unit of length is assumed by the bi-dimensional flow hypothesis (J. N. NEWMAN, 1977). This value, well known in the literature for axis-symmetric shapes that may be combined to fins, is then used in the integration along the vehicle length to produce the added mass coefficient. Like any other analytical approach, all those approximations neglect the presence of appendices commonly found in typical AUVs (transponders, antennas, cameras, etc.) and geometric differences to the pure ellipsoidal or axis-symmetric form. Therefore, viscous effects are probably present in the real AUV, that are not taken into account by the ideal flow hypothesis. Moreover, in the case of the strip method approach, the method carries the inaccuracy due to the finite length of fins and hull.

In this study, the relevant added mass and propeller coefficients are identified as well as the lift and drag related coefficients. The “parallel processing” is adopted, while the “over and under” identification technique is replaced by the preliminary analytical estimations of all parameters, using physical and geometry knowledge (fluid mechanics, slender body theory, lift and drag semi-empirical expressions). These estimates are then finely adjusted to experimental data by the EKF or UKF estimator. A physical insight of accuracy of the analytical methods allows a spiral identification process, holding invariant the most reliable model parameters in an estimation cycle and fitting the less reliable model parameters. In the next identification cycle, the non-varied model parameters are tuned. Finally, all the coefficients are estimated simultaneously in the system identification phase.

The ASE estimates are obtained by using physical concepts and empirical results collected over the years for typical geometries adopted in underwater vehicles. Therefore, they provide important reference values to assess magnitudes and signals for most of the hydrodynamic coefficients produced by system identification methods. It is also claimed, in this work, that using the ASE approach for initialization, one can help the convergence of the EKF and UKF estimation.

1.3. Objectives

This thesis intends to address the following challenges related to modeling and identification of an AUV dynamics

- To define a nonlinear mathematical model, for the AUV dynamics, able to reproduce a diversity of maneuvers types on the horizontal plane.
- To develop an analytic method to approximate the hydrodynamic coefficients of the defined AUV dynamic model.

- To present a new method to identify the complete set of the hydrodynamic coefficients, including the added-mass parameters, of the AUV dynamic model using the measurements of motion variables during sea trials experiments.
- Validate the ASE and the identification method for the following maneuvers: a straight line with a stepped thrust, turning in circles and zigzags maneuvers.
- To comparatively analyze the performance of nonlinear Kalman estimators (EKF and UKF) applied to the identification of AUV dynamics.

1.4. Manuscript Structure

This work is presented as follows. Section 2 presents the equations of motion adopted for the AUV maneuvering in the horizontal plane as well as the vertical plane, the measurement model, and the representation of hydrodynamic efforts in the parametric form. These parameters are estimated by the analytical and semi-empirical formulas presented in section 3. The new parameters identification method based on nonlinear Kalman estimators is presented in section 4. Section 5 presents results and discussions of parameter identification based on the sea trials with the “Pirajuba” AUV. Finally, section 6 presents the final comments and conclusions about the proposed method.

2. Mathematical Modeling of the AUV Motion

In this section, an AUV is modeled as a rigid body and the equations of motion are derived for the horizontal-plane as well as for the vertical-plane. These motion equations are expressed in a vector form, initially considering six degrees of freedom (DOF) of motion, then these equations are decomposed on the $\hat{x}$, $\hat{y}$ and $\hat{z}$ axis of the vehicle frame (figure 2.1.1), assuming the SNAME (1950) nomenclature such as seen in table 2.1. Finally, the equations are decoupled into motion in the horizontal plane and vertical plane, and some simplifications for each motion plane are carried out.

Table 2.1 – SNAME nomenclature

Axis	$\hat{x}$	$\hat{y}$	$\hat{z}$
Positions	x	y	z
Linear velocities	u	v	w
Angular positions	ϕ	θ	ψ
Angular velocities	p	q	r
Forces	X	Y	Z
Moments	K	M	N

In Figure 2.1.1 it is represented the vehicle reference frame centered at "O". The DVL transducers are located in the vehicle at "D", the IMU sensor is at "U", the center of pressure of the Sail is at "S", and the center of pressure of the rudders at "R".

2.1. Kinematics of an Underwater Vehicle

In a rigid body, the velocity of any point of the body relative to the center of the vehicle frame is calculated as the vector product of the angular velocity of the vehicle and the relative position. Let it be a point "A" fixed on the vehicle. Hence, the absolute velocity of the point "A" is obtained as the sum of the velocity on the point "O" and the velocity of the point "A" relative to the center of the vehicle frame. Such as:

$$\mathbf{v}_A = \mathbf{v}_O + \boldsymbol{\omega}_O \times \mathbf{p}_{A/O} \quad (2.1.1)$$

Decomposing each term of the vector equation (2.1.1) on the $\hat{x}$, $\hat{y}$ and $\hat{z}$ axis of the vehicle frame, the expressions (2.1.2 – 2.1.4) are obtained:

Figure 2.1.1: The vehicle's reference frame, its sensors and fins location.
Source: Unmanned Vehicles Laboratory of the University of São Paulo.

$$u_A = u + z_A \cdot q - y_A \cdot r \quad (2.1.2)$$

$$v_A = v + x_A \cdot r - z_A \cdot p \quad (2.1.3)$$

$$w_A = w + y_A \cdot p - x_A \cdot q \quad (2.1.4)$$

The absolute acceleration of any point fixed on the vehicle can be obtained by taking the time derivative of the expression (2.1.1):

$$\mathbf{a}_A = \dot{\mathbf{v}}_A = \dot{\mathbf{v}}_O + \dot{\boldsymbol{\omega}}_O \times \mathbf{p}_{A/O} + \boldsymbol{\omega}_O \times \dot{\mathbf{p}}_{A/O} \quad (2.1.5)$$

The term $\dot{\mathbf{p}}_{A/O}$ is the velocity of the point “A” relative to the center of the vehicle frame “O”, such as expressed in the equation (2.1.1). Hence, the absolute acceleration of a point “A”, fixed on the vehicle, can be expressed as:

$$\mathbf{a}_A = \dot{\mathbf{v}}_O + \dot{\boldsymbol{\omega}}_O \times \mathbf{p}_{A/O} + \boldsymbol{\omega}_O \times (\boldsymbol{\omega}_O \times \mathbf{p}_{A/O}) \quad (2.1.6)$$

The absolute acceleration of the center of the vehicle frame can be expressed relative to the vehicle reference frame as:

$$\mathbf{a}_O = \frac{d\mathbf{v}_O}{dt} \Big|_I = \underbrace{\frac{d\mathbf{v}_O}{dt} \Big|_O}_{\dot{u}\hat{x} + \dot{v}\hat{y} + \dot{w}\hat{z}} + \boldsymbol{\omega}_O \times \mathbf{v}_O \quad (2.1.7)$$

Replacing the expression (2.1.7) in the expression (2.1.6) and decomposing the last one in the vehicle coordinate frame, the absolute acceleration of a point “A” fixed on the vehicle can be expressed as a function of the linear and angular velocities of the vehicle as:

$$a_{x_A} = \dot{u} + wq - vr - x_A(q^2 + r^2) + y_A(pq - \dot{r}) + z_A(pr + \dot{q}) \quad (2.1.8)$$

$$a_{y_A} = \dot{v} + ur - wp + x_A(pq + \dot{r}) - y_A(p^2 + r^2) + z_A(qr - \dot{p}) \quad (2.1.9)$$

$$a_{z_A} = \dot{w} + vp - uq + x_A(pr - \dot{q}) + y_A(qr + \dot{p}) - z_A(p^2 + q^2) \quad (2.1.10)$$

2.2. Dynamics of an Underwater Vehicle

In this work, the equations of motion for an AUV were derived considering the following assumptions:

- The vehicle is a rigid body which is moving in an infinity fluid.
- The environmental disturbances can be disregarded.
- The Earth is considered as an inertial reference frame. That is, the Earth movement can be disregarded.
- In the vehicle's trimming process, the buoyancy and gravitational forces are compensated for each other, such as the vehicle is neutral in the water. Moreover, the vehicle's center of gravity is aligned underneath the buoyancy center. Therefore, the AUV is passively stable to inclinations around the $\hat{x}$ and $\hat{y}$ axis.
- The vehicles carries out 2D maneuvers. Those are maneuvers in the horizontal plane or in the vertical plane.

- Any inclination of the vehicle around the roll axis ($\hat{\mathbf{x}}$) is disregarded.

The relationship between the motion variables of the vehicle and the external forces and moments can be expressed with the first and second Newton-Euler's equations for rigid bodies.

A. Motion Equations in the 6 DOF

The translational equation to an underwater vehicle can be expressed as:

$$m \cdot \mathbf{a}_G = \sum \mathbf{F}_{ext} = \mathbf{F}_{HD} + \mathbf{F}_{GB} + \mathbf{F}_{TH} \quad (2.2.1)$$

Expressing the absolute acceleration of the center of gravity as a relative acceleration (2.1.6) is obtained:

$$m \cdot (\mathbf{a}_O + \dot{\boldsymbol{\omega}}_O \times \mathbf{p}_{G/O} + \boldsymbol{\omega}_O \times (\boldsymbol{\omega}_O \times \mathbf{p}_{G/O})) = \sum \mathbf{F}^{ext} \quad (2.2.2)$$

Decomposing in the vehicle coordinates frame, as the expressions (2.1.8-2.1.10), the translational equations of an underwater vehicle can be expressed as:

$$m \cdot (\dot{u} + wq - vr - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pr + \dot{q})) = X \quad (2.2.3)$$

$$m \cdot (\dot{v} + ur - wp + x_G(pq + \dot{r}) - y_G(p^2 + r^2) + z_G(qr - \dot{p})) = Y \quad (2.2.4)$$

$$m \cdot (\dot{w} + vp - uq + x_G(pr - \dot{q}) + y_G(qr + \dot{p}) - z_G(p^2 + q^2)) = Z \quad (2.2.5)$$

The second Euler's equation in terms of the moment of inertia tensor of the vehicle relative to the vehicle reference frame ($\mathbf{I}_O$), which is fixed to the vehicle, can be expressed as the expression (2.2.6); (RAO, 2006).

$$\sum \mathbf{M}_O^{ext} - \mathbf{p}_{G/O} \times m \cdot \mathbf{a}_O = \mathbf{I}_O \cdot \dot{\boldsymbol{\omega}}_O + \boldsymbol{\omega}_O \times (\mathbf{I}_O \cdot \boldsymbol{\omega}_O) \quad (2.2.6)$$

In which, $\sum \mathbf{M}_O$ is the resultant moment, relative to the point “O”, of the external forces acting on the vehicle;

The moment of inertia tensor can be expressed in the vehicle coordinate system as an inertia matrix ($\mathbf{I}_O^o$), such as:

$$\mathbf{I}_O^o = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} \quad (2.2.7)$$

Decomposing the terms of the vector equation (2.2.6), on the vehicle coordinate system, the rotational motion equations for an underwater vehicle can be expressed as:

$$I_{xx}\dot{p} + (I_{zz} - I_{yy})qr - (\dot{r} + pq)I_{zx} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} + m(y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)) = K \quad (2.2.8)$$

$$I_{yy}\dot{q} + (I_{xx} - I_{zz})rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} + (qp - \dot{r})I_{yz} + m(z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)) = M \quad (2.2.9)$$

$$I_{zz}\dot{r} + (I_{yy} - I_{xx})pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx} + m(x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)) = N \quad (2.2.10)$$

B. AUV Equations of Motion in the Horizontal Plane

When the vehicle is moving in the horizontal plane, simplifications can be assumed:

$$(w \approx p \approx q \approx \dot{w} \approx \dot{p} \approx \dot{q} \approx 0) \quad (2.2.11)$$

Considering these approximations in the expressions (2.2.3-2.2.4 and 2.2.10) the AUV equations of motion in the horizontal plane are reduced to:

$$m \cdot (\dot{u} - vr - x_G r^2 - y_G \dot{r}) = X^{hd} + X^{th} \quad (2.2.12)$$

$$m \cdot (\dot{v} + ur + x_G \dot{r} - y_G r^2) = Y^{hd} \quad (2.2.13)$$

$$I_{zz}\dot{r} + mx_G(\dot{v} + ur) - my_G(\dot{u} - vr) = N^{hd} \quad (2.2.14)$$

As the position of the center of gravity is assumed aligned to the position of the buoyancy center, the transversal position will be disregarded ($y_G \simeq 0$). Such that, in terms of the hydrodynamic coefficients identification, the vehicle motion equations can be decoupled in a surge system and a yaw-sway system. Therefore, the surge movement equation is:

$$m \cdot (\dot{u} - vr - x_G r^2) = X^{hd} + X^{th} \quad (2.2.15)$$

And, the Yaw-Sway movement equations are:

$$m \cdot (\dot{v} + ur + x_G \dot{r}) = Y^{hd} \quad (2.2.16)$$

$$I_{zz} \dot{r} + mx_G (\dot{v} + ur) = N^{hd} \quad (2.2.17)$$

Considering an underwater vehicle as a slender body which is symmetric, with relation to the plane $(\hat{x}-\hat{z})$, the hydrodynamic forces, and their moments, over the vehicle can be approximated as the second-order Taylor series terms of the linear and angular velocities. As seen in Sen (2000), Feldman (1979) and Imlay (1961) these hydrodynamic forces and moments are the sums of the added mass, lift and drag effects, acting over the vehicle's hull and fins, and the propeller thrust. In the case of horizontal-plane motion, the resulting forces and moments that act upon the vehicle can be expressed as:

$$X = X_{\dot{u}} \dot{u} + X_{uu} u^2 + X_{vv} v^2 + X_{vr} vr + X_{rr} r^2 + X_{u^2 \delta_R \delta_R} u^2 \delta_R^2 + X_{u^2 \delta_E \delta_E} u^2 \delta_E^2 + X_{u\eta} u\eta + X_{\eta\eta} \eta^2 \quad (2.2.18)$$

$$Y = Y_{\dot{v}} \dot{v} + Y_{\dot{r}} \dot{r} + Y_{uv} uv + Y_{ur} ur + Y_{|v|v} |v|v + Y_{u^2 \delta_R} u^2 \delta_R \quad (2.2.19)$$

$$N = N_{\dot{v}} \dot{v} + N_{\dot{r}} \dot{r} + N_{uv} uv + N_{ur} ur + N_{|v|v} |v|v + N_{u^2 \delta_R} u^2 \delta_R \quad (2.2.20)$$

Substituting the expressions (2.2.18-2.2.20) into the equations of motion in the horizontal plane (2.2.15 - 2.2.17), moving the added mass terms to the left side of the equations, and conveniently rearranging the system of differential equations, the vehicle dynamic model of the horizontal maneuvers is expressed as a matrix equation:

$$\begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_G - Y_{\dot{r}} \\ 0 & mx_G - N_{\dot{v}} & I_z - N_{\dot{r}} \end{bmatrix} \cdot \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} f_X \\ f_Y \\ f_N \end{bmatrix}$$

$$\dot{u} = \frac{1}{(m - X_{\dot{u}})} \cdot f_X \quad (2.2.21)$$

$$\dot{v} = \frac{(I_{zz} - N_{\dot{r}})}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)} f_Y + \frac{(Y_{\dot{r}} - mx_G)}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)} f_N \quad (2.2.22)$$

$$\dot{r} = \frac{(N_{\dot{v}} - mx_G)}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)} f_Y + \frac{(m - Y_{\dot{v}})}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)} f_N \quad (2.2.23)$$

In which, f_X , f_Y and f_N are functions of the vehicle velocities and control inputs, expressed as:

$$f_X = X_{uu}u^2 + X_{vv}v^2 + (m + X_{vr})vr + (mx_G + X_{rr})r^2 + X_{u^2\delta\delta}u^2(\delta_R^2 + \delta_E^2) + X_{u\eta}u\eta + X_{\eta\eta}\eta^2 \quad (2.2.24)$$

$$f_Y = Y_{uv}uv + (Y_{ur} - m)ur + Y_{|v|v}|v|v + Y_{u^2\delta_R}u^2\delta_R \quad (2.2.25)$$

$$f_N = N_{uv}uv + (N_{ur} - mx_G)ur + N_{|v|v}|v|v + N_{u^2\delta_R}u^2\delta_R \quad (2.2.26)$$

It is assumed that $(X_{u^2\delta\delta} = X_{u^2\delta_R\delta_R} = X_{u^2\delta_E\delta_E})$, since the AUV has a cruciform configuration for the fins and the vertical and horizontal hydroplanes are the same.

Expressing in matrix form:

C. AUV Equations of Motion in the Vertical Plane

When the vehicle is moving in the vertical plane, approximations are assumed as follows:

$$(v \approx p \approx r \approx \dot{v} \approx \dot{p} \approx \dot{r} \approx 0) \quad (2.2.27)$$

Analogously to the horizontal plane, the AUV is passively stable to inclinations on the roll axis and only rotations around the “ $\hat{y}$ ” axis are relevant to the motion in the vertical plane.

Therefore, replacing these approximations in the expressions (2.2.3, 2.2.5 and 2.2.9) the AUV equations of motion to the vertical plane are reduced to:

$$m \cdot (\dot{u} + wq - x_G q^2 + z_G \dot{q}) = X^{hd} + X^{th} \quad (2.2.28)$$

$$m \cdot (\dot{w} - uq - x_G \dot{q} - z_G q^2) = Z^{hd} \quad (2.2.29)$$

$$I_{yy}\dot{q} + mz_G(\dot{u} + wq) - mx_G(\dot{w} - uq) = M^{hd} + M^{gb} \quad (2.2.30)$$

Analogously to the horizontal-plane motion, the resulting forces and moments that act upon the vehicle moving on the vertical plane can be expressed as:

$$X = X_{\dot{u}}\dot{u} + X_{uu}u^2 + X_{ww}w^2 + X_{wq}wq + X_{qq}q^2 + X_{u^2\delta_R\delta_R}u^2\delta_R^2 + X_{u^2\delta_E\delta_E}u^2\delta_E^2 + X_{u\eta}u\eta + X_{\eta\eta}\eta^2 \quad (2.2.31)$$

$$Z = Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uw}uw + Z_{uq}uq + Z_{|w|w}|w|w + Z_{u^2\delta_E}u^2\delta_E \quad (2.2.32)$$

$$M = M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uw}uw + M_{uq}uq + M_{|w|w}|w|w + M_{u^2\delta_E}u^2\delta_E - mz_G g \sin \theta \quad (2.2.33)$$

Substituting the expressions (2.2.31-2.2.33) into the motion equations to the vertical plane (2.2.28 - 2.2.30), moving the added mass terms to the left side of the equations, and

conveniently rearranging to a differential equations system, the vehicle dynamic model for vertical maneuvers is expressed in a matrix form as:

$$\begin{bmatrix} m - X_{\dot{u}} & 0 & mz_G \\ 0 & m - Z_{\dot{w}} & -(mx_G + Z_{\dot{q}}) \\ mz_G & -(mx_G + M_{\dot{w}}) & (I_{yy} - M_{\dot{q}}) \end{bmatrix} \cdot \begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} f_X \\ f_Z \\ f_M \end{bmatrix} \quad (2.2.34)$$

In which, f_X , f_Z and f_M are functions of the vehicle velocities and control inputs, expressed as:

$$f_X = X_{uu}u^2 + X_{ww}w^2 + (X_{wq} - m)wq + (X_{qq} + mx_G)q^2 + X_{u^2\delta\delta}u^2(\delta_R^2 + \delta_E^2) + X_{u\eta}u\eta + X_{\eta\eta}\eta^2 \quad (2.2.35)$$

$$f_Z = Z_{uw}uw + (m + Z_{uq})uq + Z_{|w|w}|w|w + mz_Gq^2 + Z_{u^2\delta_E}u^2\delta_E \quad (2.2.36)$$

$$f_M = M_{uw}uw + (M_{uq} - mx_G)uq - mz_Gwq + M_{|w|w}|w|w + M_{u^2\delta_E}u^2\delta_E - mz_Gg \sin \theta \quad (2.2.37)$$

Analogously to the horizontal-plane case, it is assumed that $(X_{u^2\delta\delta} = X_{u^2\delta_R\delta_R} = X_{u^2\delta_E\delta_E})$.

2.3 The Measurement Model for an AUV

A typical AUV is equipped with several navigation sensors, including a strapdown inertial measurement unit (IMU), composed by tri-axial accelerometers and tri-axial gyroscopes, and auxiliary sensors; such as a Doppler velocimeter (DVL) and heading magnetic sensors. However, only DVL and Gyroscopes measurements were used on the identification of the hydrodynamic coefficients:

Gyroscopes measure the angular velocity components of the vehicle, relative to the inertial reference frame and considering the noise of the sensor ($\mathbf{n}^{gyro}$), the gyroscope measurements vector ($\boldsymbol{\omega}^{gyro}$) can be expressed as:

$$\boldsymbol{\omega}^{gyro} = \boldsymbol{\omega}^0 + \mathbf{n}^{gyro} \quad (2.3.1)$$

$$p^{gyro} = p + n_x^{gyro} \quad (2.3.2)$$

$$q^{gyro} = q + n_y^{gyro} \quad (2.3.3)$$

$$r^{gyro} = r + n_z^{gyro} \quad (2.3.4)$$

The Doppler Velocity Logger (DVL) measures the velocity's components of the point “D” (location in the vehicle where the DVL transducers are fixed) relative to the Earth's reference frame and considering the noise of the sensor ($\mathbf{n}^{dvl}$), the DVL measurements vector ($\mathbf{v}^{dvl}$) can be expressed as:

$$\mathbf{v}^{dvl} = \mathbf{v}_D^o + \mathbf{n}^{dvl} \quad (2.3.5)$$

Expressing the velocity of the point “D” relative to the vehicle frame center, such as seen in the expressions (2.1.2):

$$\mathbf{v}^{dvl} = \mathbf{v}_o^o + \boldsymbol{\omega}^o \times \mathbf{p}_{D/o}^o + \mathbf{n}^{dvl} \quad (2.3.6)$$

Decompounding in the vehicles frame are obtained the expressions (2.3.7 – 2.3.9):

$$u^{dvl} = u + z_D q - y_D r + n_x^{dvl} \quad (2.3.7)$$

$$v^{dvl} = v + x_D r - z_D p + n_y^{dvl} \quad (2.3.8)$$

$$w^{dvl} = w + y_D p - x_D q + n_z^{dvl} \quad (2.3.9)$$

Considering the approximations to motion in the horizontal plane ($z_D q \approx z_D p \approx y_D p \approx x_D q \approx 0$) the DVL measures model to the horizontal plane can be expressed as:

$$u^{dvl} = u - y_D r + n_x^{dvl} \quad (2.3.10)$$

$$v^{dvl} = v + x_D r + n_y^{dvl} \quad (2.3.11)$$

Analogously, considering the approximations to motion in the vertical plane ($y_D r \approx x_D r \approx z_D p \approx y_D p \approx 0$) the DVL measures model to the vertical plane can be expressed as:

$$u^{dvl} = u + z_D q + n_x^{dvl} \quad (2.3.12)$$

$$w^{dvl} = w - x_D q + n_z^{dvl} \quad (2.3.13)$$

3. ASE Estimation of the Hydrodynamic Coefficients

Analytical and semi-empirical methods to estimate the hydrodynamic coefficients of underwater and surface vessels are widely studied. Some hydrodynamic coefficients, such as the added-mass coefficients, can be reasonably estimated using fluid mechanics theory. However, for most of the hydrodynamic coefficients, a combination of analytical and semi-empirical expressions should be used to obtain a useful dynamic model. The semi-empirical expressions are generally obtained from experimental tests using captive-models. Some of them were borrowed from subsonic aeronautics and applied to underwater vehicles.

In this section, analysis and modeling of the hydrodynamic forces and moments that act on underwater vehicles are approached. Hydrodynamic coefficient expressions are deduced to the AUV motion in the horizontal plan as well as in the vertical plane. In both cases, it is considered that the vehicle is passively stabilized and there are not rotations or inclination around the vehicle's longitudinal axis.

Based on the SNAME notation, the hydrodynamic forces are expressed in a dimensionally form. That is, they are a function of the fluid density, the square of the flow velocity, an area of a reference surface, an angle of the attack of the body and a dimensionless and constant coefficient. The last one contains the geometry and flow characteristics of the vehicle.

3.1. Added-Mass Coefficients

A study on added-mass coefficients can be found in Imlay (1961) and Korotkin (2009) wherein a potential analysis of an underwater vehicle in an ideal fluid (frictionless fluid) is made. The surge-added-mass coefficient (X_u) is calculated by approximating the hull vehicle as an ellipsoid with an equal volume. Hence, the hull-vehicle volume is equal to the volume of the equivalent ellipsoid:

$$V^{hull} = \frac{4}{3}\pi ab^2 \quad (3.1.1)$$

The axial semi-edge of the prolate spheroid will be considered as half of the total length of the vehicle ($a = \frac{L}{2}$), and the radial semi-edge can be calculated from the equation (3.1.1).

Such as seen in Imlay (1961), the added-mass coefficient on surge can be calculated as:

$$X_{\dot{u}} = -\frac{\alpha_0}{2 - \alpha_0} \cdot \rho V^{hull} \quad (3.1.2)$$

In which:

$$\alpha_0 = \frac{2(1 - e^2)}{e^3} \left(\frac{1}{2} \ln \left(\frac{1 + e}{1 - e} \right) - e \right) \quad (3.1.3)$$

In the case of lateral added mass coefficients, a simple estimation approach is applied to the bare hull and to the combination of hull and fins (rudders and sail). In Barros et al. (2008), expressions to calculate the added-mass coefficients per unit length are presented for the hull-fins assembly. The added-mass coefficients can be calculated integrating these longitudinal coefficients along of the axial edge, for a motion on the horizontal plane, such as:

$$Y_{\dot{v}} = \int_L -c_{(x)}^{ad} \cdot dx \quad ; \quad Y_{\dot{r}} = N_{\dot{v}} = \int_L -c_{(x)}^{ad} \cdot x dx \quad ; \quad N_{\dot{r}} = \int_L -c_{(x)}^{ad} \cdot x^2 dx \quad (3.1.4)$$

In which x is the longitudinal position of a hull transversal station with relation to the vehicle coordinate frame and $c_{(x)}^{ad}$ is the added-mass coefficient per unit length which depends on the longitudinal position “ x ”. If the vehicle hull is sectioned along the $\hat{x}$ axis, the added-mass coefficient per unit length is expressed as:

$$c_{(x)}^{ad} = \begin{cases} \pi \rho r_{(x)}^2 & , \forall r_{(x)} \in \mathcal{R}_1 \\ \pi \rho s_{rudd}^2 \cdot \left(1 - \frac{r_{(x)}^2}{s_{rudd}^2} + \frac{r_{(x)}^4}{s_{rudd}^2} \right) & , \forall r_{(x)} \in \mathcal{R}_2 \\ \frac{\pi \rho s_{sail}^2}{4} \cdot \left(1 + \frac{r_{(x)}^2}{s_{sail}^2} \right) \cdot \left(-1 - \frac{r_{(x)}^2}{s_{sail}^2} + 2^{3/2} \cdot \sqrt{1 + \frac{r_{(x)}^4}{s_{sail}^4}} \right) & , \forall r_{(x)} \in \mathcal{R}_3 \end{cases} \quad (3.1.5)$$

Where:

$r_{(x)}$ is the radio of the hull cross-section as a function of the longitudinal position “ x ”.

$\mathcal{R}_1$ are the bare hull sections;

$\mathcal{R}_2$ is the hull section with rudders;

$\mathcal{R}_3$ is the hull section with sail;

ρ is the density of the surrounding fluid;

s_{rudd} is the rudder semi-span calculated as the sum of the vehicle radius and the exposed semi-span of the rudders.

and s_{sail} is the sail semi-span calculated as the sum of the bare hull radius and the exposed semi-span of the sail. In the vertical plane similar conditions are derived:

$$Z_{\dot{w}} = \int_L -c_{(x)}^{ad} \cdot dx \quad ; \quad Z_{\dot{q}} = M_{\dot{w}} = \int_L c_{(x)}^{ad} \cdot x dx \quad ; \quad M_{\dot{q}} = \int_L -c_{(x)}^{ad} \cdot x^2 dx \quad (3.1.6)$$

Only a section with elevators is considered in these integrals, since the sail effects are disregarded on the vertical plane motion.

From (H. IMLAY, 1961b), in a prolate spheroid with a dorsal fin near the maximum diameter of the body, lying in the xz-plane, and with four cruciform fins located at the rear of the body; the following equalities are fulfilled for the motion in the horizontal plane:

$$X_{vrr} = -Y_{\dot{v}} \quad ; \quad X_{rrr} = -Y_{\dot{r}} \quad ; \quad Y_{urr} = X_{\dot{u}} \quad ; \quad Y_{\dot{r}} = N_{\dot{v}} \quad ; \quad N_{uv} = Y_{\dot{v}} - X_{\dot{u}} \quad ; \quad N_{ur} = Y_{\dot{r}} \quad (3.1.7)$$

Similar equalities follow for the vertical plane:

$$X_{wq} = Z_{\dot{w}} \quad ; \quad X_{qq} = Z_{\dot{q}} \quad ; \quad Z_{uq} = -X_{\dot{u}} \quad ; \quad Z_{\dot{q}} = M_{\dot{w}} \quad ; \quad M_{uw} = X_{\dot{u}} - Z_{\dot{w}} \quad ; \quad M_{uq} = -Z_{\dot{q}} \quad (3.1.8)$$

3.2. Hydrodynamic Coefficients Related to the Bare Hull

There are external forces and their moments applied on the vehicle that are produced by the combination of viscous and potential effects. Semi-empirical expressions are commonly used to estimate these efforts.

A. External Forces and Moments Due to Viscous and Potential Effects

Allen and Perkins (1952) derived analytical expressions for the cross forces produced by potential and viscous effects acting on missiles at incidence, i.e. when the flow is at an angle of attack to the vehicle.

The potential term, produced by the bare hull lift, is calculated from the expression of the normal force on a slender body derived by Munk (1924). This force is given by integration of the cross force per unit length $\left(\frac{dF_{Munk}^{hull}}{dx}\right)$, along the longitudinal axis of the vehicle. The cross force and moment per unit length are expressed, respectively, as:

$$\frac{dF_{Munk}^{hull}}{dx} = -(k_2 - k_1) \cdot \frac{1}{2} \rho U_o^2 \cdot \frac{dA_{cs}^{hull}}{dx} \cdot \sin(2\alpha) \quad (3.2.1)$$

$$\frac{dM_{Munk}^{hull}}{dx} = -x \cdot (k_2 - k_1) \cdot \frac{1}{2} \rho U_o^2 \cdot \frac{dA_{cs}^{hull}}{dx} \cdot \sin(2\alpha) \quad (3.2.2)$$

Where:

A_{cs}^{hull} is the area of the hull's cross section which is a function of x .

U_o is the velocity of the vehicle relative to the fluid.

α is the angle of attack, which is defined between the vehicle velocity vector relative to the fluid ($\mathbf{U}_o$) and the longitudinal axis of the vehicle;

$(k_2 - k_1)$ is the Munk apparent-mass factor, used to compensate the finite length of the body, and can be approximated as the difference between the transversal and the longitudinal added mass coefficients, of an equivalent ellipsoid, and divided by the total mass. It is calculated as:

$$k_2 - k_1 = \frac{(X_u^{ellip} - Y_v^{ellip})}{\rho V_{ellip}} \quad (3.2.3)$$

Integrating the equations (3.2.1-3.2.2) along the axial edge of the vehicle, see Appendix A., the potential force and the moment are expressed as:

$$F_{Munk}^{hull} = -\frac{1}{2}\rho U_o^2(k_2 - k_1)A_{cs}^{base} \sin(2\alpha) \quad (3.2.4)$$

$$M_{Munk}^{hull} = \frac{1}{2}\rho U_o^2(k_2 - k_1)V^{hull} \sin(2\alpha) \quad (3.2.5)$$

In which α is the angle of attack of the flow to the vehicle longitudinal axis. The term A_{cs}^{base} is the area of the hull base, the area of the cross section at the end of the stern. The moment of the potential force is the famous instability moment of Munk.

For supersonic flow, the classical work of Ward (WARD, 1948) has shown that the cross force at small angles of attack acts at an angle midway between the normal to the axis of revolution of the body and the normal to the free-stream velocity. Thus, expressions (3.2.4-3.2.5) would be multiplied by $\cos(\alpha/2)$ for higher angles of attack. This result has been adopted by the missile community for the subsonic flow too (ALLEN; PERKINS, 1952); (LELAND HOWARD JORGENSEN, 1977). In his book, Karamcheti (1966) developed a subsonic slender body theory, achieving the same result derived by Munk. Considering this simpler result, which is a component of the total normal force expressed by the hydrodynamic coefficients to be identified, this work keeps the above expressions for further developments as follows.

In conventional AUV vehicles, the area of the hull base has a negligible value which would produce a null force on the hull. In fact, this feature is in accordance with a pointed body hypothesis assumed by a number of applications of the slender body theory. However, typically, due to viscous effects, the real flow can no longer be considered as potential at a section of the length between the base and a station ahead of it, when the angle of attack is different from zero. This fact motivates to adopt an empirical adjustment, choosing a greater cross-section area in (3.2.4) and (3.2.5) in order to improve the accuracy of the estimates. Barros et al. (2008) proposed the integration of the cross force per unit length, from the nose tip until the station located at the position x^* , in the middle point between the beginning of the

stern and its end. Therefore, the magnitude of the force due to the potential effects and its moment can be calculated as:

$$F_{pot}^{hull} = -\frac{1}{2}\rho U_o^2(k_2 - k_1)A^* \sin(2\alpha) \quad (3.2.6)$$

$$M_{pot}^{hull} = \frac{1}{2}\rho U_o^2(k_2 - k_1)(V^* + x^*A^*) \sin(2\alpha) \quad (3.2.7)$$

Where:

x^* is the axial position of the "station" of the body, at which the flow can no longer be considered as potential. Such station is assumed to be in the middle of the tail length, in order to produced a better agreement between theory and towing tank experiments than other estimates of x^* proposed by the other references adopted in the aircraft design, such as Datcom method (HOAK; FINK, 1960).

A^* is the cross-section area in the point x^* .

V^* is the hull's volume from the nose tip to x^* . Note that the value of V^* always is higher than the product ($x^* \cdot A^*$);

Expressing the potential force and its moment in a simpler form, it follows:

$$F_{pot}^{hull} = -\frac{1}{2}\rho U_o^2 A_{ref} c_{pot}^{hull} \sin(2\alpha) \quad (3.2.8)$$

$$M_{pot}^{hull} = \frac{1}{2}\rho U_o^2 L_{ref} A_{ref} c_{m_{pot}}^{hull} \sin(2\alpha) \quad (3.2.9)$$

Where:

A_{ref} is the reference area of the vehicle hull. In the SNAME standard, it is equal to the square of the vehicle length ($A_{ref} = L^2$);

L_{ref} is the reference length of the vehicle hull, in the SNAME standard it is equal to the vehicle length ($L_{ref} = L$);

c_{pot}^{hull} and $c_{m_{pot}}^{hull}$ are the dimensionless coefficients of the potential force acting on the hull and its moment, from the expressions (3.2.6 and 3.2.7), they can be calculated as:

$$c_{pot}^{hull} = (k_2 - k_1) \frac{A^*}{A_{ref}} \quad ; \quad c_{m_{pot}}^{hull} = (k_2 - k_1) \frac{V^* + x^* A^*}{L_{ref} A_{ref}} \quad (3.2.10)$$

The axial component of the potential force at incidence ($\alpha \neq 0$) is based on the result derived in the slender body theory by Karamcheti (1966). The increase of the axial force per unit of length is given by:

$$\frac{dX_{pot}^{hull}}{dx} = -\frac{1}{2} \rho U_o^2 \left[\alpha^2 + \frac{u_{x1}}{U_o} (r_{(x)}, x) + \left(\frac{dr_{(x)}}{dx} \right)^2 \right] \frac{dA}{dx} \quad (3.2.11)$$

The term u_{x1} is the disturbance velocity of the axis-symmetric flow past the body, and R is the body surface radius at the x position. Taking $\frac{dr_{(x)}}{dx} \ll 1$, which is the usual assumption in the slender body theory, it is reasonable to neglect the second and third terms in (3.2.11). Therefore, integrating the axial force per unit of length and adopting the correction factor, such as in the normal component, the axial component of the potential force results in:

$$X_{pot}^{hull} = \frac{1}{2} \rho U_o^2 A_{ref} c_{pot}^{hull} \alpha^2 \quad (3.2.12)$$

Considering greater angles of attack, it follows:

$$X_{pot}^{hull} = \frac{1}{2} \rho U_o^2 A_{ref} c_{pot}^{hull} \sin^2 \alpha \quad (3.2.13)$$

The resultant of the viscous drag forces is decomposed on the axial drag force (F_{d0}^{hull}) and the cross-flow drag force (F_{dc}^{hull}). They can be estimated from the dimensional form by decomposition of the vehicle velocity on the axial and the cross-flow directions. Such as:

$$F_{d0}^{hull} = -\frac{1}{2} \rho (U_o \cos \alpha)^2 A_{ref} c_{d0}^{hull} \quad (3.2.14)$$

$$F_{dc}^{hull} = -\frac{1}{2} \rho U_o^2 A_{ref} c_{dc}^{hull} |\sin \alpha| \sin \alpha \quad (3.2.15)$$

In which, c_{d0}^{hull} and c_{dc}^{hull} are the axial drag coefficient and the cross-flow drag coefficient of the hull. They represent the dimensionless drag forces due to the viscous effect over the hull

when $\alpha = 0^\circ$ and $\alpha = 90^\circ$ respectively. These dimensionless coefficients can be estimated by the following semi-empirical expressions (HOAK; FINK, 1960); (BARROS et al., 2008)

$$c_{d0}^{hull} = c_{sf}^{hull} (1 + 60f^{-3} + 0.0025f) \frac{A_{wett}}{A_{ref}} \quad ; \quad c_{dc}^{hull} = \eta c_{dc}^{cylinder} \frac{A_p}{A_{ref}} \quad (3.2.16)$$

Where:

f is the fineness ratio of the vehicle hull ($f = L/D$).

A_{wett} is the wetted surface area, in the case of a submersible vehicle it is equal to the total surface of the hull.

c_{sf}^{hull} is the skin friction drag coefficient of the hull and it is calculated as:

$$c_{sf}^{hull} = \frac{0.075}{(\log_{10} Re - 2)^2} + 0.00025 \quad (3.2.17)$$

A_p is the plan-form area of the hull in the $\hat{x}$ - $\hat{y}$ plane;

$c_{dc}^{cylinder}$ and η are the cross-flow drag coefficient, in an infinitely long cylinder, and its correction factor for a finite cylinder respectively. Their values can be obtained from the (figures 3.2.1 and 3.2.2);

Figure 3.2.1: The cross-flow drag coefficient against the cross-flow Reynolds number for circular cylinders at subcritical Mach numbers ($M_n \leq 0.4$). It was reproduced from Jorgensen (1977, pp. 76)

In which, the cross-flow Reynolds number to a cylinder can be calculated as:

$$Re_{cross} = \frac{U_o \sin(\alpha) \cdot D}{\nu} \quad (3.2.18)$$

ν is the kinematic viscosity of the seawater ($1.022 \cdot 10^{-6} \text{ m}^2/\text{s}$);

When considered the symmetry of the vehicle, the moment on the vehicle produced by the viscous forces has a direction perpendicular to the maneuver plane. In order to calculate it, only the normal component is necessary due to the fact that the axial component crosses the center of the vehicle coordinate frame and it does not contribute to the resultant moment.

Figure 3.2.2: Correction factor of the cross-flow drag coefficient for a finite-length cylinder.

It was reproduced from Jorgensen (1977), Figure N°4, pp 77).

The moment produced by the cross flow on the vehicle, due to the viscous-drag forces on the hull, can be calculated as:

$$M_{drag}^{hull} = -x_p \cdot F_{dc}^{hull} = \frac{1}{2} \rho U_o^2 A_{ref} x_p c_{dc}^{hull} |\sin \alpha| \sin \alpha \quad 3.2.19$$

In which x_p is the Longitudinal position of the plan-form-geometric center of the hull, and it is calculated by:

$$x_p = \frac{1}{A_p} \int_{x_{stern}}^{x_{bow}} (2r_{(x)}x) dx \quad (3.2.20)$$

The signal of the torque depends on the signal of the x_p and the signal of the cross-flow-drag force (F_{dc}^{hull}) with relation to the vehicle coordinate frame.

B. Hull's Hydrodynamic Coefficients Related to Motion on the Vertical Plane

Expressions for the hydrodynamic coefficients relative to maneuvering in the vertical plane can be derived directly from the potential and viscous efforts described previously, and the relationship between the angle of attack and the vehicle velocity as follows:

$$\sin(\alpha) = \frac{w}{U_o} \quad ; \quad \cos(\alpha) = \frac{u}{U_o} \quad (3.2.21)$$

Using (3.2.21) in the relations expressed by (3.2.8-3.2.9), (3.2.13-3.2.15) and (3.2.19), for the forces and moment over the hull, decomposed in the vehicle reference frame, are given as follows:

$$X^{hull} = \frac{1}{2} \rho A_{ref} (c_{pot}^{hull} w^2 - c_{d0}^{hull} u^2) \quad (3.2.22)$$

$$Z^{hull} = -\frac{1}{2} \rho A_{ref} (2c_{pot}^{hull} uw + c_{dc}^{hull} |w|w) \quad (3.2.23)$$

$$M^{hull} = \frac{1}{2} \rho L_{ref} A_{ref} \left(2c_{mpot}^{hull} uw + \frac{x_p}{L_{ref}} c_{dc}^{hull} |w|w \right) \quad (3.2.24)$$

Table 3.2.1 - Hull's hydrodynamic coefficients to motion on the vertical plane.

Axial Coefficients	Surge-Heave Coefficients	
$X_{uu}^{hull} = -\frac{1}{2} \rho A_{ref} c_{d0}^{hull}$	$Z_{uw}^{hull} = -\frac{1}{2} \rho A_{ref} (2c_{pot}^{hull})$	$M_{uw}^{hull} = \frac{1}{2} \rho L_{ref} A_{ref} (2c_{mpot}^{hull})$
$X_{ww}^{hull} = \frac{1}{2} \rho A_{ref} c_{pot}^{hull}$	$Z_{ w w}^{hull} = -\frac{1}{2} \rho A_{ref} c_{dc}^{hull}$	$M_{ w w}^{hull} = -x_p \cdot Z_{ w w}^{hull}$
$X_{qw}^{hull} = Z_{\dot{w}}^{hull}$	$Z_{uq}^{hull} = X_{\dot{u}}^{hull}$	$M_{uq}^{hull} = -Z_{\dot{q}}^{hull}$
$X_{qq}^{hull} = Z_{\dot{q}}^{hull}$	—	—

The hydrodynamic coefficients of the hull, when the vehicle maneuvers in the vertical plane, can be obtained from the expression (3.2.22-3.2.24), such as seen in table 3.2.1.

The table also includes the hydrodynamic coefficients related to rotations (velocities and accelerations) in ideal flow as seen in the expression (3.1.7).

C. Hull Hydrodynamic Coefficients Related to Motion in the Horizontal Plane

Expressions of the hydrodynamic coefficients related to the efforts acting on the vehicle when maneuvering in the horizontal plane, are derived by replacing the angle of attack by the sideslip angle, considering the correct signal in the vehicle frame, in the expressions of the potential and viscous forces and moments (3.2.8-3.2.9), (3.2.13-3.2.15) and (3.2.19). In the vehicle reference frame, the components are given as follows:

$$X^{hull} = \frac{1}{2} \rho U_o^2 A_{ref} (c_{pot}^{hull} \sin^2(\beta) - c_{d0}^{hull} \cos^2(\beta)) \quad (3.2.25)$$

$$Y^{hull} = \frac{1}{2} \rho U_o^2 A_{ref} (c_{pot}^{hull} \sin(2\beta) + c_{dc}^{hull} |\sin \beta| \sin \beta) \quad (3.2.26)$$

$$N^{hull} = \frac{1}{2} \rho U_o^2 L_{ref} A_{ref} \left(c_{m_{pot}}^{hull} \sin(2\beta) + \frac{x_p}{L_{ref}} c_{dc}^{hull} |\sin(\beta)| \sin(\beta) \right) \quad (3.2.27)$$

The relations of the vehicle velocities with the sideslip angle are expressed as:

$$\sin(\beta) = -\frac{v}{U_o} \quad ; \quad \cos(\beta) = \frac{u}{U_o} \quad (3.2.28)$$

Substituting (3.2.28) in the equations (3.2.25-3.2.27) are obtained the final expressions for the potential and viscous effects forces and moment over the hull, due to translation in the horizontal plane:

$$X^{hull} = \frac{1}{2} \rho A_{ref} (c_{pot}^{hull} v^2 - c_{d0}^{hull} u^2) \quad (3.2.29)$$

$$Y^{hull} = -\frac{1}{2} \rho A_{ref} (2c_{pot}^{hull} uv + c_{dc}^{hull} |v|v) \quad (3.2.30)$$

$$N^{hull} = -\frac{1}{2} \rho L_{ref} A_{ref} \left(2c_{m_{pot}}^{hull} uv + \frac{x_p}{L_{ref}} c_{dc}^{hull} |v|v \right) \quad (3.2.31)$$

Table 3.2.2 - Hull coefficients to motion on the horizontal plane

Axial Coefficients	Yaw-Sway Coefficients	
$X_{uu}^{hull} = -\frac{1}{2}\rho A_{ref} c_{d0}^{hull}$	$Y_{uv}^{hull} = -\frac{1}{2}\rho A_{ref} (2c_{pot}^{hull})$	$N_{uv}^{hull} = -\frac{1}{2}\rho L_{ref} A_{ref} (2c_{mpot}^{hull})$
$X_{vv}^{hull} = \frac{1}{2}\rho A_{ref} c_{pot}^{hull}$	$Y_{ v v}^{hull} = -\frac{1}{2}\rho A_{ref} c_{dc}^{hull}$	$N_{ v v}^{hull} = x_p \cdot Y_{ v v}^{hull}$
$X_{vr}^{hull} = -Y_v^{hull}$	$Y_{ur}^{hull} = X_u^{hull}$	$N_{ur}^{hull} = Y_r^{hull}$
$X_{rr}^{hull} = -Y_r^{hull}$	–	–

The hydrodynamic coefficients of the hull related to the translational motion in the horizontal plane can be obtained from the expressions (3.2.29-3.2.31). They are presented in table 3.2.2.

Moreover, efforts related to the vehicle rotations in the ideal flow are also taken into account.

The corresponding coefficients $(X_{ur}^{hull}, X_{rr}^{hull}, Y_{ur}^{hull}, N_{ur}^{hull})$ can be approximated using the equalities (3.1.6), such as seen in the table 3.2.2.

3.3. Hydrodynamic Coefficients Related to Vehicle Fins

In this work, a fin can depict a sail as well as control surfaces such as rudders and elevators. The analysis presented in this sub-section considers only isolated fins without the hull interference.

A. Lift and Drag Forces Acting on Vehicle Fins

The resultant hydrodynamic forces that act on a fin can be decomposed in lift and drag components. In a dimensional form (SNAME, 1950) these forces are expressed as:

$$F_L^{fin} = \frac{1}{2}\rho U_F^2 A^{fin} C_L^{fin} \quad ; \quad F_D^{fin} = \frac{1}{2}\rho U_F^2 A^{fin} C_D^{fin} \quad (3.3.1)$$

Where:

A^{fin} is the area of the fin's plan-form.

U_F is the velocity of the fin's hydrodynamic center relative to the fluid.

C_L^{fin} and C_D^{fin} are the lift and drag coefficients of the vehicle fins respectively, they depend on the geometrical and hydrodynamic characteristics of the fins as well as the fin's angle of attack. The lift and drag coefficients can be calculated using the expressions given by Whicker and Fehlner (1958), to low-aspect-ratio fin (effective aspect ratio of 3 or less):

$$C_L^{fin} = c_l^{fin} \alpha_F + \frac{c_{dc}^{fin}}{a^{fin}} \alpha_F^2 \quad ; \quad C_D^{fin} = c_{d0}^{fin} + \frac{1}{\pi a^{fin} \varepsilon} (C_L^{fin})^2 \quad (3.3.2)$$

Where:

α_F is the attack angle of the fin relative to the flow around to the fin. In these expressions, it is considered an Euclidean angle ($\alpha_F > 0$).

a^{fin} is the geometrical aspect ratio of the fin and it is calculated as:

$$a^{fin} = \frac{(h^{fin})^2}{A^{fin}} \quad (3.3.3)$$

h^{fin} is the height of the fin.

c_l^{fin} is the linear part of the lift coefficient, it is the curve's slope of the lift coefficient as a function of the angle of attack when the angle of attack is zero. This dimensionless coefficient is calculated as:

$$c_l^{fin} = \left. \frac{\partial C_L^{fin}}{\partial \alpha} \right|_{\alpha=0} = \frac{2\pi}{\frac{2}{a^{fin}} + \frac{1}{\varepsilon} \sqrt{\frac{1}{\cos^2 \Lambda_{c/4}^{fin}} + \frac{4 \cos^2 \Lambda_{c/4}^{fin}}{(a^{fin})^2}}} \quad (3.3.4)$$

$\varepsilon = 0.9$ is the efficiency factor obtained from experimental observations.

$\Lambda_{c/4}^{fin}$ is the Sweep Angle for a quarter chord in the fin.

c_{d0}^{fin} and c_{dc}^{fin} are the axial and cross-flow drag coefficients of the fin. They depict the viscous effect on the fin. They depend on the geometrical characteristic of the fin. Whicker and Fehlner (1958) presented an experimental plot, see figure 3.3.1, that the fin cross-flow drag coefficient can be approximated to be a linear relation with the fin taper ratio, for fins with square tips. It is given as:

$$c_{dc}^{fin} = 1.63 * \frac{\text{tip chord}}{\text{root chord}} + 0.08 \quad (3.3.5)$$

Developing the drag coefficients from the expressions (3.3.2), and considering terms until second order, the drag coefficient of the fin can be expressed as:

$$C_D^{fin} = c_{d0}^{fin} + c_{id}^{fin} \alpha_F^2 \quad (3.3.6)$$

In which c_{id}^{fin} is the induced drag coefficient due to potential effects, for that reason, it depends on the potential term of the lift coefficient (c_l^{fin}). The induced drag coefficient is calculated by:

$$c_{id}^{fin} = \frac{1}{\pi \alpha^{fin} \epsilon} (c_l^{fin})^2 \quad (3.3.7)$$

Finally, the lift and drag components of the resultant force over the fin can be expressed as a function of the dimensionless coefficients and the attack angle, such as:

$$F_L^{fin} = \frac{1}{2} \rho U_F^2 A^{fin} \left(c_l^{fin} \alpha_F + \frac{c_{dc}^{fin}}{\alpha^{fin}} \alpha_F^2 \right) \quad (3.3.8)$$

$$F_D^{fin} = \frac{1}{2} \rho U_F^2 A^{fin} (c_{d0} + c_{id}^{fin} \alpha_F^2) \quad (3.3.9)$$

B. Fin Coefficients Related to Motion in The Vertical Plane

Due to the hydrodynamic shape of the fins, the only fins that affect the vehicle dynamic on the vertical plane are the elevators. The elevators are control surfaces. Hence they could have a deflected angle relative to the vehicle's axial edge (δ_E). The angle of attack of the elevators (α_E^{elev}) is the angle between the velocity vector, of the elevators' hydrodynamic center, and the root chord of the elevators. It is a trigonometric angle with signal defined in figure 3.3.2. The angle of attack of the elevators can obtained as:

$$\alpha_E^{elev} = \alpha_E + \delta_E \quad (3.3.10)$$

Figure 3.3.1: Effect of the Taper Ratio on Cross-flow Drag Coefficient.

Reproduce from Whicker and Fehlner (1958) , fig. 28, pp. 21.

The elevator's velocity and the drag force vectors are decomposed on the vehicle frame, according to the sign of the angle of attack, such as seen in figure 3.3.2:

Figure 3.3.2: Side View of the Forces on the Elevators on Vertical-Plane Motion

$$\mathbf{U}_E = \underbrace{U_E \cos \alpha_E}_{u_E} \hat{\mathbf{x}} + \underbrace{U_E \sin \alpha_E}_{w_E} \hat{\mathbf{z}} \quad (3.3.11)$$

$$\mathbf{F}_D^{elev} = F_D^{elev} \cos(-\alpha_E + \pi) \hat{\mathbf{x}} - F_D^{elev} \sin(-\alpha_E + \pi) \hat{\mathbf{z}} \quad (3.3.12)$$

Considering that, the signal of the lift force depends on the signal of the attack angle of the elevators (α_E^{elev}), such as seen in figure (3.3.2), the lift force can be expressed as:

$$\mathbf{F}_L^{elev} = F_L^{elev} \cos\left(-\alpha_E + \text{sign}(\alpha_E^{elev}) \cdot \frac{\pi}{2}\right) \hat{\mathbf{x}} - F_L^{elev} \sin\left(-\alpha_E + \text{sign}(\alpha_E^{elev}) \cdot \frac{\pi}{2}\right) \hat{\mathbf{z}} \quad (3.3.13)$$

Applying basic trigonometry identities, the components on $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$ of the lift and drag forces can be expressed as:

$$X^{elev} = \text{sign}(\alpha_E^{elev}) \cdot F_L^{elev} \cdot \sin(\alpha_E) - F_D^{elev} \cdot \cos(\alpha_E) \quad (3.3.14)$$

$$Z^{elev} = -\text{sign}(\alpha_E^{elev}) \cdot F_L^{elev} \cdot \cos(\alpha_E) - F_D^{elev} \cdot \sin(\alpha_E) \quad (3.3.15)$$

The resultant moment over the vehicle, due to the lift and drag forces on the sail, can be calculated considering only the components on $\hat{\mathbf{z}}$, since the component on $\hat{\mathbf{x}}$ is aligned with the center of the vehicle frame, and it does not contribute to the resultant moment in pitch.

Therefore, the resultant torque on the vehicle due to the forces on the elevators is calculated as:

$$M^{elev} = -x_E \cdot Z^{elev} \quad (3.3.16)$$

In (3.3.16), x_E is the axial position of the hydrodynamic center of the elevators.

Using the derived expression of the lift and drag forces (3.3.8) and (3.3.9) in the expressions (3.3.14) and (3.3.15), and considering that the angle of attack in these expressions is the angle of attack relative to the deflected elevators (α_E^{elev}), the components on $\hat{\mathbf{x}}$ and $\hat{\mathbf{z}}$ of the lift and drag forces are expressed as:

$$\begin{aligned} X^{elev} &= \frac{1}{2} \rho U_E^2 A^{elev} \left(c_l^{elev} \alpha_E^{elev} + \frac{c_{dc}^{elev}}{\alpha^{elev}} |\alpha_E^{elev}| \alpha_E^{elev} \right) \sin(\alpha_E) \\ &\quad - \frac{1}{2} \rho U_E^2 A^{elev} \left(c_{d0}^{elev} + c_{id}^{elev} (\alpha_E^{elev})^2 \right) \cos(\alpha_E) \end{aligned} \quad (3.3.17)$$

$$\begin{aligned}
Z^{elev} = & -\frac{1}{2}\rho U_E^2 A^{elev} \left(c_l^{elev} \alpha_E^{elev} + \frac{c_{dc}^{elev}}{\alpha^{elev}} |\alpha_E^{elev}| \alpha_E^{elev} \right) \cos(\alpha_E) \\
& -\frac{1}{2}\rho U_E^2 A^{elev} \left(c_{d0}^{elev} + c_{id}^{elev} (\alpha_E^{elev})^2 \right) \sin(\alpha_E)
\end{aligned} \quad (3.3.18)$$

The angle of attack relative to the elevators root chord (α_E^{elev}) is replaced such as on the expression (3.3.10). It is considered the approximation for small angle of attack ($\alpha_E \approx \sin(\alpha_E)$; $\cos(\alpha_E) \approx 1$) and the terms higher than second order are disregarded. Therefore, the components on the $\hat{x}$ and $\hat{z}$ of the lift and drag forces can be expressed as:

$$X^{elev} = \frac{1}{2}\rho U_E^2 A^{elev} \left((c_l^{elev} - c_{id}^{elev}) \alpha_E^2 + (c_l^{elev} - 2c_{id}^{elev}) \alpha_E \delta_E - c_{d0}^{elev} - c_{id}^{elev} \delta_E^2 \right) \quad (3.3.19)$$

$$Z^{elev} = \frac{1}{2}\rho U_E^2 A^{elev} \left((-c_l^{elev} + c_{d0}^{elev}) \alpha_E - c_l^{elev} \delta_E - \frac{c_{dc}^{elev}}{\alpha^{elev}} |\alpha_E + \delta_E| (\alpha_E + \delta_E) \right) \quad (3.3.20)$$

Moreover, from the equation (3.3.16), the resultant torque on the vehicle due to the forces on the elevators is expressed as:

$$M^{elev} = -\frac{1}{2}\rho U_E^2 A^{elev} x_E \left((-c_l^{elev} + c_{d0}^{elev}) \alpha_E - c_l^{elev} \delta_E - \frac{c_{dc}^{elev}}{\alpha^{elev}} |\alpha_E + \delta_E| (\alpha_E + \delta_E) \right) \quad (3.3.21)$$

From the equations (2.1.2-2.1.4), derived in Section 2.1. The velocity components of the elevators' hydrodynamic center "E" are expressed as:

$$u_E = u + z_E q - y_E r \quad (3.3.22)$$

$$v_E = v + x_E r - z_E p \quad (3.3.23)$$

$$w_E = w + y_E p - x_E q \quad (3.3.24)$$

Considering maneuvers on the vertical plane ($p \approx r \approx v \approx 0$) and that, the elevators generally are symmetrically allocated in the $\hat{x}$ - $\hat{y}$ plane of the hull ($z_E = 0$). The angle of attack can be expressed as:

$$\alpha_E \approx \sin \alpha_E = \frac{w_E}{U_E} = \frac{w - x_E q}{U_E} \quad ; \quad \cos \alpha_E = \frac{u_E}{U_E} = \frac{u}{U_E} \approx 1 \quad (3.3.25)$$

Table 3.3.1 - Elevator coefficients to motion on the vertical plane

Coefficients Related to $\hat{\mathbf{x}}$	Coefficients Related to $\hat{\mathbf{z}}$	Coefficients Related to $\hat{\mathbf{y}}$
$X_{uu}^{elev} = -\frac{1}{2}\rho A^{elev} c_{d0}^{elev}$	$Z_{uw}^{elev} = -\frac{1}{2}\rho A^{elev} (c_l^{elev} - c_{d0}^{elev})$	$M_{uw}^{elev} = -x_E \cdot Z_{uw}^{elev}$
$X_{ww}^{elev} = \frac{1}{2}\rho A^{elev} (c_l^{elev} - c_{id}^{elev})$	$Z_{uq}^{elev} = -x_E \cdot Z_{uw}^{elev}$	$M_{uq}^{elev} = x_E^2 \cdot Z_{uw}^{elev}$
$X_{wq}^{elev} = -2x_E \cdot X_{ww}^{elev}$	$Z_{ w w}^{elev} = -\frac{1}{2}\rho A^{elev} \frac{c_{dc}^{elev}}{a^{elev}}$	$M_{ w w}^{elev} = -x_E \cdot Z_{ w w}^{elev}$
$X_{qq}^{elev} = x_E^2 \cdot X_{ww}^{elev}$	$Z_{u^2\delta_E}^{elev} = -\frac{1}{2}\rho A^{elev} c_l^{elev}$	$M_{u^2\delta_E}^{elev} = -x_E \cdot Z_{u^2\delta_E}^{elev}$
$X_{u^2\delta_E^2}^{elev} = -\frac{1}{2}\rho A^{elev} c_{id}^{elev}$	—	—

The expressions (3.3.25) are replaced in the forces and moments expression (3.3.19 - 3.3.21) and considering that the dynamic model derived on section 2.2 (2.2.29 – 2.2.31) disregards the terms $u(w - x_E q)\delta_E$ in the axial force and the terms containing $(q^2, uw\delta_E, uq\delta_E$ or $u^2\delta_E^2)$ in the heave force and moment. Hence, the resultant force and moment on the elevators can be expressed as:

$$X^{elev} = \frac{1}{2}\rho A^{elev} \left((c_l^{elev} - c_{id}^{elev})(w^2 - 2x_E wq + x_E^2 q^2) - c_{d0}^{elev} u^2 - c_{id}^{elev} u^2 \delta_E^2 \right) \quad (3.3.26)$$

$$Z^{elev} = \frac{1}{2}\rho A^{elev} \left(-(c_l^{elev} - c_{d0}^{elev})(uw - x_E uq) - c_l^{elev} u^2 \delta_E - \frac{c_{dc}^{elev}}{a^{elev}} |w|w \right) \quad (3.3.27)$$

$$M^{elev} = \frac{1}{2}\rho A^{elev} x_E \left((c_l^{elev} - c_{d0}^{elev})(uw - x_E uq) + c_l^{elev} u^2 \delta_E + \frac{c_{dc}^{elev}}{a^{elev}} |w|w \right) \quad (3.3.28)$$

Finally, the hydrodynamic coefficients of the elevators, to a motion on the vertical plane, can be obtained as seen in table 3.3.1.

C. Fin Coefficients Related to Motion in the Horizontal Plane

Considering the hydrodynamic shape of the fins; only the sail, and the rudders affect the horizontal plan dynamics.

Figure 3.3.3: Top View of the Velocity, Lift and Drag forces over the Sail

The sail is fixed to the vehicle hull. Hence, the sideslip angle is relative to it, such as seen in figure 3.3.3. The velocity on the hydrodynamic center of the sail (S), when it is moving on the horizontal plane, can be decomposed on the vehicle frame, such as:

$$\mathbf{U}_S = \underbrace{U_S \cos \beta_s}_{u_s} \hat{\mathbf{x}} + \underbrace{U_S \sin(-\beta_s)}_{v_s} \hat{\mathbf{y}} \quad (3.3.29)$$

Lift and drag forces that act on the sail can be decomposed, to a motion on the horizontal plane, on the vehicle frame as:

$$\mathbf{F}_L^{sail} = F_L^{sail} \cos\left(-\beta_s \pm \frac{\pi}{2}\right) \hat{\mathbf{x}} + F_L^{sail} \sin\left(-\beta_s \pm \frac{\pi}{2}\right) \hat{\mathbf{y}} \quad (3.3.30)$$

$$\mathbf{F}_D^{sail} = F_D^{sail} \cos(-\beta_s + \pi) \hat{\mathbf{x}} + F_D^{sail} \sin(-\beta_s + \pi) \hat{\mathbf{y}} \quad (3.3.31)$$

Applying fundamental trigonometry relations and considering that, the lift force depends on the sideslip angle signal, such as seen in figure (3.3.3). The components on the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ of the lift and drag forces over the sail can be expressed as:

$$X^{sail} = \text{sign}(\beta_s) \cdot F_L^{sail} \cdot \sin(\beta_s) - F_D^{sail} \cdot \cos(\beta_s) \quad (3.3.32)$$

$$Y^{sail} = \text{sign}(\beta_s) \cdot F_L^{sail} \cdot \cos(\beta_s) + F_D^{sail} \cdot \sin(\beta_s) \quad (3.3.33)$$

The resultant moment over the vehicle, due to the lift and drag forces on the sail, can be calculated using the force components in $\hat{\mathbf{y}}$, since the components in $\hat{\mathbf{x}}$ can be considered as aligned with the center of the vehicle reference frame. Therefore, the resultant moment on the vehicle, due to the forces on the sail, is calculated as:

$$N^{sail} = x_s \cdot Y^{sail} \quad (3.3.34)$$

In which x_s is the axial position of the sail's hydrodynamic center.

Substituting the derived expression to the lift and drag forces (3.3.8) and (3.3.9) in the expressions (3.3.32) and (3.3.33), according to the correct signal of the sideslip angle as in figure (3.3.3), the components on the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ of the lift and drag forces over the sail can be expressed as:

$$X^{sail} = \frac{1}{2} \rho U_s^2 A^{sail} \left(\left(c_l^{sail} \beta_s + \frac{c_{dc}^{sail}}{a^{sail}} |\beta_s| \beta_s \right) \sin(\beta_s) - (c_{d0}^{sail} + c_{id}^{sail} \beta_s^2) \cos(\beta_s) \right) \quad (3.3.35)$$

$$Y^{sail} = \frac{1}{2} \rho U_s^2 A^{sail} \left(\left(c_l^{sail} \beta_s + \frac{c_{dc}^{sail}}{a^{sail}} |\beta_s| \beta_s \right) \cos(\beta_s) + (c_{d0}^{sail} + c_{id}^{sail} \beta_s^2) \sin(\beta_s) \right) \quad (3.3.36)$$

Considering approximation for small sideslip angles ($\beta_s \approx \sin(\beta_s)$; $\cos(\beta_s) \approx 1$) and disregarding terms higher than second order, the components on the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ of the lift and drag forces over the sail are expressed as:

$$X^{sail} = \frac{1}{2} \rho U_s^2 A^{sail} \left((c_l^{sail} - c_{id}^{sail}) \beta_s^2 - c_{d0}^{sail} \right) \quad (3.3.37)$$

$$Y^{sail} = \frac{1}{2} \rho U_s^2 A^{sail} \left((c_l^{sail} + c_{d0}^{sail}) \beta_s + \frac{c_{dc}^{sail}}{a^{sail}} |\beta_s| \beta_s \right) \quad (3.3.38)$$

Moreover, from the equation (3.3.34), the resultant moment on the vehicle due to the forces on the sail is expressed as:

$$N^{sail} = \frac{1}{2} \rho U_s^2 A^{sail} x_s \left((c_l^{sail} + c_{d0}^{sail}) \beta_s + \frac{c_{dc}^{sail}}{a^{sail}} |\beta_s| \beta_s \right) \quad (3.3.39)$$

From the expressions (2.1.2 – 2.1.4) derived in Section 2.1. The velocity components, of the Sail's hydrodynamic center "S", are expressed as:

$$u_s = u + z_s q - y_s r \quad (3.3.40)$$

$$v_s = v + x_s r - z_s p \quad (3.3.41)$$

$$w_s = w + y_s p - x_s q \quad (3.3.42)$$

Considering motion in the horizontal plane ($p \approx q \approx w \approx 0$) and that the sail is located in the symmetrical $\hat{x}$ - $\hat{z}$ plane of the hull ($y_s = 0$). The sideslip angle can be replaced by the following expressions:

$$\beta_s \approx -\frac{v_s}{U_s} = -\frac{v + x_s r}{U_s} \quad ; \quad \frac{u_s}{U_s} = \frac{u}{U_s} \approx 1 \quad (3.3.43)$$

Replacing the expressions (3.3.43) on the expressions (3.3.37 - 3.3.39), and considering that the dynamic model derived in section 2.2, expressions (2.2.18 – 2.2.20), disregards the terms vr and r^2 for the sideslip force, the components on the $\hat{x}$ and $\hat{y}$ of the lift and drag forces over the sail and their resulting moment over the vehicle can be expressed as:

$$X^{sail} = \frac{1}{2} \rho A^{sail} \left((c_l^{sail} - c_{id}^{sail}) (v^2 + 2x_s vr + x_s^2 r^2) - c_{d0}^{sail} u^2 \right) \quad (3.3.44)$$

$$Y^{sail} = -\frac{1}{2} \rho A^{sail} \left((c_l^{sail} + c_{d0}^{sail}) (uv + x_s ur) + \frac{c_{dc}^{sail}}{a^{sail}} |v|v \right) \quad (3.3.45)$$

$$N^{sail} = -\frac{1}{2} \rho A^{sail} x_s \left((c_l^{sail} + c_{d0}^{sail}) (uv + x_s ur) + \frac{c_{dc}^{sail}}{a^{sail}} |v|v \right) \quad (3.3.46)$$

Finally, the hydrodynamic coefficients related to the sail, are presented in table 3.3.2:

Table 3.3.2 - Sail coefficients to motion on the horizontal plane.

Axial Coefficients	Yaw-Sway Coefficients	
$X_{uu}^{sail} = -\frac{1}{2}\rho A^{sail} c_{d0}^{sail}$	$Y_{uv}^{sail} = -\frac{1}{2}\rho A^{sail} (c_l^{sail} + c_{d0}^{sail})$	$N_{uv}^{sail} = -\frac{1}{2}\rho A^{sail} x_s (c_l^{sail} + c_{d0}^{sail})$ $= x_s Y_{uv}^{sail}$
$X_{vv}^{sail} = \frac{1}{2}\rho A^{sail} (c_l^{sail} - c_{id}^{sail})$	$Y_{ur}^{sail} = -\frac{1}{2}\rho A^{sail} x_s (c_l^{sail} + c_{d0}^{sail})$ $= x_s Y_{uv}^{sail}$	$N_{ur}^{sail} = -\frac{1}{2}\rho A^{sail} x_s^2 (c_l^{sail} + c_{d0}^{sail})$ $= x_s^2 \cdot Y_{uv}^{sail}$
$X_{vr}^{sail} = \frac{1}{2}\rho A^{sail} (c_l^{sail} - c_{id}^{sail}) 2x_s$ $= 2x_s \cdot X_{vv}^{sail}$	$Y_{ v v}^{sail} = -\frac{1}{2}\rho A^{sail} \frac{c_{dc}^{sail}}{a^{sail}}$	$N_{ v v}^{sail} = -\frac{1}{2}\rho A^{sail} x_s \frac{c_{dc}^{sail}}{a^{sail}} = x_s \cdot Y_{ v v}^{sail}$
$X_{rr}^{sail} = \frac{1}{2}\rho A^{sail} (c_l^{sail} - c_{id}^{sail}) x_s^2$ $= x_s^2 \cdot X_{vv}^{sail}$	—	—

In the case of the rudders, they could have a deflected angle relative to the vehicle axial edge (δ_R). Hence for the rudders, the angle in the expressions of the lift and drag forces (3.3.8 and 3.3.9) is the resultant value of the sideslip angle of the rudders (β_R^{rudd}), which is the angle between the velocity vector of the rudders hydrodynamic center and the root chord of the rudders. This angle is algebraically composed as defined in figure (3.3.4). Therefore, the sideslip angle of the rudders can be expressed as:

$$\beta_R^{rudd} = \beta_R + \delta_R \quad 3.3.47$$

The velocity on the hydrodynamic center of the rudders (R), when it is moving on the horizontal plane, can be decomposed on the vehicle frame, such as:

$$\mathbf{U}_R = \underbrace{U_R \cos(-\beta_R)}_{u_R} \hat{\mathbf{x}} + \underbrace{U_R \sin(-\beta_R)}_{v_R} \hat{\mathbf{y}} \quad (3.3.48)$$

The drag force acting on the sail, to a motion on the horizontal plane, is decomposed on the vehicle frame such as seen in the figure 3.3.4:

$$\mathbf{F}_D^{rudd} = F_D^{rudd} \cos(-\beta_R + \pi) \hat{\mathbf{x}} + F_D^{rudd} \sin(-\beta_R + \pi) \hat{\mathbf{y}} \quad (3.3.49)$$

Figure 3.3.4: Top-View of the Velocity, Lift and Drag forces of the Rudders

The lift force acting on the sail, to a motion on the horizontal plane, is decomposed on the vehicle frame such as seen in the figure 3.3.4:

$$\mathbf{F}_L^{rudd} = F_L^{rudd} \cos\left(-\beta_R \pm \frac{\pi}{2}\right) \hat{\mathbf{x}} + F_L^{rudd} \sin\left(-\beta_R \pm \frac{\pi}{2}\right) \hat{\mathbf{y}} \quad (3.3.50)$$

Considering that the signal of the lift force depends on the signal of the sideslip angle of the rudders (β_R^{rudd}), such as seen in figure (3.3.4, the components on $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ of the lift and drag forces over the rudders can be expressed as:

$$X^{rudd} = \text{sign}(\beta_R^{rudd}) \cdot F_L^{rudd} \cdot \sin(\beta_R) - F_D^{rudd} \cdot \cos(\beta_R) \quad (3.3.51)$$

$$Y^{rudd} = \text{sign}(\beta_R^{rudd}) F_L^{rudd} \cos(\beta_R) + \text{sign}(\beta_R^{rudd}) \text{sign}(\beta_R) F_D^{rudd} \sin(\beta_R) \quad (3.3.52)$$

The resultant moment over the vehicle, due to the lift and drag forces on the rudders, can be calculated considering only the components on $\hat{\mathbf{y}}$, because the component on $\hat{\mathbf{x}}$ is considered aligned with the center of the vehicle frame.

Therefore, the resultant torque on the vehicle due to the forces on the rudders is calculated as:

$$N^{rudd} = x_R \cdot Y^{rudd} \quad (3.3.53)$$

In which x_R is the axial position of the hydrodynamic center of the rudders.

Substituting the derived expressions of the lift and drag forces (3.3.8) and (3.3.9) in the expressions (3.3.51) and (3.3.52), and considering the correct signal of the sideslip angle, the components on $\hat{x}$ and $\hat{y}$ of the lift and drag forces over the sail can be expressed as:

$$X^{rudd} = \frac{1}{2} \rho U_R^2 A^{rudd} \left(c_l^{rudd} \beta_R^{rudd} + \frac{c_{dc}^{rudd}}{a^{rudd}} |\beta_R^{rudd}| \beta_R^{rudd} \right) \sin(\beta_R) - \frac{1}{2} \rho U_R^2 A^{rudd} \left(c_{d0}^{rudd} + c_{id}^{rudd} (\beta_R^{rudd})^2 \right) \cos(\beta_R) \quad (3.3.54)$$

$$Y^{rudd} = \frac{1}{2} \rho U_R^2 A^{rudd} \left(c_l^{rudd} \beta_R^{rudd} + \frac{c_{dc}^{rudd}}{a^{rudd}} |\beta_R^{rudd}| \beta_R^{rudd} \right) \cos(\beta_R) + \frac{1}{2} \rho U_R^2 A^{rudd} s \left(c_{d0}^{rudd} + c_{id}^{rudd} (\beta_R^{rudd})^2 \right) \sin(\beta_R) \quad (3.3.55)$$

Substituting the sideslip angle of the rudder such as defined in (3.3.47), considering approximation for small sideslip angles ($\beta_R \approx \sin(\beta_R)$; $\cos(\beta_R) \approx 1$) and disregarding terms higher than second order, the components on the $\hat{x}$ and $\hat{y}$ of the lift and drag forces over the sail can be expressed as:

$$X^{rudd} = \frac{1}{2} \rho U_R^2 A^{rudd} \left((c_l^{rudd} - c_{id}^{rudd}) \beta_R^2 + (c_l^{rudd} - 2c_{id}^{rudd}) \beta_R \delta_R - c_{d0}^{rudd} - c_{id}^{rudd} \delta_R^2 \right) \quad (3.3.56)$$

$$Y^{rudd} = \frac{1}{2} \rho U_R^2 A^{rudd} \left((c_l^{rudd} + c_{d0}^{rudd}) \beta_R + c_l^{rudd} \delta_R + \frac{c_{dc}^{rudd}}{a^{rudd}} |\beta_R + \delta_R| (\beta_R + \delta_R) \right) \quad (3.3.57)$$

Besides, from the equation (3.3.53), the resultant torque on the vehicle due to the forces on the rudders is expressed as:

$$N^{rudd} = \frac{1}{2} \rho U_R^2 A^{rudd} x_R \left((c_l^{rudd} + c_{d0}^{rudd}) \beta_R + c_l^{rudd} \delta_R + \frac{c_{dc}^{rudd}}{a^{rudd}} |\beta_R + \delta_R| (\beta_R + \delta_R) \right) \quad (3.3.58)$$

Moreover, from the equations (2.1.2 – 2.1.4) derived in Section 2.1. The velocity components of the rudders' hydrodynamic center "R" are expressed as:

$$u_R = u + z_R q - y_R r \quad (3.3.59)$$

$$v_R = v + x_R r - z_R p \quad (3.3.60)$$

$$w_R = w + y_R p - x_R q \quad (3.3.61)$$

Considering motion on the horizontal plane ($p \approx q \approx w \approx 0$) and the rudders allocated in the symmetrical $\hat{x}$ - $\hat{z}$ plane of the hull. ($y_R = 0$). The sideslip angle can be replaced by the following expressions:

$$\beta_R \approx -\frac{v_R}{U_R} = -\frac{v + x_R r}{U_R} \quad ; \quad \frac{u_R}{U_R} = \frac{u}{U_R} \quad (3.3.62)$$

Table 3.3.3 - Rudder coefficients to motion on the horizontal plane

Axial Coefficients	Yaw-Sway Coefficients	
$X_{uu}^{rudd} = -\frac{1}{2}\rho A^{rudd} c_{d0}^{rudd}$	$Y_{uv}^{rudd} = -\frac{1}{2}\rho A^{rudd} (c_l^{rudd} + c_{d0}^{rudd})$	$N_{uv}^{rudd} = x_R Y_{uv}^{rudd}$
$X_{vv}^{rudd} = \frac{1}{2}\rho A^{rudd} (c_l^{rudd} - c_{id}^{rudd})$	$Y_{ur}^{rudd} = x_R Y_{uv}^{rudd}$	$N_{ur}^{rudd} = x_R^2 \cdot Y_{uv}^{rudd}$
$X_{vr}^{rudd} = 2x_R \cdot X_{vv}^{rudd}$	$Y_{ v v}^{rudd} = -\frac{1}{2}\rho A^{rudd} \frac{c_{dc}^{rudd}}{a^{rudd}}$	$N_{ v v}^{rudd} = x_R \cdot Y_{ v v}^{rudd}$
$X_{rr}^{rudd} = x_R^2 \cdot X_{vv}^{rudd}$	$Y_{u^2\delta_R}^{rudd} = \frac{1}{2}\rho A^{rudd} c_l^{rudd}$	$N_{u^2\delta_R}^{rudd} = x_R Y_{u^2\delta_R}^{rudd}$
$X_{u^2\delta_R^2}^{rudd} = -\frac{1}{2}\rho A^{rudd} c_{id}^{rudd}$	–	–

Replacing (3.3.62) on (3.3.56 - 3.3.58) and considering that the dynamic model derived in Section 2.2, expressions (2.2.18 – 2.2.20), disregards the terms $u(v + x_R r)\delta_R$ for the axial force, and the terms containing r^2 , $uv\delta_R$, $ur\delta_R$ or $u^2\delta_R^2$ for the sideslip force and their moment, the components on the $\hat{x}$ and $\hat{y}$ of the lift and drag forces over the rudders and their resulting moment over the vehicle can be expressed as:

$$X^{rudd} = \frac{1}{2}\rho A^{rudd} \left((c_l^{rudd} - c_{id}^{rudd})(v^2 + 2x_R vr + x_R^2 r^2) - c_{d0}^{rudd} u^2 - c_{id}^{rudd} u^2 \delta_R^2 \right) \quad (3.3.63)$$

$$Y^{rudd} = -\frac{1}{2}\rho A^{rudd} \left((c_l^{rudd} + c_{d0}^{rudd})uv + x_R (c_l^{rudd} + c_{d0}^{rudd})ur - c_l^{rudd} u^2 \delta_R + \frac{c_{dc}^{rudd}}{a^{rudd}} |v|v \right) \quad (3.3.64)$$

$$N^{rudd} = -\frac{1}{2}\rho A^{rudd} x_R \left((c_l^{rudd} + c_{d0}^{rudd})uv + x_R (c_l^{rudd} + c_{d0}^{rudd})ur - c_l^{rudd} u^2 \delta_R + \frac{c_{dc}^{rudd}}{a^{rudd}} |v|v \right) \quad (3.3.65)$$

Finally, the hydrodynamic coefficients for the rudders, to a motion on the horizontal plane, can be obtained as seen in Table 3.3.3:

3.4. Hydrodynamic Coefficients of the Hull-Fins Combination

The influence of the hull presence on the hydrodynamic forces acting on the fins can be considered through interference factors. The deduction of the interference factor expressions generally is based on the slender body theory. Moreover, it considers that the interference is with a cylindrical body, small angles of attack, and low aspect ratio of the fins.

A. Interference Factors

Due to the interference factor are derived from potential theory, they are applied only on the linear term of the lift force (c_l^{fin}) and on the induced drag coefficient (c_{id}^{fin}). Such as:

$$c_l^{fh} = K_{fh} \cdot c_l^{fin} \quad ; \quad c_{id}^{fh} = \frac{1}{\pi a^{fin}_\varepsilon} (K_{fh} \cdot c_l^{fin})^2 = K_{fh}^2 \cdot c_{id}^{fin} \quad (3.4.1)$$

The estimation of the influence on the sail due to the hull is based on the added mass formulation for a slender body, such as seen in (de BARROS et al. 2008).

K_{sh} is the interference factor of the sail due to the hull and it can be calculated as:

$$K_{sh} = \frac{\frac{\pi \rho s_{sail}^2}{4} \cdot \left(1 + \frac{R^2}{s_{sail}^2}\right) \cdot \left(-1 - \frac{R^2}{s_{sail}^2} + 2^{3/2} \cdot \sqrt{\left(1 + \frac{R^4}{s_{sail}^4}\right)}\right) - \pi \rho R^2}{\frac{\pi \rho (s_{sail} - R)^2}{2}} \quad (3.4.2)$$

As shown in de Barros et al.(2008), the semi-empirical expression developed by Pitts, Nielsen and Kaattary, (PITTS; NIELSEN; KAATTARI, 1957) can be used to calculate the interference factor between the hull and the control surfaces. For the rudders, the interference factor can be calculated as:

$$K_{rh} = K_{h(r)} + K_{r(h)} = \left(1 + \frac{R}{s_{rudd}}\right)^2 \quad (3.4.3)$$

In which:

$K_{h(r)}$ is the interference factor of the hull due to the rudders.

$K_{r(h)}$ is the interference factor of the rudders due to the hull.

As proposed in Barros et al.(2008), to the deflected term should be considerate $K_{r(h)}$ as interference factor.

Table 3.4.1 - Fins-hull coefficients to motion on the vertical plane.

Coefficients Related to $\hat{\mathbf{x}}$	Coefficients Related to $\hat{\mathbf{z}}$	Coefficients Related to $\hat{\mathbf{y}}$
$X_{ww}^{eh} = \frac{1}{2} \rho A^{elev} (K_{eh} c_l^{elev} - K_{eh}^2 c_{id}^{elev})$	$Z_{uw}^{eh} = -\frac{1}{2} \rho A^{elev} (K_{eh} c_l^{elev} + c_{d0}^{elev})$	$M_{uw}^{eh} = -x_E \cdot Z_{uw}^{eh}$
$X_{wq}^{eh} = -2x_E \cdot X_{ww}^{eh}$	$Z_{uq}^{eh} = -x_E \cdot Z_{uw}^{eh}$	$M_{uq}^{eh} = x_E^2 \cdot Z_{uw}^{eh}$
$X_{qq}^{eh} = x_E^2 \cdot X_{ww}^{eh}$	$Z_{u^2\delta_E}^{eh} = -\frac{1}{2} \rho A^{elev} K_{e(h)} c_l^{elev}$	$M_{u^2\delta_E}^{eh} = -x_E \cdot Z_{u^2\delta_E}^{eh}$
$X_{u^2\delta_E^2}^{eh} = -\frac{1}{2} \rho A^{elev} K_{e(h)}^2 c_{id}^{elev}$	—	—

Due to the symmetry of the vehicle and the cruciform of the rudders and elevators, the same interference factors used on the rudders can be used to the elevators. Such as:

$$K_{eh} = K_{h(e)} + K_{e(h)} = \left(1 + \frac{R}{S_{elev}}\right)^2 \quad (3.4.4)$$

In which:

$K_{h(e)}$ is the interference factor of the hull due to the elevators.

$K_{e(h)}$ is the interference factor of the elevators due to the hull.

Table 3.4.2 - Sail-hull coefficients to motion on the horizontal plane.

Coefficients Related to $\hat{x}$	Coefficients Related to $\hat{y}$	Coefficients Related to $\hat{z}$
$X_{vv}^{sh} = \frac{1}{2} \rho A^{sail} (K_{sh} c_l^{sail} - K_{sh}^2 c_{id}^{sail})$	$Y_{uv}^{sh} = -\frac{1}{2} \rho A^{sail} (K_{sh} c_l^{sail} + c_{d0}^{sail})$	$N_{uv}^{sh} = x_s \cdot Y_{uv}^{sh}$
$X_{vr}^{sh} = 2x_s \cdot X_{vv}^{sh}$	$Y_{ur}^{sh} = x_s \cdot Y_{uv}^{sh}$	$N_{ur}^{sh} = x_s^2 \cdot Y_{uv}^{sh}$
$X_{rr}^{sh} = x_s^2 \cdot X_{vv}^{sh}$	—	—

Table 3.4.3 - Rudders-hull coefficients to motion on the horizontal plane.

Coefficients Related to $\hat{x}$	Coefficients Related to $\hat{y}$	Coefficients Related to $\hat{z}$
$X_{vv}^{rh} = \frac{1}{2} \rho A^{rudd} (K_{rh} c_l^{rudd} - K_{rh}^2 c_{id}^{rudd})$	$Y_{uv}^{rh} = -\frac{1}{2} \rho A^{rudd} (K_{rh} c_l^{rudd} + c_{d0}^{rudd})$	$N_{uv}^{rh} = x_R \cdot Y_{uv}^{rh}$
$X_{vr}^{rh} = 2x_R \cdot X_{vv}^{rh}$	$Y_{ur}^{rh} = x_R \cdot Y_{uv}^{rh}$	$N_{ur}^{rh} = x_R^2 \cdot Y_{uv}^{rh}$
$X_{rr}^{rh} = x_R^2 \cdot X_{vv}^{rh}$	$Y_{u^2 \delta_R}^{r(h)} = \frac{1}{2} \rho A^{rudd} K_{r(h)} c_l^{rudd}$	$N_{u^2 \delta_R}^{rh} = x_R \cdot Y_{u^2 \delta_R}^{rh}$
$X_{u^2 \delta_R}^{rh} = -\frac{1}{2} \rho A^{rudd} K_{r(h)} c_{id}^{rudd}$	—	—

B. Hydrodynamic Coefficients of the Hull-Fins Combination

The resultant force over the hull-fins combination is obtained from the vector sum of the forces over the hull and the fins for each motion plane. Hence, the hydrodynamic coefficients on the horizontal plane can be expressed as the sum of the contributions of the hull, sail and rudders coefficients. Likewise, the hydrodynamic coefficients on the vertical plane can be expressed as the sum of the hull and elevators coefficients. Such as seen in table 3.4.4.

Table 3.4.4 - Hydrodynamics coefficients of the hull-fins combination.

Maneuvers on the Horizontal plane		
$X_{uu} = X_{uu}^{hull} + X_{uu}^{sail} + X_{uu}^{rudd} + X_{uu}^{elev}$	$Y_{uv} = Y_{uv}^{hull} + Y_{uv}^{sh} + Y_{uv}^{rh}$	$N_{uv} = N_{uv}^{hull} + N_{uv}^{sh} + N_{uv}^{rh}$
$X_{vv} = X_{vv}^{hull} + X_{vv}^{sh} + X_{vv}^{rh}$	$Y_{ur} = Y_{ur}^{hull} + Y_{ur}^{sh} + Y_{ur}^{rh}$	$N_{ur} = N_{ur}^{hull} + N_{ur}^{sh} + N_{ur}^{rh}$
$X_{vr} = X_{vr}^{hull} + X_{vr}^{sh} + X_{vr}^{rh}$	$Y_{v v} = Y_{v v}^{hull} + Y_{v v}^{sail} + Y_{v v}^{rudd}$	$N_{v v} = N_{v v}^{hull} + N_{v v}^{sail} + N_{v v}^{rudd}$
$X_{rr} = X_{rr}^{hull} + X_{rr}^{sh} + X_{rr}^{rh}$	$Y_{u^2\delta_R} = Y_{u^2\delta_R}^{r(h)}$	$N_{u^2\delta_R} = N_{u^2\delta_R}^{r(h)}$
$X_{u^2\delta\delta} = X_{u^2\delta_R}^{r(h)}$	—	—
Maneuvers on the Vertical Plane		
$X_{uu} = X_{uu}^{hull} + X_{uu}^{sail} + X_{uu}^{rudd} + X_{uu}^{elev}$	$Z_{uw} = Z_{uw}^{hull} + Z_{uw}^{eh}$	$M_{uw} = M_{uw}^{hull} + M_{uw}^{eh}$
$X_{ww} = X_{ww}^{hull} + X_{ww}^{eh}$	$Z_{uq} = Z_{uq}^{hull} + Z_{uq}^{eh}$	$M_{uq} = M_{uq}^{hull} + M_{uq}^{eh}$
$X_{wq} = X_{wq}^{hull} + X_{wq}^{eh}$	$Z_{w w} = Z_{w w}^{hull} + Z_{w w}^{elev}$	$M_{w w} = M_{w w}^{hull} + M_{w w}^{elev}$
$X_{qq} = X_{qq}^{hull} + X_{qq}^{eh}$	$Z_{u^2\delta_E} = Z_{u^2\delta_E}^{e(h)}$	$M_{u^2\delta_E} = M_{u^2\delta_E}^{e(h)}$
$X_{u^2\delta\delta} = X_{u^2\delta_E}^{e(h)}$	—	—

3.5. Propeller Thrust Coefficients

The classical semi-empirical expression for the thrust force, as seen in (Lewis, 1988) is adopted:

$$X^{th} = \rho D_{sc}^4 \eta^2 C_{th}(J) \quad (3.5.1)$$

where:

C_{th} is the propeller's thrust coefficient;

$J = \frac{u}{D_{sc}|\eta|}$ is the advance rate;

D_{sc} is the screw diameter;

and η is the screw rotation rate.

The relation between the thrust coefficient (C_{th}) and the advance ratio (J) depends on the kind of screw, it is commonly depicted in screw-curves data. Once set a vehicle's operation

range, can be adjusted the screw-curve $C_{th}(J)$ in this range to a linear function. Therefore, the thrust coefficient can be expressed as:

$$C_{th}(J) = a_{sc} + b_{sc} \cdot J = a_{sc} + b_{sc} \left(\frac{u}{D_{sc}|\eta|} \right) \quad (3.5.2)$$

Substituting the thrust coefficient expression (3.5.2) into the thrust force expression (3.5.1) results in:

$$X^{th} = \rho D_h^4 a_{sc} \eta^2 + \rho D_h^3 b_{sc} u \eta \quad (3.5.3)$$

Hence, the hydrodynamic coefficients of the propeller can be calculated as seen in Table 3.5.1.

Table 3.5.1 - Thrust coefficients

Thrust Coefficients
$X_{u\eta} = b_{sc} \rho D_{sc}^3$
$X_{\eta\eta} = a_{sc} \rho D_{sc}^4$

4. Model Parameters Identification of an AUV

In this study, the relevant added-mass and propeller coefficients are identified as well as the lift and drag related coefficients. The “parallel processing” is adopted, while the “over and under” identification technique is replaced by the preliminary analytical estimations of all parameters, using physical and geometry knowledge (fluid mechanics, slender body theory, lift and drag semi-empirical expressions). These estimates are then finely adjusted to experimental data by the EKF or the UKF estimators. The relative performances of both filters are compared. A physical insight of accuracy of the analytical methods allows a spiral identification process, holding invariant the most reliable model parameters in an estimation cycle and fitting the less reliable model parameters. In the next identification cycle, the non-varied model parameters are tuned. Finally, all the coefficients are estimated simultaneously by the state estimator. The next sections present the EKF and UKF algorithms applied to the AUV parameter estimation. Then, the identification method proposed in this work is presented in details.

4.1. Structure of a Model Parameters Estimator

Let be a nonlinear system expressed as:

$$\dot{\mathbf{x}} = \mathbf{f}(\boldsymbol{\Theta}, \mathbf{x}, \mathbf{u}) + \mathbf{n}_f \quad ; \quad \mathbf{y} = \mathbf{h}(\boldsymbol{\Theta}, \mathbf{x}, \mathbf{u}) + \mathbf{n}_h \quad (4.1.1)$$

Where:

$\boldsymbol{\Theta}$ is the parameters vector composed by " p " model parameters.

$\mathbf{x}$ is the simple state vector composed by " n " movement variables, and it is modeled as a random variables vector with $\mathbf{P}$ covariance matrix;

$\mathbf{u}$ is the system input vector composed by " l " known variables (control inputs and sensor measurements);

$\mathbf{n}_f$ is the process noise vector which represents the disturbances in the dynamic system. It is considered one noise variable for each dynamic equation (n) and all the noise variables are modeled as independent Gaussian random variables with zero-means and covariance matrix $\mathbf{Q}_f$;

$\mathbf{f}(\boldsymbol{\Theta}, \mathbf{x}, \mathbf{u})$ is the vector of nonlinear functions representing the system's dynamic model.

$\mathbf{y}$ is the measurements vector composed by " m " sensor measurements;

$\mathbf{n}_h$ is the measurements noise vector composed by " l " random variables. These noises are considerate as white noise and are modeled as independent Gaussian random variables with zero-means and covariance matrix " $\mathbf{R}$ ";

And $\mathbf{h}(\boldsymbol{\Theta}, \mathbf{x}, \mathbf{u})$ is the nonlinear function representing the system's measurements model.

The nonlinear states-space system, for an AUV moving on the steering plane, can be decoupled into Surge and Yaw-Sway dynamics to identification purposes.

A. The Surge System

This state-space system is obtained from the expressions (2.2.21, 2.2.24 and 2.3.10) corresponding to the vehicle surge dynamics, and the measurement equation for the surge velocity.

The parameters, state variables, inputs and measurements vectors are:

$$\boldsymbol{\Theta} = \begin{bmatrix} m - X_{\dot{u}} \\ X_{uu} \\ X_{vv} \\ m + X_{vr} \\ mx_G + X_{rr} \\ X_{u^2\delta\delta} \\ X_{u\eta} \\ X_{\eta\eta} \end{bmatrix} ; \quad \mathbf{x} = [u] \quad ; \quad \mathbf{u} = \begin{bmatrix} v \\ r \\ \delta_R \\ \delta_E \\ \eta \end{bmatrix} ; \quad \mathbf{y} = u^{dvl} + y_D \cdot r^{gyro} \quad (4.1.2)$$

The nonlinear functions modeling the dynamics and the measurements are:

$$\mathbf{f}(\boldsymbol{\Theta}, \mathbf{x}, \mathbf{u}) = \frac{1}{(m - X_{\dot{u}})} \cdot f_X \quad ; \quad \mathbf{h}(\boldsymbol{\Theta}, \mathbf{x}, \mathbf{u}) = u \quad (4.1.3)$$

In which:

$$f_X = X_{uu}u^2 + X_{vv}v^2 + (m + X_{vr})vr + (mx_G + X_{rr})r^2 + X_{u^2\delta\delta}u^2(\delta_R^2 + \delta_E^2) + X_{u\eta}u\eta + X_{\eta\eta}\eta^2 \quad (4.1.4)$$

The process and measurement noise vectors are:

$$\mathbf{n}_f = n_{\dot{u}} \quad ; \quad \mathbf{n}_y = n_X^{dvl} + y_D \cdot n_Z^{gyro} = [1 \quad y_D] \begin{bmatrix} n_X^{dvl} \\ n_Z^{gyro} \end{bmatrix} \quad (4.1.5)$$

The covariance matrices of the process and measurements noise vectors are:

$$\mathbf{Q}_f = \text{var}(n_{\dot{u}}) = (\sigma^{\dot{u}})^2 \quad ; \quad \mathbf{R} = \text{cov}(\mathbf{n}_y) = [1 \quad y_D] \begin{bmatrix} (\sigma_X^{dvl})^2 & 0 \\ 0 & (\sigma_Z^{gyro})^2 \end{bmatrix} [1 \quad y_D]^T \quad (4.1.6)$$

B. The Yaw-Sway System

This state-space system is obtained from the expressions (2.2.22, 2.2.23, 2.2.25, 2.2.26, 2.3.3 and 2.3.11) corresponding to the equations of motion and those ones representing the measurement of the yaw and sway velocities. The parameters, state variables, inputs and measures vectors are:

$$\Theta = \begin{bmatrix} m - Y_{\dot{v}} \\ Y_{\dot{r}} - mx_G \\ Y_{uv} \\ Y_{ur} - m \\ Y_{v|v|} \\ Y_{u^2\delta_R} \\ N_{\dot{v}} - mx_G \\ I_{zz} - N_{\dot{r}} \\ N_{uv} \\ N_{ur} - mx_G \\ N_{v|v|} \\ N_{u^2\delta_R} \end{bmatrix} \quad ; \quad \mathbf{x} = \begin{bmatrix} v \\ r \end{bmatrix} \quad ; \quad \mathbf{u} = \begin{bmatrix} u \\ \delta_R \end{bmatrix} \quad ; \quad \mathbf{y} = \begin{bmatrix} v^{dvl} - x_D r^{gyro} \\ r^{gyro} \end{bmatrix} \quad (4.1.7)$$

The nonlinear vector function modeling the dynamic is:

$$\mathbf{f}(\Theta, \mathbf{x}, \mathbf{u}) = \begin{bmatrix} \frac{(I_{zz} - N_{\dot{r}})}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)} \cdot f_Y + \frac{(Y_{\dot{r}} - mx_G)}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)} \cdot f_N \\ \frac{(N_{\dot{v}} - mx_G)}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)} \cdot f_Y + \frac{(m - Y_{\dot{v}})}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)} \cdot f_N \end{bmatrix} \quad (4.1.8)$$

In which:

$$f_Y = Y_{uv}uv + (Y_{ur} - m)ur + Y_{v|v}|v|v| + Y_{u^2\delta_R}u^2\delta_R \quad (4.1.9)$$

$$f_N = N_{uv}uv + (N_{ur} - mx_G)ur + N_{v|v}|v|v| + N_{u^2\delta_R}u^2\delta_R \quad (4.1.10)$$

The process noise vector and its covariance matrix are:

$$\mathbf{n}_f = \begin{bmatrix} n_{\dot{v}} \\ n_{\dot{r}} \end{bmatrix} \quad ; \quad \mathbf{Q}_f = \text{cov}(\mathbf{n}_f) = \begin{bmatrix} \sigma_{\dot{v}}^2 & 0 \\ 0 & \sigma_{\dot{r}}^2 \end{bmatrix} \quad (4.1.11)$$

The nonlinear vector function modeling the measurements is:

$$\mathbf{h} = \begin{bmatrix} v \\ r \end{bmatrix} \quad (4.1.12)$$

The measurements noise vector and its covariance matrix are:

$$\mathbf{n}_y = \begin{bmatrix} n_Y^{dvl} - x_D n_Z^{gyro} \\ n_Z^{gyro} \end{bmatrix} = \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} n_Y^{dvl} \\ n_Z^{gyro} \end{bmatrix} \quad ; \quad \mathbf{R} = \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} (\sigma_Y^{dvl})^2 & 0 \\ 0 & (\sigma_Z^{gyro})^2 \end{bmatrix} \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix}^T \quad (4.13)$$

In order to identify the parameters of a dynamic system, using a state estimator, it is necessary to add the model parameters to the state variables in such a way that an augmented state system is built. Moreover, if it is required to process multiples maneuver data, the complete set of state variables for each maneuver should be added to the augmented state vector.

Therefore, a nonlinear system expressed as an augmented state system, which includes “ k ” maneuvers to be processed, is expressed such as:

$$\dot{\mathbf{x}}_a = \mathbf{f}_a(\mathbf{x}_a, \mathbf{u}_a) + \mathbf{n}_{fa} \quad ; \quad \mathbf{y}_a = \mathbf{h}_a(\mathbf{x}_a, \mathbf{u}_a) + \mathbf{n}_{ha} \quad (4.14)$$

Where:

$\mathbf{x}_a$ is the augmented state vector composed by model parameters and the motion variables vector of each proceeded maneuver;

$\mathbf{u}_a$ is the augmented system inputs vector composed by the system inputs vectors of each maneuver;

$\mathbf{y}_a$ is the augmented measurements vector composed by the measurements vectors of each maneuver;

$$x_a = \begin{bmatrix} \Theta \\ x_1 \\ x_2 \\ \vdots \\ x_k \end{bmatrix} ; \quad u_a = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_k \end{bmatrix} ; \quad n_{fa} = \begin{bmatrix} n_{\dot{\Theta}} \\ n_{f1} \\ n_{f2} \\ \vdots \\ n_{fk} \end{bmatrix} ; \quad y_a = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} ; \quad n_{ya} = \begin{bmatrix} n_{y1} \\ n_{y2} \\ \vdots \\ n_{yk} \end{bmatrix} \quad (4.1.15)$$

f_a and h_a are the augmented-nonlinear functions of the dynamic and measurements model and they are expressed as

$$f_a(x_a, u_a) = \begin{bmatrix} \mathbf{0}_{p,1} \\ f(\Theta, x_1, u_1) \\ \vdots \\ f(\Theta, x_k, u_k) \end{bmatrix} ; \quad h_a(x_a, u_a) = \begin{bmatrix} h(\Theta, x_1, u_1) \\ \vdots \\ h(\Theta, x_k, u_k) \end{bmatrix} \quad (4.1.16)$$

$\mathbf{0}_{(p,1)}$ is a null vector composed by "p" zeros. It represents a zero order dynamics due to the model parameters being constant in the time;

$n_{\dot{\Theta}}$ is the parameters noise vector which represents the uncertainty in the values of the parameters. This noise vector is composed by independent random variables with zero-mean and a covariance matrix Q_p expressed as:

$$Q_p = \text{cov}(n_{\dot{\Theta}}) = \text{Diag}((S * \Theta_0)^2) \quad (4.1.17)$$

In which, S is a vector containing tuning factors for each parameter. They are manipulated by the user until a stable convergence is achieved in the estimated parameters.

Q_a and R_a are the process and measurement noise covariance matrices respectively of the augmented state system. They are given by:

$$Q_a = \begin{bmatrix} Q_p & \mathbf{0}_{(p,n)} & \cdots & \mathbf{0}_{(p,n)} \\ \mathbf{0}_{(n,p)} & Q_{f1} & \cdots & \mathbf{0}_{(n,n)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0}_{(n,n)} & \cdots & Q_{fk} \end{bmatrix} ; \quad R_a = \begin{bmatrix} R & \mathbf{0}_{(l,l)} & \cdots & \mathbf{0}_{[l,l]} \\ \mathbf{0}_{(l,l)} & R & \cdots & \mathbf{0}_{l,l} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{(l,l)} & \mathbf{0}_{(l,l)} & \cdots & R \end{bmatrix} \quad (4.1.18)$$

The covariance matrix of the initial states is expressed as:

$$(P_a)_0 = \text{cov}(x_a(0)) = \begin{bmatrix} P_{\Theta}(0) & [\mathbf{0}]_{p,n} & \cdots & \mathbf{0}_{(p,n)} \\ \mathbf{0}_{(N,P)} & P_1(0) & \cdots & \mathbf{0}_{(n,n)} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{(n,p)} & \mathbf{0}_{(n,n)} & \cdots & P_k(0) \end{bmatrix} \quad (4.1.19)$$

4.2. The Extended Kalman Filter

The extended Kalman filter (EKF) is the nonlinear version of the original Kalman Filter and it was originally proposed by Stanley Schmidt to deal with the spacecraft navigation problem (BELLANTONI; DODGE, 1967). It is the most widely applied state estimation algorithm for a nonlinear system (SIMON, 2006), due to its effective performance and a relatively low computational cost. The extended Kalman filter is based on the linearization of the nonlinear functions of the dynamic and measurements model ($\mathbf{f}$ e $\mathbf{h}$) on each iteration step. A first-order-discrete algorithm of the EKF, on the stabilized form, is shown in Table 4.2.1. A detailed explication of such algorithm can be found in Simon (SIMON, 2006).

Table 4.2.1- The Extended Kalman filter algorithm for discrete systems

<p>Initialization:</p> $\mathbf{x}_i^- = \hat{\mathbf{x}}_0 \quad ; \quad \mathbf{P}_i^- = \mathbf{P}_0$
<p>Updating Phase:</p> $\mathbf{K}_i = \mathbf{P}_i^- \mathbf{H}_i^T \cdot (\mathbf{H}_i \mathbf{P}_i^- \mathbf{H}_i^T + \mathbf{R})^{-1}$ $\mathbf{x}_i^+ = \mathbf{x}_i^- + \mathbf{K}_i \cdot (\mathbf{y}_i - \mathbf{h}(\mathbf{x}_i^-, \mathbf{u}_i))$ $\mathbf{P}_i^+ = (\mathbf{I} - \mathbf{K}_i \mathbf{H}_i) \cdot \mathbf{P}_i^- \cdot (\mathbf{I} - \mathbf{K}_i \mathbf{H}_i)^T + \mathbf{K}_i \mathbf{R} \mathbf{K}_i^T$
<p>Prediction Phase:</p> $\mathbf{x}_{i+1}^- = \mathbf{x}_i^+ + T \mathbf{f}(\mathbf{x}_i^+, \mathbf{u}_i)$ $\mathbf{P}_{i+1}^- = (\mathbf{I} + T \mathbf{F}_i) \cdot \mathbf{P}_i^+ \cdot (\mathbf{I} + T \mathbf{F}_i)^T + T^2 \mathbf{Q}$

In order to simplify the calculations of the Jacobian matrices ($\mathbf{F}$ and $\mathbf{H}$), in this work it is proposed a change of variables, replacing the added mass coefficients expression by the formulas containing the m_{ij} terms, such as seen in Table 4.2.2. In order to calculate the

original added-mass coefficients, each m_{ij} term should be multiplied with the following factor:

$$m_{21}m_{32} - m_{22}m_{31} = \frac{1}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)} \quad (4.2.1)$$

Table 4.2.2 - Change of variable of the added mass coefficients

$m_{11} = \frac{1}{(m - X_{\dot{u}})}$	$(m - X_{\dot{u}}) = \frac{1}{m_{11}}$
$m_{21} = \frac{(I_{zz} - N_{\dot{r}})}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)}$	$(I_{zz} - N_{\dot{r}}) = \frac{m_{21}}{(m_{21}m_{32} - m_{22}m_{31})}$
$m_{22} = \frac{(Y_{\dot{r}} - mx_G)}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)}$	$(Y_{\dot{r}} - mx_G) = \frac{m_{22}}{(m_{21}m_{32} - m_{22}m_{31})}$
$m_{31} = \frac{(N_{\dot{v}} - mx_G)}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)}$	$(N_{\dot{v}} - mx_G) = \frac{m_{31}}{(m_{21}m_{32} - m_{22}m_{31})}$
$m_{32} = \frac{(m - Y_{\dot{v}})}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)(N_{\dot{v}} - mx_G)}$	$(m - Y_{\dot{v}}) = \frac{m_{32}}{(m_{21}m_{32} - m_{22}m_{31})}$

Considering these m_{ij} terms, the surge and yaw-sway modeling in subsections (3.2.A) and (3.2.B) are modified to the expressions presented as follows:

C. The EKF Surge system

The parameters, state variables, inputs and measurement vectors are given by:

$$\Theta = \begin{bmatrix} m_{11} \\ X_{uu} \\ X_{vv} \\ m + X_{vr} \\ mx_G + X_{rr} \\ X_{u^2\delta\delta} \\ X_{u\eta} \\ X_{\eta\eta} \end{bmatrix} ; \quad \mathbf{x} = \mathbf{u} \quad ; \quad \mathbf{u} = \begin{bmatrix} v \\ r \\ \delta_R \\ \delta_E \\ \eta \end{bmatrix} ; \quad \mathbf{y} = \mathbf{u}^{dvl} + \mathbf{y}_D \cdot r^{gyro} \quad (4.2.2)$$

The nonlinear function describing the surge dynamics is:

$$f(\Theta, x, u) = m_{11} \cdot f_X \quad (4.2.3)$$

In which:

$$f_X = X_{uu}u^2 + X_{vv}v^2 + (m + X_{vr})vr + (mx_G + X_{rr})r^2 + X_{u^2\delta\delta}u^2(\delta_R^2 + \delta_E^2) + X_{u\eta}u\eta + X_{\eta\eta}\eta^2 \quad (4.2.4)$$

The Jacobian matrices of the nonlinear function f are given by:

$$\frac{\partial f}{\partial \Theta} = [f_X \quad m_{11}u^2 \quad m_{11}v^2 \quad m_{11}vr \quad m_{11}r^2 \quad m_{11}u^2(\delta_R^2 + \delta_E^2) \quad m_{11}u\eta \quad m_{11}\eta^2] \quad (4.2.5)$$

$$\frac{\partial f}{\partial x} = m_{11} \frac{\partial f_X}{\partial u} = m_{11}(2X_{uu}u + 2X_{u^2\delta\delta}u \cdot (\delta_R^2 + \delta_E^2) + X_{u\eta}\eta) \quad (4.2.6)$$

The nonlinear functions modeling the measurements and the Jacobians matrices of it are:

$$h(\Theta, x, u) = u \quad ; \quad \frac{\partial h}{\partial \Theta} = \mathbf{0}_{(1,p)} \quad ; \quad \frac{\partial h}{\partial x} = 1 \quad (4.2.7)$$

The process and measurements noise vectors are:

$$\mathbf{n}_f = n_{\dot{u}} \quad ; \quad \mathbf{n}_y = n_X^{dvl} + y_D \cdot n_Z^{gyro} = [1 \quad y_D] \begin{bmatrix} n_X^{dvl} \\ n_Z^{gyro} \end{bmatrix} \quad (4.2.8)$$

The covariance matrices of the process and measurements noise vectors are:

$$\mathbf{Q}_f = cov(\mathbf{n}_f) = \sigma_{\dot{u}}^2 \quad ; \quad \mathbf{R} = cov(\mathbf{n}_y) = [1 \quad y_D] \begin{bmatrix} (\sigma_X^{dvl})^2 & 0 \\ 0 & (\sigma_Z^{gyro})^2 \end{bmatrix} [1 \quad y_D]^T \quad (4.2.9)$$

The covariance matrices of the parameters are:

$$\mathbf{P}_{p0} = Diag((\mathbf{S}_o * \Theta_0)^2) \quad ; \quad \mathbf{Q}_{\Theta} = Diag((\mathbf{S} * \Theta_0)^2) \quad (4.2.10)$$

In which $\mathbf{S}_o$ and $\mathbf{S}$ are the adjust parameters of the filter.

D. The EKF Yaw-Sway system

The parameters, state variables, inputs and measures vectors are:

$$\Theta = \begin{bmatrix} m_{32} \\ m_{22} \\ Y_{uv} \\ Y_{ur} - m \\ Y_{v|v|} \\ Y_{u^2\delta_r} \\ m_{31} \\ m_{21} \\ N_{uv} \\ N_{ur} - mx_G \\ N_{v|v|} \\ N_{u^2\delta_r} \end{bmatrix} ; \quad \mathbf{x} = \begin{bmatrix} v \\ r \end{bmatrix} ; \quad \mathbf{u} = \begin{bmatrix} u \\ \delta_r \end{bmatrix} ; \quad \mathbf{y} = \begin{bmatrix} v^{dvl} - x_D r^{gyro} \\ r^{gyro} \end{bmatrix} \quad (4.2.11)$$

The nonlinear function of the dynamic model is:

$$\mathbf{f}(\Theta, \mathbf{x}, \mathbf{u}) = \begin{bmatrix} m_{21} \cdot f_Y + m_{22} \cdot f_N \\ m_{31} \cdot f_Y + m_{32} \cdot f_N \end{bmatrix} \quad (4.2.12)$$

In which:

$$f_Y = Y_{uv}uv + (Y_{ur} - m)ur + Y_{v|v|}v|v| + Y_{u^2\delta_R}u^2\delta_R \quad (4.2.13)$$

$$f_N = N_{uv}uv + (N_{ur} - mx_G)ur + N_{v|v|}v|v| + N_{u^2\delta_R}u^2\delta_R \quad (4.2.14)$$

The Jacobians matrices of the nonlinear function $\mathbf{f}$ are:

$$\frac{\partial \mathbf{f}}{\partial \Theta} = \begin{bmatrix} 0 & f_N & m_{21}uv & m_{21}ur & m_{21}v|v| & m_{21}u^2\delta_R & 0 & f_Y & m_{22}uv & m_{22}ur & m_{22}v|v| & m_{22}u^2\delta_R \\ f_N & 0 & m_{31}uv & m_{31}ur & m_{31}v|v| & m_{31}u^2\delta_R & f_Y & 0 & m_{32}uv & m_{32}ur & m_{32}v|v| & m_{32}u^2\delta_R \end{bmatrix} \quad (4.2.15)$$

and

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} m_{21}(Y_{uv}u + 2Y_{v|v|}|v|) + m_{22}(N_{uv}u + 2N_{v|v|}|v|) & m_{21}(Y_{ur} - m)u + m_{22}(N_{ur} - mx_G)u \\ m_{31}(Y_{uv}u + 2Y_{v|v|}|v|) + m_{32}(N_{uv}u + 2N_{v|v|}|v|) & m_{31}(Y_{ur} - m)u + m_{32}(N_{ur} - mx_G)u \end{bmatrix} \quad (4.2.16)$$

The nonlinear functions modeling the measurements and their Jacobians matrices of it are:

$$\mathbf{h} = \begin{bmatrix} v \\ r \end{bmatrix} ; \quad \frac{\partial \mathbf{h}}{\partial \Theta} = \mathbf{0}_{(m,p)} ; \quad \frac{\partial \mathbf{h}}{\partial \mathbf{x}} = \mathbf{I}_{(n)} ; \quad \frac{\partial \mathbf{h}}{\partial \mathbf{n}_y} = \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix} \quad (4.2.17)$$

The process noise vector and its covariance matrix are:

$$\mathbf{n}_f = \begin{bmatrix} n_{\dot{v}} \\ n_{\dot{r}} \end{bmatrix} ; \quad \mathbf{Q}_f = \begin{bmatrix} \sigma_{\dot{v}}^2 & 0 \\ 0 & \sigma_{\dot{r}}^2 \end{bmatrix} \quad (4.2.18)$$

The measurements noise vector and its covariance matrix are:

$$\mathbf{n}_y = \begin{bmatrix} n_Y^{dvl} - x_D n_Z^{gyro} \\ n_Z^{gyro} \end{bmatrix} = \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} n_Y^{dvl} \\ n_Z^{gyro} \end{bmatrix} ; \quad \mathbf{R} = \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} (\sigma_Y^{dvl})^2 & 0 \\ 0 & (\sigma_Z^{gyro})^2 \end{bmatrix} \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix}^T \quad (4.2.19)$$

The covariance matrices of the parameters are:

$$\mathbf{P}_{p0} = \text{Diag}((\mathbf{S}_o * \boldsymbol{\Theta}_0)^2) \quad ; \quad \mathbf{Q}_p = \text{Diag}((\mathbf{S} * \boldsymbol{\Theta}_0)^2) \quad (4.2.20)$$

In which $\mathbf{S}_o$ and $\mathbf{S}$ are the tuning factors of the filter.

Finally, the Jacobian matrices of the augmented nonlinear functions to " k " maneuvers

$(\mathbf{F}_a, \mathbf{H}_a)$ can be expressed as in Table 4.2.3;

Table 4.2.3- Jacobian Matrices of the nonlinear model functions

$\mathbf{F}_a = \frac{\partial \mathbf{f}_a}{\partial \mathbf{x}_a} =$	$\mathbf{H}_a = \frac{\partial \mathbf{h}_a}{\partial \mathbf{x}_a} =$
$\begin{bmatrix} \mathbf{O}_{p,p} & \mathbf{O}_{(p,n)} & \mathbf{O}_{p,n} & \cdots & \mathbf{O}_{(p,n)} \\ \left. \frac{\partial \mathbf{f}}{\partial \boldsymbol{\Theta}} \right _{(\boldsymbol{\Theta}, x_1, u_1)} & \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right _{(\boldsymbol{\Theta}, x_1, u_1)} & \mathbf{O}_{(n,n)} & \cdots & \mathbf{O}_{(n,n)} \\ \left. \frac{\partial \mathbf{f}}{\partial \boldsymbol{\Theta}} \right _{(\boldsymbol{\Theta}, x_2, u_2)} & \mathbf{O}_{(n,n)} & \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right _{(\boldsymbol{\Theta}, x_2, u_2)} & \cdots & \mathbf{O}_{(n,n)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \left. \frac{\partial \mathbf{f}}{\partial \boldsymbol{\Theta}} \right _{(\boldsymbol{\Theta}, x_k, u_k)} & \mathbf{O}_{(n,n)} & \mathbf{O}_{(n,n)} & \cdots & \left. \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \right _{(\boldsymbol{\Theta}, x_k, u_k)} \end{bmatrix}$	$\begin{bmatrix} \left. \frac{\partial \mathbf{h}}{\partial \boldsymbol{\Theta}} \right _{(\boldsymbol{\Theta}, x_1, u_1)} & \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right _{(\boldsymbol{\Theta}, x_1, u_1)} & \mathbf{O}_{(m,n)} & \cdots & \mathbf{O}_{(m,n)} \\ \left. \frac{\partial \mathbf{h}}{\partial \boldsymbol{\Theta}} \right _{(\boldsymbol{\Theta}, x_2, u_2)} & \mathbf{O}_{(m,n)} & \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right _{(\boldsymbol{\Theta}, x_2, u_2)} & \cdots & \mathbf{O}_{(m,n)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \left. \frac{\partial \mathbf{h}}{\partial \boldsymbol{\Theta}} \right _{(\boldsymbol{\Theta}, x_k, u_k)} & \mathbf{O}_{(m,n)} & \mathbf{O}_{(m,n)} & \cdots & \left. \frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right _{(\boldsymbol{\Theta}, x_k, u_k)} \end{bmatrix}$

$\mathbf{O}_{(i,j)}$ is a null matrix with " i " files and " j " columns of zeros;

4.3. The Unscented Kalman Filter

The EKF is the most widely applied algorithm for nonlinear systems. However, it could display poor accuracy estimations if the nonlinearities on the system are strong. It due to, the EKF linearizes the nonlinear functions on each step of the iteration. Another drawback in this estimator is the need of calculate the Jacobian matrix of the nonlinear functions.

The unscented Kalman filter (UKF) is an algorithm developed to reduce the linearization errors of the EKF, as originally presented by Julier et al. (2000). The principal insight of the UKF is that, from a known mean of the state vector and their known covariance matrix $(\bar{\mathbf{x}}, \mathbf{P})$; it is possible to find a set of deterministic vectors called sigma points whose ensemble mean and covariance are equal to $(\bar{\mathbf{x}}, \mathbf{P})$. These sigma point vectors are propagated

from the mean state vector one sigma on the positive and negative directions for each state in the state space; then it is applied the nonlinear function ($\mathbf{y} = \mathbf{h}(\mathbf{x})$) to each deterministic vector in order to achieve the transformed vectors. The ensemble mean and covariance of these transformed vectors give a good estimation of the true mean and covariance of $\mathbf{y}$, equivalent to the third order of the Taylor series. A first-order-discrete algorithm of the UKF is shown in Table 4.2.4. and a detailed explication of it can be found in Simon (2006).

Table 4.2.4-The unscented Kalman filter algorithm for discrete systems.

<p>Initialization:</p> $\mathbf{x}_i^- = \hat{\mathbf{x}}_0 \quad ; \quad \mathbf{P}_i^- = \mathbf{P}_0$
<p>Updating Phase:</p> $\mathbf{x}_i^k = \begin{cases} \mathbf{x}_i^- + \left(\sqrt{n\mathbf{P}_i^-} \right)_{[k,:]}^T & \forall k = 1, \dots, n \\ \mathbf{x}_i^- - \left(\sqrt{n\mathbf{P}_i^-} \right)_{[k-n,:]}^T & \forall k = n+1, \dots, 2n \end{cases}$ $\mathbf{y}_i^- = \frac{1}{2n} \sum_{k=1}^{2n} \mathbf{y}_i^k \quad ; \quad \mathbf{y}_i^k = \mathbf{h}(\mathbf{x}_i^k, \mathbf{u}_i) \quad \forall k = 1, \dots, 2n$ $\mathbf{P}_{yy} = \frac{1}{2n} \sum_{k=1}^{2n} (\mathbf{y}_i^k - \mathbf{y}_i^-)(\mathbf{y}_i^k - \mathbf{y}_i^-)^T$ $\mathbf{P}_{xy} = \frac{1}{2n} \sum_{k=1}^{2n} (\mathbf{x}_i^k - \mathbf{x}_i^-)(\mathbf{y}_i^k - \mathbf{y}_i^-)^T$ $\mathbf{K}_i = \mathbf{P}_{xy}(\mathbf{P}_{yy} + \mathbf{R})^{-1}$ $\mathbf{x}_i^+ = \mathbf{x}_i^- + \mathbf{K}_i(\mathbf{y}_i - \mathbf{y}_i^-) \quad ; \quad \mathbf{P}_i^+ = \mathbf{P}_i^- - \mathbf{K}_i(\mathbf{P}_{yy} + \mathbf{R})\mathbf{K}_i^T$

Prediction Phase:

$$\mathbf{x}_i^k = \begin{cases} \mathbf{x}_i^+ + \left(\sqrt{n\mathbf{P}_i^+} \right)_{[k,:]}^T & \forall k = 1, \dots, n \\ \mathbf{x}_i^+ - \left(\sqrt{n\mathbf{P}_i^+} \right)_{[k-n,:]}^T & \forall k = n + 1, \dots, 2n \end{cases}$$

$$\mathbf{x}_{i+1}^- = \frac{1}{2n} \sum_{k=1}^{2n} \mathbf{x}_{i+1}^k \quad ; \quad \mathbf{x}_{i+1}^k = \mathbf{x}_i^k + T \cdot \mathbf{f}(\mathbf{x}_i^k, \mathbf{u}_i)$$

$$\forall k = 1, \dots, 2n$$

$$\mathbf{P}_{i+1}^- = \frac{1}{2n} \sum_{k=1}^{2n} (\mathbf{x}_{i+1}^k - \mathbf{x}_{i+1}^-) \cdot (\mathbf{x}_{i+1}^k - \mathbf{x}_{i+1}^-)^T + T^2 \mathbf{Q}$$

The surge and yaw-sway systems used in the UKF estimator are the same as defined in section 4.1.

4.4. A New Method to Identify the Hydrodynamic Coefficients

The identification method proposed in this work was based on Abkowitz (1980) and it was improved using ASE approximations of the parameters in order to deal with the over-parametrization problem. The insight of this method is the parameter identification on a spiral process. That is, the coefficients are gradually estimated according to the maneuver type and the confidence level of their initial values, decreasing the probability of mutual divergence between two or more coefficients. The method is presented as follows.

A. Identification According to the Maneuver type

This technique groups the hydrodynamic coefficients according to a specific type of maneuver in which they have a relevant effect. In this way, only the minimal quantity of coefficients are identified by each type of maneuver, such as seen on tables 4.4.1 and 4.4.2. For the straight-line maneuvers, such as stepped thrust, the coefficients related to surge

dynamics $(X_{\dot{u}}, X_{uu}, X_{u\eta}, X_{\eta\eta})$ are identified. Turning and zigzag maneuvers are used to identify the remaining coefficients of the surge dynamics $(X_{vv}, (m + X_{vr}), (mx_G + X_{rr}), X_{u^2\delta\delta})$. For the Yaw-Sway dynamics, the turning and zigzag maneuvers are adopted since these motions are excited by rudder deflections.

The turning maneuver is a motion response of the vehicle when the rudder is held on a deflected angle different to zero and the vehicle achieves a stationary turning motion. Hence, the lift, drag and rudder forces are compensated for each other; such that, only centripetal acceleration is experimented by the vehicle. In this kind of maneuver the added mass coefficients don't have any effect. Hence, only the coefficients corresponding to the lift, drag and control forces $(Y_{uv}, (Y_{ur} - m), Y_{|v|v}, Y_{u^2\delta_R}, N_{uv}, (N_{ur} - mx_G), N_{|v|v}, N_{u^2\delta_R})$ were identified with the turning maneuvers.

The zigzag maneuver is a dynamic response of the vehicle when the rudder deflection is alternated between positive and negative angles. With this type of maneuver were identified the remained added-mass coefficients $(m - Y_{\dot{v}}), (Y_{\dot{r}} - mx_G), (N_{\dot{v}} - mx_G)$ and $(I_{zz} - N_{\dot{r}})$ or with the EKF estimator would be $(m_{21}, m_{22}, m_{31}, m_{32})$.

Moreover, the turning maneuvers can be subdivided into soft and sharp turning, that have different hydrodynamic characteristics. The soft turnings are produced by small rudder deflections (when the lift force is linear with the incidence flow angle). Hence, the coefficients corresponding to the linear part of the lift efforts can be identified $(Y_{uv}, (Y_{ur} - m), Y_{u^2\delta_R}, N_{uv}, (N_{ur} - mx_G), N_{u^2\delta_R})$. The sharp turnings are produced by high deflection of the rudders (when the lift force is nonlinear with the incidence flow angle), Hence, with this type of maneuver, it is possible to identify the coefficients related to the linear and nonlinear range of the lift efforts $(Y_{uv}, (Y_{ur} - m), Y_{|v|v}, Y_{u^2\delta_R}, N_{uv}, (N_{ur} - mx_G), N_{|v|v}, N_{u^2\delta_R})$.

Table 4.4.1 - Surge coefficients identified according to the type of maneuver.

	Step	Maneuver type	Coefficients to be adjusted
Rough adjustment	1°	Straight line and constant thrust	X_{uu}
	2°	Straight line and constant thrust	$X_{uu}, X_{u\eta}, X_{\eta\eta}$
	3°	Straight line and stepped thrust	$X_{\dot{u}}$
	4°	Turnings and Zigzags	$X_{u^2\delta\delta}$
	5°	Turnings and Zigzags	$X_{vv}, (m + X_{vr}), (mx_G + X_{rr}), X_{u^2\delta\delta}$
Fine adjustment	6°	Straight line and constant thrust	$X_{uu}, X_{u\eta}, X_{\eta\eta}$
	7°	Right line with stepped thrust	$X_{\dot{u}}$
	8°	Turnings and Zigzags	$X_{vv}, (m + X_{vr}), (mx_G + X_{rr}), X_{u^2\delta\delta}$

Table 4.4.2 - Identified coefficients by type of maneuver in the yaw-sway System

	Step	Maneuver type	Coefficients to be adjusted
Rough adjustment	1°	Soft turnings	$Y_{u^2\delta_R}$
	2°	Soft turnings	$(Y_{ur} - m), (N_{ur} - mx_G), Y_{u^2\delta_R}$
	3°	Soft and sharp turnings	$Y_{ v v}, N_{ v v}, Y_{u^2\delta_R}$
Fine adjustment	4°	Soft turnings	$Y_{uv}, (Y_{ur} - m), Y_{u^2\delta_R}, N_{uv}, (N_{ur} - mx_G), N_{u^2\delta_R}$
	5°	Soft and sharp turnings	$Y_{ v v}, N_{ v v}, Y_{u^2\delta_R}, N_{u^2\delta_R}$

	Step	Maneuver type	Coefficients to be Identified
Rough adjustment	1°	Zigzags	$(I_{zz} - N_{\dot{r}})$
	2°	Zigzags	$(m - Y_{\dot{v}}), (Y_{\dot{r}} - mx_G), (I_{zz} - N_{\dot{r}})$
Fine adjustment	3°	Zigzags	$(m - Y_{\dot{v}}), (Y_{\dot{r}} - mx_G), (N_{\dot{v}} - mx_G), (I_{zz} - N_{\dot{r}})$

B. Parameters Initialization using an ASE Approach

The proposed identification method initializes the hydrodynamic coefficients, estimated previously to the identification process, using the ASE method described in section 3. It has been very useful in the identification process because, when large uncertainties occur in all the initial estimates, there is not a general criterion to the initializing of the covariance matrices of the filter ($\mathbf{Q}$ and $\mathbf{P}_o$). Several trials and combinations are adopted until the estimation achieves appropriate results. However, if the values of $\mathbf{Q}$ and $\mathbf{P}_o$ matrices are widely varied; the probability of mutual divergence between two or more parameters is increased. On the other side, if initial parameters closer to the true values are adopted, it is not necessary to widely vary the values of the $\mathbf{Q}$ and $\mathbf{P}_o$ matrices and their values can be initialized with close-to-zero values.

Another advantage of the ASE initialization is the accuracy of some estimated hydrodynamic coefficients. They can be used as reference values of magnitudes and signals for each coefficient provided by the ASE approach since these are physically and empirically grounded. Moreover, the ASE method provides an insight of the estimated coefficient accuracy. Therefore, they can be classified according to the confidence level of the ASE estimation. The high confidence level is assigned, for example, to the added mass coefficients and those ones related to the stationary lift efforts. While low confidence levels are related to the rotation movement and the control surfaces. This sorting of the hydrodynamic

coefficients, according to their confidence level, is applied to the next characteristic of the proposed method.

C. Identification of Hydrodynamic Coefficients Using a Spiral Process.

The groups of coefficients formed according to the confidence levels allow us to carry out a spiral identification process. In a first iteration, the coefficients with the least level of confidence are adjusted by the parameters estimator while the other coefficients are kept fixed to the initial values. In this iteration the values of the $\mathbf{Q}$ and $\mathbf{P}_o$ matrices are manipulated without limit until the model output fits on the experimental data. In a second iteration, the most reliable coefficients and the already adjusted coefficients are simultaneously adjusted. In this iteration the values of the $\mathbf{Q}$ and $\mathbf{P}_o$ matrices are gradually increased, starting from close-to-zero values. Finally, in order to reduce, even more, the number of coefficients to be estimated in the first and second iteration; it could be adopted the following equalities, assumed in the analytic relationships presented in section 3, the expression (3.1.6) and table 3.3.3.

$$N_{u^2\delta_R} = x_R \cdot Y_{u^2\delta_R} \quad ; \quad Y_{\dot{r}} - mx_G = N_{\dot{v}} - mx_G \quad \text{or} \quad m_{22} = m_{31} \quad (4.4.1)$$

Considering these equalities, a yaw-sway system with two fewer parameters can be built, using the EKF as well the UKF, which will be named the yaw-sway-reduced systems such as seen on Appendix B. From this stage, a third iteration should be added, considering now the complete yaw-sway systems, in which all the coefficients are independently estimated.

Finally, in the complete identification process, the covariance matrices are easily constructed. The matrices $\mathbf{Q}$ and $\mathbf{P}_o$ are proportional to the corresponding initial values of the coefficients to be identified and a tuning factor, such as seen in the expressions (4.1.17), (4.2.10) and (4.2.20). When their values are set to zero, the corresponding estimates are not allowed to change. In the section of appendices (B) a criterion to tuning the matrices $\mathbf{Q}$ and $\mathbf{P}_o$ is presented, based on the equations of the conventional Kalman filter.

5. Experimental Validation of the ASE and Parameters

Identification Methods

The proposed method was validated by experimental sea trials. Inside this identification method framework, two parameter estimators, the Extended Kalman Filter (EKF), and the Unscented Kalman Filter (UKF) are combined with an Analytical and Semi-Empirical method for estimating the hydrodynamic coefficients. Experimental conditions together with validation results showing the adherence between predicted and measured velocities during AUV trials are presented in this chapter. The convergence analysis of the estimates and a final discussion of results are also included.

5.1. Sea-Trial Data

The experimental data used on this work was obtained through the Pirajuba AUV. It is a vehicle developed in the Unmanned Vehicles Laboratory at the University of São Paulo. This AUV was conceived originally for studies on dynamics and navigation of underwater vehicles. Nowadays, it can carry up payloads like sonars and oceanographic sensors. The experimental data, used to validate the identification method proposed in this work, was obtained with the academic version of the vehicle (2016), shown in figures 1.1.1 and 5.1.1. The physical features of this version are presented in tables (5.1.2) and its sensor specification in table 5.1.1

Table 5.1.1 - Sensor's specifications

Sensors	Type	bias	Noise	Scale Factor
Gyroscopes	FOG	$1^\circ/h_r$ (1σ)	$0.0667^\circ/\sqrt{h_r}$ (1σ)	1500 ppm (1σ)
Accelerometers	MEMS	0.25 mg (1σ)	$55 \mu g/\sqrt{Hz}$ (1σ)	4000 ppm (1σ)
DVL	Acoustic	-	± 1 mm/s	0.2 %
Compass	Magnetic	$\pm 0.5^\circ$	-	-

In order to identify the hydrodynamic coefficients of the academic version of the Pirajuba, sea-trials were carried out in Ubatuba Bay (Brazil, December 2016). During these tests, the vehicle performed classical submarine maneuvers in the horizontal plane such as zigzags, turning and stepped thrust. For the last type of maneuver, a number of consecutive steps of the propeller's RPM (revolutions per minutes), are imposed to the AUV. Different variations of the executed maneuvers during the trials are presented in table 5.1.3.

Table 5.1.2 - Physical characteristics of the Pirajuba (academic version, 2016)

Magnitude	Symbol	Value	Unit
Length	L	1.880	(m)
Hull maximal diameter	D	0.234	(m)
Underwater vehicle mass	m	72.8	(Kg)
Moment of inertia in "z"	I_{zz}	28	(Kg·m ²)
Tail Myring Angle	θ_M	25	(°)
Vehicle nose length	l_n	0.217	(m)
Vehicle Tail length	l_T	0.279	(m)
Cylindrical section length	l_c	1.384	(m)
Rudder fin height	h_R	0.16	(m)
Rudders tip chord	Ct	0.06	(m)
Rudders root chord	Cr	0.09	(m)
Rudder's leading edge position	x_{le}^{rudd}	-0.611	(m)
Sail length	l_s	0.17	(m)
Sail height	h_s	0.105	(m)
Maximum sail width	D_s	0.032	(m)
Sail's leading edge position	x_{le}^{sail}	0.523	(m)
Sea water density	ρ	1023	(Kg/m ³)
Buoyancy center measured since bow end	l_B	0.9	(m)

Figure 5.1.1: Academic version of the Pirajuba AUV in sea-trials.

Table 5.1.3: Kind of maneuvers in sea-tests with the Pirajuba

Used to	ID Ride	Maneuver kind
Model Identification	Zigzag_1	Zigzag with 5° rudder to 5° heading
Model Validation	Zigzag_2	Zigzag with 5° rudder to 5° heading
Model Validation	Zigzag_3	Zigzag with 10° rudder to 10° heading
Model Validation	Zigzag_4	Zigzag with 20° rudder to 20° heading
Model Identification	Zigzag_5	Zigzag with 20° rudder to 20° heading
Model Identification	Turning_1	Turning with +5° rudder
Model Validation	Turning_2	Turning with +10° rudder
Model Identification	Turning_3	Turning with -10° rudder
Model Identification and Validation	Turning_4	Turning with +20° rudder
Model Identification	Turning_5	Turning with -20° rudder
Model Identification	Stepped_Thrust_1	15 seconds of RPM step
Model Validation	Stepped_Thrust_2	30 seconds of RPM step
Model Identification	Stepped_Thrust_3	45 seconds of RPM step

In the DVL measurements, some "spikes" in the signals were observed. It was concluded that they are due to some misreading of the sensor when, after sending the acoustic wave, the sensor does not register the signal reflected from the seafloor. In order to cope with this problem, a Fourier spectral analysis was performed in the “ u ” and “ v ” measurements.

Afterwards, a low-pass filter based on a Discrete Fourier Transform was performed to filter these spikes. In the filtered velocity readings, an irregular oscillation of the signal amplitude was observed in all the analyzed maneuvers. A power spectral analysis was applied to " v " velocities measurements as well as the control surfaces inputs (rudders and elevators), in all maneuvers. Peaks were observed at frequencies in a range between 0.023 to 0.293 (Hz), which could not be produced by any vehicle actuator. Therefore, this kind of noise was considered as an environmental disturbance, such as ocean waves, or currents.

5.2. Estimated Hydrodynamic Coefficients

In this subsection, the hydrodynamic coefficients estimated with the ASE method and the parameters identification methods, using EKF as well as UKF, are presented in table 5.2.1. It is also included the relative differences of the EKF and UKF estimator from the ASE values.

Table 5.2.1 - Hydrodynamic coefficient estimates through ASE and parameters identification methods.

Coefficient Symbol	ASE Values	EKF Values	UKF Values	Relative Differences From ASE	
				% EKF	% UKF
$m - X_{\dot{u}}$	75.42	75.42	75.42	0.0	0
X_{uu}	-4.018	-5.165	-6.455	28.5	60.7
X_{vv}	40.06	44.5	47.7	11.1	19.1
$m + X_{vr}$	98.44	74.2	79.3	24.6	19.4
$mx_G + X_{rr}$	13.45	16.7	18.4	24.2	36.8
$X_{u^2\delta\delta}$	-56.94	-39.23	-46.0	31.1	19.2
$X_{u\eta}$	-0.0276	-0.02763	-0.016	0.1	42.0
$X_{\eta\eta}$	0.0033	0.0034	0.00383	3.0	16.1
$m - Y_{\dot{v}}$	167.4	145.5	146	13.1	12.8
$Y_r - mx_G$	6.858	5.775	6.858	15.8	0
Y_{uv}	-126.4	-135	-135	6.8	6.8
$Y_{ur} - m$	-25.85	-37.0	-40.0	43.1	54.7
$Y_{v v }$	-189.2	-130	-150	31.3	20.7
$Y_{u^2\delta_R}$	62.25	51.78	51.9	16.8	16.6
$N_{\dot{v}} - mx_G$	6.858	6.548	6.858	4.5	0
$I_{zz} - N_r$	41.69	31.51	31.0	24.4	25.6
N_{uv}	-1.461	-1.2	-1.05	17.9	28.1
$N_{ur} - mx_G$	-41.43	-41.9	-41.68	1.1	0.6
$N_{v v }$	-8.865	-11.25	-11.4	26.9	28.6
$N_{u^2\delta_R}$	-39.61	-32.94	-33.02	16.8	16.6
Average	-	-	-	17.1	21.2

During the hydrodynamic coefficient estimation, it is considered that they are identified when the corresponding state variable achieve a constant value and oscillate around it, as observed in figures 5.2.1, 5.2.3, 5.2.5 and 5.2.7. Moreover, the convergence to zero of the standard deviation of the estimated velocities (σ_u , σ_v or σ_r) indicate that the hydrodynamic coefficients converge to the estimated values, since the convergence of the estimated velocity variables represent the convergence of the model parameters presented in their dynamics (MAYBECK P. S., 1979).

The standard deviations of the velocity variables (σ) were calculated as the square root of the corresponding variance obtained from the diagonal of the state covariance matrix ($\mathbf{P}$). Multiple maneuvers are parallel processed in the state variable estimation process. Hence, a velocity variable is estimated for each processed maneuver. The mean of these velocity signals are presented in figures 5.2.2, 5.2.4, 5.2.6 and 5.2.8 to show the convergence of these variables.

Figure 5.2.1: Convergence of the identified-surge coefficients using EKF.

The graphics of the convergence of the surge coefficients are showed in the figure 5.2.1. They correspond to the fine adjustment phase in the identification process of the surge coefficients using the EKF, such as described in table 4.4.1. An exceptional case is the graphic of estimation of the coefficient m_{11} , which shows that this value does not vary from this initial value along to the maneuver data process (the graphic scale has been changed to show the very small estimate variation). Therefore, the identified value of this coefficient is assumed to be the same as the corresponding ASE estimate.

The normalized-standard deviation of the velocity u for each phase of the surge-coefficients identification is showed in the figure 5.2.2.

Figure 5.2.2: Convergence of the standard deviation of the velocity $\mathbf{u}$

The graphics of the convergence of the yaw-sway coefficients are showed in the figure 5.2.3. They correspond to the fine adjustment phase in the identification process of the yaw-sway coefficients using the EKF as described in table 4.4.2.

Figure 5.2.3: Convergence of the identified-yaw-sway coefficients using EKF.

The normalized-standard deviation of the velocities v and r for each phase of the yaw-sway coefficients identification are showed in the figure 5.2.4.

Figure 5.2.4: Convergence of the standard deviation of the velocities v and r

The graphics of the convergence of the surge coefficients are showed in the figure 5.2.5. They correspond to the fine adjustment phase in the identification process of the surge coefficients using the UKF as described in table 4.4.1. An exceptional case is the graphic of the estimated coefficient $(m - X_{\dot{u}})$, which shows that this value does not vary from this initial value along to the maneuver data process. Therefore, the identified value of this coefficient is assumed to be the same as its ASE estimate.

Figure 5.2.5: Convergence of the identified surge coefficients using UKF.

The normalized-standard deviation of the velocity u for each phase of the surge-coefficients identification is showed in the figure 5.2.6.

Figure 5.2.6: Convergence of the standard deviation of the velocity u

The graphics of the convergence of the yaw-sway coefficients are showed in the figure 5.2.7. They correspond to the fine adjustment phase in the identification process of the yaw-sway coefficients using the UKF as described in table 4.4.2. Exceptional cases are the graphics of the estimated coefficients $(Y_{\dot{r}} - mx_G)$ and $(N_{\dot{v}} - mx_G)$, that show that these values do not significantly vary from initial values along to the maneuver data processing. Therefore, the identified values of these coefficients are assumed to be equal to the corresponding ASE values.

Figure 5.2.7: Convergence of the identified yaw-sway coefficients using UKF

The normalized-standard deviation of the velocities v and r for each phase of the yaw-sway coefficients identification, are showed in the figure 5.2.6.

Figure 5.2.8: Convergence of the standard deviation of the velocities v and r

5.3. Simulation of Maneuvers Using the Identified Models

Simulations of the Pirajuba dynamics were carried out using the input data of the validation maneuvers and the ASE model (black dashed line) as well as the identified models, obtained with the EKF (continuous black lines) and UKF (continuous gray lines) estimators. The results of these simulations are depicted by the linear and angular velocities signals that were plotted together with the sensor measurements (gray dashed line) for comparison purposes.

A. Stepped Thrust 2 Maneuver (30 seconds of RPM step)

Figure 5.3.1 – Simulated velocity u in the Stepped_Thrust_2 maneuver.

B. Turning 2 maneuver (10° rudder)

Figure 5.3.2 – Simulated velocity u in the Turning_2 maneuver.

Figure 5.3.3 – Simulated velocity v in the Turning_2 maneuver.

Figure 5.3.4 – Simulated angular velocity r in the Turning_2 maneuver.

B. Turning 4 maneuver (20° rudder)

Figure 5.3.5 – Simulated velocity u in the Turning_4 maneuver.

Figure 5.3.6 – Simulated velocity v in the Turning_4 maneuver.

Figure 5.3.7 – Simulated angular velocity r in the Turning_4 maneuver.

C. Zigzag_2 maneuver (5° rudder)

Figure 5.3.8 – Simulated velocity u in the Zigzag_2 maneuver.

Figure 5.3.9 – Simulated velocity v in the Zigzag_2 maneuver.

Figure 5.3.10 – Simulated velocity r in the Zigzag_2 maneuver.

D. Zigzag 3 maneuver (10° rudder)

Figure 5.3.11 – Simulated velocity u in the Zigzag_3 maneuver.

Figure 5.3.12 – Simulated velocity v in the Zigzag_3 maneuver.

Figure 5.3.13 – Simulated velocity r in the Zigzag_3 maneuver.

E. Zigzag_5 maneuver (20° rudder)

Figure 5.3.14 – Simulated velocity u in the Zigzag_5 maneuver.

Figure 5.3.15 – Simulated velocity v in the Zigzag_5 maneuver.

Figure 5.3.16 – Simulated velocity r in the Zigzag_5 maneuver.

5.4. Discussion About the Results

From the hydrodynamic coefficients estimated with the ASE and identification methods, showed in table 5.2.1, it can be observed that:

- The coefficients estimated using the ASE method, in average, need to be adjusted around 20% to reach the identified coefficients.
- the added-mass coefficients, except for the coefficient $(I_{zz} - N_{\dot{r}})$, estimated by the ASE method differ by less than 15.8% to the identified coefficients.
- The linear lift coefficient (Y_{uv}) estimated with the ASE method differs by 6.8% to the identified coefficients.
- The coefficients $(X_{uu}, Y_{ur} - m \text{ and } I_{zz} - N_{\dot{r}})$ estimated with the ASE method showed the largest differences relative to the identified ones.

The last result is not surprising. In the case of the coefficient X_{uu} , it was not possible to estimate, by ASE expressions, the contribution of the axial drag on the sail, rudders and elevators. In order to roughly compensate for these effects, this estimate was increased by 20% in the values of the hull axial-drag. The coefficient Y_{ur} represents the lift and drag forces during rotations, and by estimating without considering viscous effects on the hull. Finally, to calculate the term I_{zz} a rough approximation was made, considering the hull filled by an homogeneous distributions of water.

Therefore, it can be concluded that the previous classification of the coefficients according to each level of confidences, obtained from the ASE method, is confirmed. Moreover, based on validation maneuvers, it can be concluded that:

- The velocities obtained with the ASE model present a coherent behavior against the measurements.
- The velocities obtained with the identified models present a behavior appreciably closer to the measurements.

Concerning the forward velocity, in the figures (5.3.2) and (5.3.8) it is observed that the " u " signal of the identified models (black and gray-continuous lines) achieves a higher cruising velocity than the measurement signal (gray dashed line). The source of this stationary error wasn't determined in this work, but some hypotheses could explain such a difference. As seen in the previous discussion, the identified dynamic model does not reproduce some non-commanded oscillations in the vehicle produced by the environment. They may produce damping in the vehicle advance velocity, which is not reproduced by the identified model.

The EKF and UKF models show an equivalent performance on all simulated velocities plots. A slight difference could be highlighted in the figure 5.3.1, of the stepped thrust maneuver, in which the u signal of the UKF model (gray-continuous line) slightly drift from black-continuous line to the measurement line (gray-dashed line) on the steepest curve of the signal (at 215 seconds). However, in general, no differences were observed between the EKF and UKF models. This fact can be explained by the slow dynamic behavior shown by an autonomous underwater vehicle in typical maneuvers. In these conditions, the nonlinearities of the system are not accentuated.

The ASE model, in general, presents a less damped behavior than the identified model, in translational as well as in rotational movements. This characteristic could be explained by the fact that the ASE model does not take into account the effect of appendages in the hull (antennas, acoustic transducers for underwater communication, sensors, and so forth) and the hull imperfections that introduce viscous effects that are difficult to model and predict.

6. Conclusions and Future Works

Numerical simulations of the identified dynamic models were compared with the experimental data, and they show visible agreement between estimated and measured variables. Therefore, a nonlinear dynamic model can satisfactorily reproduce the AUV's different types of maneuvers. The proposed identification method, through the selective identification of the coefficients according to their level of confidence and the type of the maneuver data used and their initialization using ASE estimates, proved to be able to identify the complete set of the hydrodynamic coefficients of the AUV, even the added-mass coefficients. This identification method deals with the over-parameterization problem, decreasing the probability of mutual divergence of the estimates. It is possible due to the ASE estimations and their physical meaning. On the other hand, this method requires the manual adjustment of the filter parameters to reach the convergence of all the coefficients. This still requires some criterion and experience of the analyst, mainly in the cases when several coefficients have to be estimated at the same time. Therefore, the total automation of the identification process of the hydrodynamic coefficients was not reached in this work.

When compared with the behavior of the models estimated by the EKF and UKF algorithms, no significant differences were observed between them. Such as mentioned in section 4.3, the UKF presents a significantly lower error against the EKF when the nonlinearities are severe. From the results obtained in this work, it can be concluded that the slow dynamics of an autonomous underwater vehicle does not produce accentuated nonlinearities effects in the efforts on the AUV. In this case, it is supposed that performances of EKF and UKF are practically equivalent.

Future works should complement the testing of the proposed method by the identification of the vertical-plane hydrodynamic coefficients. Moreover, improvements in the dynamic model complexity can be considered. Those suggested improvements should be combined in

order to automatize the identification process. Finally, it should be considered the establishment of a methodology to be adopted during the carrying out of the trials and for the pre-processing of the experimental data.

Bibliography References

ABKOWITZ, M. A. Measurement of hydrodynamic characteristics from ship maneuvering trials by system identification. 1980.

ALLEN, H. J.; PERKINS, E. W. **A Study of Effects of Viscosity On Flow Over Slender Inclined Bodies of Revolution**. Ames Aeronautical Laboratory Moffett Field. Calif.: National Advisory Committee for Aeronautics, ago. 1952.

AVILA, J.; PABLO, J. **Modelagem e identificação de parâmetros hidrodinâmicos de um veículo robótico submarino**. text—[s.l.] Universidade de São Paulo, 17 out. 2008.

BARROS, E. A., DANTAS J. L. D., PASCOAL A. M., SÁ Elgar. Investigation of Normal Force and Moment Coefficients for an AUV at Nonlinear Angle of Attack and Sideslip Range. **IEEE Journal of Oceanic Engineering**, v. 33, n. 4, p. 538–549, out. 2008.

BELLANTONI, J. F.; DODGE, K. W. A Square Root Formulation of the Kalman -Schmidt Filter. **AIAA Journal**, v. 5, n. 7, jul. 1967.

HUMPHREYS, D. E.. **Development of the Equations of Motion and Transfer Functions for Underwater Vehicles**. Panama city, Florida 32401 USA: Naval Coastal Systems Laboratory, 1 jul. 1976. Disponível em: <http://archive.org/details/DTIC_ADA033882>. Acesso em: 20 jun. 2018.

HUMPHREYS, D. E.; WATKINSON, K. W.. **Prediction of Acceleration Hydrodynamic Coefficients for Underwater Vehicles from Geometric Parameters**. Panama city, Florida 32407: Naval Coastal Systems Laboratory, fev. 1978.

DANTAS, J. L. D., CAETANO W. S., VALE R. T. S., BARROS E. A. **Analysis of Identification Methods Applied to Free Model Tests of the Pirajuba AUV**. IFAC Proceedings Volumes, 9th IFAC Conference on Control Applications in Marine Systems. v. 46, n. 33, p. 185–190, 1 jan. 2013.

BARROS, E. A.; MIYAJIMA, S.; MAEDA, H. **A system identification experiment for an underwater vehicle**. Ocean Engineering Symposium. **Anais...** In: 11TH OCEAN ENGINEERING SYMPOSIUM. 1992

GERTLER, M.; HAGEN, G. R. **Standard equations of motion for submarine simulation**. Bethesda MD: David W. Taylor Naval Ship Research and Development Center, jun. 1967. Disponível em: <<http://www.dtic.mil/docs/citations/AD0653861>>. Acesso em: 20 jun. 2018.

IMLAY, F. H. **The Complete Expressions for Added Mass of a Rigid Body Moving in an Ideal Fluid**. [s.l.] Department of the Navy David Taylor Model Basin, 1 jul. 1961.

HASSANEIN, O.; ANAVATTI, S. G.; RAY, T. **Fuzzy modeling and control for Autonomous Underwater Vehicle**. The 5th International Conference on Automation, Robotics and Applications. **Anais...** In: THE 5TH INTERNATIONAL CONFERENCE ON AUTOMATION, ROBOTICS AND APPLICATIONS. dez. 2011

HEGRENAES, O.; HALLINGSTAD, O.; JALVING, B. **Comparison of Mathematical Models for the HUGIN 4500 AUV Based on Experimental Data**. 2007 Symposium on Underwater Technology and Workshop on Scientific Use of Submarine Cables and Related Technologies. **Anais...** In: 2007 SYMPOSIUM ON UNDERWATER TECHNOLOGY AND WORKSHOP ON SCIENTIFIC USE OF SUBMARINE CABLES AND RELATED TECHNOLOGIES. abr. 2007

HOAK, D. E.; FINK, R. D. **USAF Stability and Control Datcom**. I ed. Flight Control Division Air Force Flight Dynamics Laboratory Wright-Patterson Air Force Base, Ohio: Douglas Aircraft Division of the McDonnell Douglas Corporation, 1960.

FELDMAN, J. **DTNSRDC Revised Standard Submarine Equations of Motion**. Bethesda, MD 20084: David W. Taylor naval ship research and development center, 1979a. Disponível em: <<https://ntrl.ntis.gov/NTRL/dashboard/searchResults/titleDetail/ADA071804.xhtml>>. Acesso em: 20 jun. 2018.

NEWMAN, J. N. **Marine Hydrodynamics**. [s.l.] The MIT Press Cambridge, Massachusetts, and London, England, 1977.

JULIER, S.; UHLMANN, J.; DURRANT-WHYTE, H. F. **A new method for the nonlinear transformation of means and covariances in filters and estimators**. IEEE Transactions on Automatic Control, v. 45, n. 3, p. 477–482, mar. 2000.

KARAMCHETI K. **Principles of ideal-fluid aerodynamics**. Wiley press, New York 1966.

KARRAS, G. C. et al. **On-line identification of autonomous underwater vehicles through global derivative-free optimization**. 2013 IEEE/RSJ International Conference on Intelligent Robots and Systems. *Anais...* In: 2013 IEEE/RSJ INTERNATIONAL CONFERENCE ON INTELLIGENT ROBOTS AND SYSTEMS. nov. 2013

KIM, J., KIM K., CHOI H. S., SEONG W. and LEE K. Y. Estimation of hydrodynamic coefficients for an AUV using nonlinear observers. *IEEE Journal of Oceanic Engineering*, v. 27, n. 4, p. 830–840, out. 2002.

KIM, K.; CHOI, H. S. Analysis on the controlled nonlinear motion of a test bed AUV–SNUUV I. *Ocean Engineering*, v. 34, n. 8, p. 1138–1150, 1 jun. 2007.

KOROTKIN, A. I. **Added Masses of Ship Structures**. [s.l.] Springer Netherlands, 2009.

LELAND HOWARD JORGENSEN. **Prediction of Static Aerodynamic Characteristics for Slender Bodies Alone and With Lifting Surfaces to Very High Angles of Attack**. Moffett Field, Calif. 94035: Ames Research Center, set. 1977.

LEWIS, E. V. **Principles of Naval Architecture Second Revision: Volume II; Resistance, Propulsion and Vibration**. Published by The Society of Naval Architects and Marine Engineers. 601 Pavonia Avenue Jersey City, NJ, 1988.

LIANG, X. LI W., LIN J., SU L. and LI H. **Model Identification for Autonomous Underwater Vehicles Based on Maximum Likelihood Relaxation Algorithm**. 2010 Second International Conference on Computer Modeling and Simulation. *Anais...* In: 2010 SECOND INTERNATIONAL CONFERENCE ON COMPUTER MODELING AND SIMULATION. jan. 2010

LUQUE, J. C. C.; DONHA, D. C. **AUV Identification and Robust Control**. IFAC Proceedings Volumes, 18th IFAC World Congress. v. 44, n. 1, p. 14735–14741, 1 jan. 2011.

LUQUE, J. C. C.; DONHA, D. C.; DE BARROS, E. A. **AUV parameter identification**. 8th IFAC Conference on Manoeuvring and Control of Marine Craft, IFAC Proceedings Volumes. v. 42, n. 18, p. 72–77, 1 jan. 2009.

MACKAY, M. **The Standard Submarine Model: A Survey of Static Hydrodynamic Experiments and Semiempirical Predictions**. [s.l.] Defence research and development of Canada-Atlantic, 2003.

MAYBECK P. S. **Stochastic models, estimation, and control**; Vol. 1. Academic Press, Inc. New-York 1979.

MUNK, M. M. **The Aerodynamic Forces on Airship Hulls**. Washington Government Printing Office: National Advisory Committee for Aeronautics, 1924.

PITTS, W. C.; NIELSEN, J. N.; KAATTARI, G. E. **Lift and center of pressure of wing-body-tail combinations at subsonic, transonic, and supersonic speeds**. USA: [s.n.]. Disponível em: <<http://ntrs.nasa.gov/search.jsp?R=19930091008>>. Acesso em: 21 jun. 2018.

PRESTERO, T. **Development of a six-degree of freedom simulation model for the REMUS autonomous underwater vehicle**. MTS/IEEE Oceans 2001. An Ocean Odyssey. Conference Proceedings (IEEE Cat. No.01CH37295). *Anais...* In: MTS/IEEE OCEANS 2001. AN OCEAN ODYSSEY. CONFERENCE PROCEEDINGS (IEEE CAT. NO.01CH37295). 2001

RAO, A. V. **Dynamics of Particles and Rigid Bodies: A Systematic Approach**. Cambridge, UK: Cambridge University Press, 2006.

RENTSCHLER, M. E.; HOVER, F. S.; CHRYSSOSTOMIDIS, C. **System identification of open-loop maneuvers leads to improved AUV flight performance**. IEEE Journal of Oceanic Engineering, v. 31, n. 1, p. 200–208, jan. 2006.

ROBERTS, G. N.; SUTTON, R. **Advances in Unmanned Marine Vehicles**. [s.l.] IET, 2006.

SABET, M. T., DANIALI H. M., FATHI A. And ALIZADEH E. **Identification of an Autonomous Underwater Vehicle Hydrodynamic Model Using the Extended, Cubature, and Transformed Unscented Kalman Filter**. IEEE Journal of Oceanic Engineering, v. 43, n. 2, p. 457–467, abr. 2018.

SABET, M. T.; POURIA SARHADI; ZARINI, M. **Extended and Unscented Kalman filters for parameter estimation of an autonomous underwater vehicle**. Ocean Engineering, v. 91, p. 329–339, 15 nov. 2014.

SANDMAN, B. E.; KELLY, J. G. **Systems Identification: Application to Underwater Vehicle Dynamics**. Journal of Hydronautics, v. 8, n. 3, p. 94–99, 1974.

SEN, D. **A Study on Sensitivity of Maneuverability Performance on the Hydrodynamic Coefficients for Submerged Bodies**. Journal of Ship Research, v. 44, p. 186–196, 1 set. 2000.

SHAHINFAR, E.; BOZORG, M.; BIDOKY, M. Parameter Estimation of an AUV Using the Maximum Likelihood method and a Kalman Filter with Fading Memory. **IFAC Proceedings Volumes**, 7th IFAC Symposium on Intelligent Autonomous Vehicles. v. 43, n. 16, p. 1–6, 1 jan. 2010.

SIMON, D. **Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches**. 1 edition ed. Hoboken, N.J: Wiley-Interscience, 2006.

SNAME. **Nomenclature for treating the motion of a submerged body through a fluid: Report of the American Towing Tank Conference**. New York: Technical and Research Committee of the Society of Naval Architects and Marine Engineers (SNAME), 1950.

TAUBERT, R., EICHHORN M., AMENT C., JACOBI M., KARIMANZIRA D., and PFUETZENREUTER T. **Model identification and controller parameter optimization for an autopilot design for autonomous underwater vehicles**. OCEANS 2014 - TAIPEI. *Anais...* In: OCEANS 2014 - TAIPEI. abr. 2014

TIANO, A., SUTTON R., LZOWICKI A., and NAEEM W. **Observer Kalman filter identification of an autonomous underwater vehicle**. Control Engineering Practice, Special Section on Control Applications in Marine Systems. v. 15, n. 6, p. 727–739, 1 jun. 2007.

WARD, G. N. **Supersonic Flow Past Slender Pointed Bodies**. Quarterly Journal of Mechanics and Applied Mathematics, jan. 1948.

WHICKER, L. F.; FEHLNER, L. F. **Free-Stream Characteristics of A Family of Low-Aspect-Ratio, All-Movable Control Surfaces for Application to Ship Design**. [s.l: s.n.].

XU, F., ZOU Z. J., YIN J. C., and CAO J. **Identification modeling of underwater vehicles' nonlinear dynamics based on support vector machines**. Ocean Engineering, v. 67, p. 68–76, 15 jul. 2013.

APPENDICES

Appendix A - Integration of the Munk force per unit length

In order to calculate the normal component of the force produced by the potential effects, the cross force per unit length $\left(\frac{dF_{pot}^{hull}}{dx}\right)$, derived by Munk (1924), should be integrated along the axial edge of the hull. The cross force per unit length was expressed in the equation (3.2.1) in section 3.2, such as:

$$\frac{dF_{pot}^{hull}}{dx} = \frac{1}{2} \rho U_o^2 (k_2 - k_1) \frac{dA_{cs}^{hull}}{dx} \sin(2\alpha)$$

Integrating this expression along to the axial edge of the hull, such as:

$$F_{pot}^{hull} = \int_{x_{bow}}^{x_{end}} \frac{dF_{pot}^{hull}}{dx} \cdot dx = \int_{x_{bow}}^{x_{end}} \frac{1}{2} \rho U_o^2 (k_2 - k_1) \sin(2\alpha) \frac{dA_{cs}^{hull}}{dx} dx$$

Considering a bidimensional-stationary flow, the attack angle and the velocity are constant along to the axial edge. Hence, the potential force is expressed as:

$$F_{pot}^{hull} = \frac{1}{2} \rho U_o^2 (k_2 - k_1) \sin(2\alpha) \cdot \int_{x_{bow}}^{x_{end}} dA_{cs}^{hull}$$

The area of the cross-section on the end of the bow is zero. Therefore, the potential force over the hull can be calculated as:

$$F_{pot}^{hull} = \frac{1}{2} \rho U_o^2 (k_2 - k_1) \sin(2\alpha) A_{cs}^{hull}(x_{end})$$

Analogously, the magnitude of the lift force moment relative to the vehicle's reference system is calculated by the integration of the longitudinal position multiplied by the cross force per unit length along of the axial edge. :

$$M_{pot}^{hull} = \int_{x_{bow}}^{x_{end}} \left(x \cdot \frac{dF_L^{hull}}{dx} \right) dx = \int_{x_{bow}}^{x_{end}} \frac{1}{2} \rho U_o^2 (k_2 - k_1) \sin(2\alpha) x \frac{dA_{cs}^{hull}}{dx} \cdot dx$$

The attack angle and the velocity are constant along to the axial edge, as in the previous case.

Hence, the potential force is expressed as:

$$M_{pot}^{hull} = \frac{1}{2} \rho U_O^2 (k_2 - k_1) \sin(2\alpha) \int_{x_{bow}}^{x_{end}} x \frac{dA_{cs}^{hull}}{dx} \cdot dx$$

Integrating by parts:

$$\begin{aligned} \int_{x_{bow}}^{x_{end}} x \frac{dA_{cs}^{hull}}{dx} \cdot dx &= x \cdot A_{cs}^{hull} \Big|_{x_{bow}}^{x_{end}} - \int_{x_{bow}}^{x_{end}} A_{cs}^{hull} dx \\ \int_{x_{bow}}^{x_{end}} x \frac{dA_{cs}^{hull}}{dx} \cdot dx &= x_{end} \cdot A_{cs}^{hull}(x_{end}) - x_{bow} \cdot \underbrace{A_{cs}^{hull}(x_{bow})}_0 + V(x_{end}) \end{aligned}$$

Replacing in the moment equation:

$$M_{Munk}^{hull} = \frac{1}{2} \rho U_O^2 (k_2 - k_1) \sin(2\alpha) V(x_{end}) + x_{end} \cdot A_{cs}^{hull}(x_{end})$$

Appendix B - Reduced yaw-sway system

For the EKF estimator:

$$\Theta = \begin{bmatrix} m_{32} \\ m_{22} \\ Y_{uv} \\ Y_{ur} - m \\ Y_{v|v|} \\ Y_{u^2\delta_r} \\ m_{21} \\ N_{uv} \\ N_{ur} - mx_G \\ N_{v|v|} \end{bmatrix} ; \quad \mathbf{x} = \begin{bmatrix} v \\ r \end{bmatrix} ; \quad \mathbf{u} = \begin{bmatrix} u \\ \delta_r \end{bmatrix} ; \quad \mathbf{n}_f = \begin{bmatrix} n_{\dot{v}} \\ n_{\dot{r}} \end{bmatrix}$$

From the dynamical model expressions (2.2.22 and 2.2.23) derived in section 2.2.

$$\dot{\mathbf{x}} = \begin{bmatrix} \dot{v} \\ \dot{r} \end{bmatrix} = \mathbf{f}(\Theta, \mathbf{x}, \mathbf{u}) = \begin{bmatrix} m_{21}f_Y + m_{22}f_N \\ m_{22}f_Y + m_{32}f_N \end{bmatrix}$$

In which:

$$f_Y = Y_{uv}uv + (Y_{ur} - m)ur + Y_{v|v|}v|v| + Y_{u^2\delta_R}u^2\delta_R$$

$$f_N = N_{uv}uv + (N_{ur} - mx_G)ur + N_{v|v|}v|v| + x_R Y_{u^2\delta_R}u^2\delta_R$$

The Jacobian matrices:

$$\frac{\partial \mathbf{f}}{\partial \boldsymbol{\Theta}} = \begin{bmatrix} 0 & f_N & m_{21}uv & m_{21}ur & m_{21}v|v| & (m_{21} + m_{22}x_R)u^2\delta_R & f_Y & m_{22}uv & m_{22}ur & m_{22}v|v| \\ f_N & f_Y & m_{22}uv & m_{22}ur & m_{22}v|v| & (m_{22} + m_{32}x_R)u^2\delta_R & 0 & m_{32}uv & m_{32}ur & m_{32}v|v| \end{bmatrix}$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix} m_{21}(Y_{uv}u + 2Y_{v|v}|v|) + m_{22}(N_{uv}u + 2N_{v|v}|v|) & m_{21}(Y_{ur} - m)u + m_{22}(N_{ur} - mx_G)u \\ m_{22}(Y_{uv}u + 2Y_{v|v}|v|) + m_{32}(N_{uv}u + 2N_{v|v}|v|) & m_{22}(Y_{ur} - m)u + m_{32}(N_{ur} - mx_G)u \end{bmatrix}$$

$$\frac{\partial \mathbf{f}}{\partial \mathbf{n}_f} = \mathbf{I}_{(n,n)} \quad ; \quad \mathbf{Q}_p = \text{Diag}((\mathbf{S} * \boldsymbol{\Theta}_0)^2) \quad ; \quad \mathbf{Q}_f = \begin{bmatrix} \sigma_v^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix}$$

$$\frac{\partial \mathbf{h}}{\partial \boldsymbol{\Theta}} = \mathbf{O}_{(2,10)} \quad ; \quad \frac{\partial \mathbf{h}}{\partial \mathbf{x}} = \mathbf{I}_{(2,2)}$$

From the measurements model expressions (2.3.3 and 2.3.6) in section 2.3.

$$\mathbf{y} = \begin{bmatrix} v^{dvl} - x_D r^{gyro} \\ r^{gyro} \end{bmatrix} \quad ; \quad \mathbf{h} = \begin{bmatrix} v \\ r \end{bmatrix}$$

$$\mathbf{n}_h = \begin{bmatrix} n_Y^{dvl} - x_D n_Z^{gyro} \\ n_Z^{gyro} \end{bmatrix} = \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} n_Y^{dvl} \\ n_Z^{gyro} \end{bmatrix}$$

$$\mathbf{R} = \text{cov}(\mathbf{n}_h) = \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} (\sigma_Y^{dvl})^2 & 0 \\ 0 & (\sigma_Z^{gyro})^2 \end{bmatrix} \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix}^T$$

For the UKF estimator:

$$\boldsymbol{\Theta} = \begin{bmatrix} m - Y_{\dot{v}} \\ Y_{\dot{r}} - mx_G \\ Y_{uv} \\ Y_{ur} - m \\ Y_{v|v|} \\ Y_{u^2\delta_R} \\ I_{zz} - N_{\dot{r}} \\ N_{uv} \\ N_{ur} - mx_G \\ N_{v|v|} \end{bmatrix} \quad ; \quad \mathbf{x} = \begin{bmatrix} v \\ r \end{bmatrix} \quad ; \quad \mathbf{u} = \begin{bmatrix} u \\ \delta_R \end{bmatrix} \quad ; \quad \mathbf{n}_f = \begin{bmatrix} n_{\dot{v}} \\ n_{\dot{r}} \end{bmatrix}$$

From the dynamical model expressions (2.2.22 and 2.2.23) derived in section 2.2.

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \frac{(I_{zz} - N_{\dot{r}})}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)^2} \cdot f_Y + \frac{(Y_{\dot{r}} - mx_G)}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)^2} \cdot f_N \\ \frac{(Y_{\dot{r}} - mx_G)}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)^2} \cdot f_Y + \frac{(m - Y_{\dot{v}})}{(m - Y_{\dot{v}})(I_{zz} - N_{\dot{r}}) - (Y_{\dot{r}} - mx_G)^2} \cdot f_N \end{bmatrix}$$

In which:

$$f_Y = Y_{uv}uv + (Y_{ur} - m)ur + Y_{v|v|}v|v| + Y_{u^2\delta_R}u^2\delta_R$$

$$f_N = N_{uv}uv + (N_{ur} - mx_G)ur + N_{v|v}|v||v| + x_R Y_{u^2 \delta_R} u^2 \delta_R$$

$$\mathbf{Q}_f = \text{cov}(\mathbf{n}_f) = \begin{bmatrix} \sigma_{\dot{v}}^2 & 0 \\ 0 & \sigma_{\dot{r}}^2 \end{bmatrix}$$

From the measurements model expressions (2.3.3 and 2.3.6) derived in section 2.3.

$$\mathbf{y} = \begin{bmatrix} v^{dvl} - x_D r^{gyro} \\ r^{gyro} \end{bmatrix} \quad ; \quad \mathbf{h} = \begin{bmatrix} v \\ r \end{bmatrix}$$

$$\mathbf{n}_h = \begin{bmatrix} n_Y^{dvl} - x_D n_Z^{gyro} \\ n_Z^{gyro} \end{bmatrix} = \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} n_Y^{dvl} \\ n_Z^{gyro} \end{bmatrix}$$

$$\mathbf{R} = \text{cov}(\mathbf{n}_h) = \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix} \begin{bmatrix} (\sigma_Y^{dvl})^2 & 0 \\ 0 & (\sigma_Z^{gyro})^2 \end{bmatrix} \begin{bmatrix} 1 & -x_D \\ 0 & 1 \end{bmatrix}^T$$

Appendix C - Tuning of the $\mathbf{Q}$ and $\mathbf{P}_0$ matrix on the Kalman Filter

In this section, an expression of the estimated state by the Kalman Filter is derived. This expression could aid the visualization of the influence of the $(\mathbf{P}_0, \mathbf{Q}$ e $\mathbf{R})$ matrices on the estimation process in the Kalman filter algorithm.

First, replacing the Kalman gain $(\mathbf{K})$, which is calculated from the covariance matrices $(\mathbf{P}_0, \mathbf{Q}$ e $\mathbf{R})$, on the state correction equation of the Kalman filter algorithm is obtained:

$$\mathbf{x}_i^+ = \mathbf{x}_i^- + \mathbf{P}_i \mathbf{C}^T \cdot (\mathbf{C} \mathbf{P}_i \mathbf{C}^T + \mathbf{R})^{-1} \cdot (\mathbf{y}_i - \mathbf{C} \mathbf{x}_i^-) \quad (\text{C. 1})$$

Multiplying both sides of (C.1) by the measurements model matrix $\mathbf{C}$ it is follows:

$$\mathbf{C} \mathbf{x}_i^+ = \mathbf{C} \mathbf{x}_i^- + \mathbf{C} \mathbf{P}_i \mathbf{C}^T \cdot (\mathbf{C} \mathbf{P}_i \mathbf{C}^T + \mathbf{R})^{-1} \cdot (\mathbf{y}_i - \mathbf{C} \mathbf{x}_i^-) \quad (\text{C. 2})$$

Factoring and grouping the terms of (C.2) results in:

$$\mathbf{C} \mathbf{x}_i^+ = (\mathbf{I} - \mathbf{C} \mathbf{P}_i \mathbf{C}^T (\mathbf{C} \mathbf{P}_i \mathbf{C}^T + \mathbf{R})^{-1}) \cdot \mathbf{C} \mathbf{x}_i^- + \mathbf{C} \mathbf{P}_i \mathbf{C}^T (\mathbf{C} \mathbf{P}_i \mathbf{C}^T + \mathbf{R})^{-1} \cdot \mathbf{y}_i \quad (\text{C. 3})$$

Using $\mathbf{I} = (\mathbf{C} \mathbf{P}_i \mathbf{C}^T + \mathbf{R})(\mathbf{C} \mathbf{P}_i \mathbf{C}^T + \mathbf{R})^{-1}$ and factoring the result it follows:

$$\mathbf{C} \mathbf{x}_i^+ = \mathbf{R}(\mathbf{C} \mathbf{P}_i \mathbf{C}^T + \mathbf{R})^{-1} \cdot \mathbf{C} \mathbf{x}_i^- + \mathbf{C} \mathbf{P}_i \mathbf{C}^T (\mathbf{C} \mathbf{P}_i \mathbf{C}^T + \mathbf{R})^{-1} \cdot \mathbf{y}_i \quad (\text{C. 4})$$

Finally, when the measures are equal to the state variables $(\mathbf{C} = \mathbf{I})$, the state vector corrected by the measures can be expressed as:

$$\mathbf{x}_i^+ = \underbrace{\mathbf{R} \cdot (\mathbf{P}_i^- + \mathbf{R})^{-1}}_{\frac{m_1}{m_1+m_2}} \cdot \mathbf{x}_i^- + \underbrace{\mathbf{P}_i^- \cdot (\mathbf{P}_i^- + \mathbf{R})^{-1}}_{\frac{m_2}{m_1+m_2}} \cdot \mathbf{y}_i \quad (\text{C.5})$$

In which: m_1 and m_2 are the weights of an average.

The state estimation in the Kalman Filter is obtained by a weighted average of the predicted state ($\mathbf{x}_i^-$) and the measurements vector ($\mathbf{y}_i$). The weights of this average (m_1 and m_2) are the prediction covariance matrix ($\mathbf{P}_i^-$) and the measurement noise covariance matrix ($\mathbf{R}$). Therefore, some interpretation can be produced in the expression (C.5):

- When the $\mathbf{R}$ matrix have values higher than the $\mathbf{P}_i^-$ matrix, the KF assigns a greater weight to the predicted state ($\mathbf{x}_i^-$) and when the $\mathbf{P}_i^-$ matrix has higher values than $\mathbf{R}$ matrix; then the KF assigns greater weight to the measurements ($\mathbf{y}_i$).
- As seen in the Kalman Filter algorithm, the prediction covariance matrix ($\mathbf{P}_i^-$) depends on how much it is amplified or attenuated from the previously correction covariance matrix $\mathbf{P}_i^+$ and of the process noise covariance matrix ($\mathbf{Q}$).
- Since the EKF and UKF algorithms are applications of the KF to nonlinear systems, they could also be tuned by the manipulation of the $\mathbf{Q}$ and $\mathbf{P}_0$ covariance matrices. The $\mathbf{P}_0$ matrix affect the estimation process at the beginning of the ride. If this matrix includes high values, then the EKF estimator assigns a greater weight to measurements at the beginning of the ride and decrease the weight in the maneuver's course as well as the $\mathbf{P}_i^-$ matrix values exponentially decrease to stabilize in a stationary value. This stationary value is an average value as a function of the $\mathbf{Q}$ and $\mathbf{R}$ values.